

EDWARD J. COGAN and
ROBERT Z. NORMAN

**Handbook of
CALCULUS, DIFFERENCE and
DIFFERENTIAL EQUATIONS**

GEOMETRIC FORMULAS

Triangle: altitude h , base b :

$$\text{Area} = hb/2.$$

Rectangle: length a , width b :

$$\text{Area} = ab.$$

Parallelogram: altitude h , base b :

$$\text{Area} = hb.$$

Trapezoid: altitude h , two parallel sides a and b :

$$\text{Area} = (a + b)h/2.$$

Circle: radius r :

$$\text{Area} = \pi r^2,$$

$$\text{Circumference} = 2\pi r.$$

Arc of a circle: radius r , central angle θ (in radians):

$$\text{Length} = r\theta.$$

Sector of a circle: radius r , central angle θ (in radians):

$$\text{Area} = r^2\theta/2.$$

Ellipse: semiaxes a and b :

$$\text{Area} = \pi ab,$$

$$\text{Circumference} = 2\pi\sqrt{(a^2 + b^2)/2}.$$

Rectangular parallelepiped: length a , width b , height c :

$$\text{Volume} = abc.$$

Pyramid: altitude h , area A :

$$\text{Volume} = \frac{1}{3} hA.$$

Sphere: radius r :

$$\text{Area of surface} = 4\pi r^2,$$

$$\text{Volume} = 4\pi r^3/3.$$

Right circular cylinder: height h , base radius r :

$$\text{Area of lateral surface} = 2\pi rh,$$

$$\text{Volume} = \pi r^2 h.$$

Right circular cone: height h , base radius r :

$$\text{Area of lateral surface} = \pi r\sqrt{r^2 + h^2},$$

$$\text{Volume} = \pi r^2 h/3.$$

IMPORTANT CONSTANTS

$\pi = 3.14159\ 26536$	$e = 2.71828\ 18284$
$\pi^2 = 9.86960$	$e^2 = 7.38906$
$\sqrt{\pi} = 1.77245$	$1/e = 0.36788$
$\sqrt{2\pi} = 2.50663$	$\ln 10 = 2.30258$
$\sqrt[3]{\pi} = 1.46459$	$\log_{10} e = 0.43429$
$1/\pi = 0.31831$	$\ln 2 = 0.69315$
$1/\sqrt{\pi} = 0.56419$	$\log_2 e = 1.44269$
$1/\sqrt{2\pi} = 0.39896$	$\gamma = 0.57722$



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Handbook of
CALCULUS, DIFFERENCE
and
DIFFERENTIAL EQUATIONS

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Handbook of
CALCULUS, DIFFERENCE
and
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PREFACE

THIS HANDBOOK offers to the student of calculus and differential equations a review of properties of often-used functions as well as tables of values of the more common transcendental functions and a table of integrals. Since the intention of the authors is that this book be a self-contained supplement to a textbook in such a course, the expository material is necessarily concise.

The key new feature of this Handbook is its tables for the solution of differential equations. To solve an elementary differential equation, the first step is to classify it so as to determine what technique to apply; the second step is to apply the indicated technique. The Handbook does just this. For each large class of differential equations, there is a classification table which refers to an associated table of techniques for solution. These tables are preceded by a general discussion of differential equations and their solutions. Thus these tables do for elementary differential equations what an integral table does for indefinite integrals. Because of the numerous analogies that exist between the difference and differential calculus, we have included tables for differences, antidifferences, difference equations, and sums.

There is an increasing awareness that the undergraduate mathematics curriculum should include a deeper study of basic concepts and more topics outside the calculus. We feel that the use of a handbook wherever possible to replace extensive memorization of techniques releases sufficient time for the introduction of these topics into the program of the first two years.

The original idea for this Handbook was conceived by the Committee on the Undergraduate Program of the Mathematical Association of America. It was intended as a companion to *Modern*

Mathematical Methods and Models, an experimental text for sophomore students of the behavioral sciences developed for the Committee by the Dartmouth College Writing Group. On writing the first version of the Handbook, it became clear that using the Handbook is a good way to teach all students of differential equations. The present Handbook is extensively rewritten from the earlier version and is now designed as well to supplement a basic calculus and differential equations course for engineering and science students.

We are most grateful to the Committee on the Undergraduate Program for their encouragement to prepare this Handbook, and for the release to us of the original version. We received a number of valuable suggestions from members of the Dartmouth College Writing Group. Special thanks go to our students Sheldon Lippe and Richard Schneider, who painstakingly helped check the Handbook's entries and who thrust upon us the viewpoint of the student, notably indispensable in a work such as this.

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Handbook of
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1

FUNCTIONS

A **FUNDAMENTAL** notion in mathematics is that of a **function**. In order to specify a function, two sets called the **domain** and the **range** of the function must be given, as well as a **law of correspondence**, or **mapping**, which assigns to each member of its domain exactly one element in its range.

1.1 DEFINITION

A **function** is a law of correspondence which assigns to each element of a set, called the **domain** of the function, exactly one element of another set, called the **range** of the function. The range is restricted by the property that at least one domain element is mapped into each range element.

The domain and range of functions treated in the calculus are generally sets of real numbers. In finite differences, the domain is usually a set of nonnegative integers.

1.2 DEFINITION

Any element of the domain of a function is called an **argument** of the function; any element of its range is called a **value** of the function.

1.3 SYMBOLISM

In this handbook, the letters **f**, **g**, and **h**, with or without subscripts, are used to stand for unspecified functions. An unspecified value of **f** is denoted by y . The value of the function **f** which corresponds to an unspecified argument x is denoted by $\mathbf{f}(x)$. Similarly, the value of a function **f** for a specified argument, such as the number 2, is denoted by $\mathbf{f}(2)$.

1.4 DEFINING FUNCTIONS EXPLICITLY

A function may be defined explicitly by specifying its domain, its range, and a rule for computing its value for any given argument. However, when only the rule is given, the domain and range are taken to be the greatest sets of real numbers for which the rule applies. Thus the formula

$$y = x^2 - x$$

defines a function whose value for each argument is found by subtracting the argument from its square. The domain of this function is the set of all real numbers; the range can be shown to be the set of all real numbers greater than or equal to $-\frac{1}{4}$.

When a function is defined by an equation in which y occurs alone in the left-hand member, we say that the function is **defined explicitly**, and we use the right-hand member of the equation as a name for the function. Thus we may call the function defined by the above equation $x^2 - x$, even though this combination of symbols actually stands for the value of the function for an unspecified argument.

1.5 DEFINITION

Given a real number a , the function **f** for which $\mathbf{f}(x) = a$ for every x is called a **constant function** and is denoted by **a**.

1.6 DEFINITION

The function **f** for which $\mathbf{f}(x) = x$ for every x is called the **identity function** and is denoted by **x**.

1.7 COMPOSITION OF FUNCTIONS

Given two functions \mathbf{f} and \mathbf{g} , we call a function \mathbf{h} the **composition of \mathbf{f} on \mathbf{g}** if, for an unspecified argument x , the value $\mathbf{h}(x)$ is found by first evaluating $\mathbf{g}(x)$ and then finding the value of \mathbf{f} for the argument $\mathbf{g}(x)$. Thus, $\mathbf{h}(x) = \mathbf{f}(\mathbf{g}(x))$ for every argument x . The composition of \mathbf{f} on \mathbf{g} will be denoted by $\mathbf{f}(\mathbf{g})$. The domain of $\mathbf{f}(\mathbf{g})$ is the set of all real numbers in the domain of \mathbf{g} for which $\mathbf{g}(x)$ is in the domain of \mathbf{f} ; the range of $\mathbf{f}(\mathbf{g})$ is the corresponding part of the range of \mathbf{f} .

1.8 EXAMPLE

Let $\mathbf{f}(x) = x^2 + x$ and $\mathbf{g}(x) = x - 1$. Then

$$\mathbf{f}(\mathbf{g}(x)) = (x - 1)^2 + (x - 1) = x^2 - x$$

and

$$\mathbf{g}(\mathbf{f}(x)) = (x^2 + x) - 1.$$

1.9 EXAMPLE

Let $\mathbf{h}(x) = (x + 2)^3 - (x + 2) + 1/(x + 2)$. We can write $\mathbf{h}(x)$ as a composite of two functions \mathbf{f} and \mathbf{g} . By choosing $\mathbf{f}(x) = x^3 - x + (1/x)$ and $\mathbf{g}(x) = x + 2$, then $\mathbf{h} = \mathbf{f}(\mathbf{g})$.

2

ALGEBRAIC FUNCTIONS

2.1 DEFINITION

If n is a nonnegative integer and a_0, a_1, \dots, a_n are real numbers, then the function f for which $f(x) = a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n$ is called a **polynomial**. If $a_0 \neq 0$, then n is called the **degree** of the polynomial. Each number a_i (where $0 \leq i \leq n$) is called the **coefficient** of the corresponding x^{n-i} .

2.2 DEFINITION

- (1) A polynomial of degree 1 is called a **linear function**.
- (2) A polynomial of degree 2 is called a **quadratic function**.
- (3) A polynomial of degree 3 is called a **cubic function**.

2.3 PRIME POLYNOMIALS

A polynomial $p(x)$ of degree n , with real coefficients which cannot be written as the product of two polynomials of degree less than n with real coefficients, is called a **prime polynomial**.

2.4 PROPERTIES OF POLYNOMIALS

- (1) The sum, difference, and product of two polynomials are polynomials.

(2) Every polynomial can be expressed as a product of prime factors which are polynomials of degree at most 2. Thus if $p(x)$ is a polynomial,

$$p(x) = a(x - b_1)(x - b_2) \cdots (x - b_k)(x^2 + c_1x + d_1) \cdots (x^2 + c_lx + d_l),$$

where the numbers a , b_i , c_i , and d_i are all real.

2.5 DEFINITION

The number of times that a given polynomial occurs as a factor in the product form of (2) of Sec. 2.4 is called the **multiplicity** of the factor.

2.6 DEFINITION

If f and g are polynomials, then the function h for which $h(x) = f(x)/g(x)$ is called a **rational function**.

2.7 PARTIAL FRACTIONS DECOMPOSITION

Let $p(x)/q(x)$ be a rational function in which the degree of the polynomial $p(x)$ is m and that of $q(x)$ is n . If $m \geq n$, then $p(x)/q(x)$ can be written in the form $s(x) + t(x)/q(x)$, where $s(x)$ is a polynomial of degree $m - n$ and $t(x)$ is a polynomial of degree less than n .

Assume that $p(x)/q(x)$ is a rational function for which the degree of $p(x)$ is less than that of $q(x)$. The denominator can be written as a product

$$[q_1(x)]^{r_1} \cdot [q_2(x)]^{r_2} \cdots [q_k(x)]^{r_k},$$

where the functions $q_i(x)$ are prime polynomials and the numbers r_i are their respective multiplicities. In this case, $p(x)/q(x)$ is expressible as a sum of fractions of the form $P(x)/Q(x)$, where $Q(x)$ is a power less than or equal to r_i of the factor $q_i(x)$ and $P(x)$ is a polynomial of degree less than that of $q_i(x)$.

2.8 EXAMPLE

We shall obtain the partial fractions decomposition of the rational function

$$\frac{x^4 - 2x^3 + 3x^2 - x + 1}{x^3 + 2x^2 + x}.$$

On dividing the numerator by the denominator, a quotient of $x - 4$ is obtained, with a remainder of $10x^2 + 3x + 1$. Hence, the fraction may be written in the form

$$x - 4 + \frac{10x^2 + 3x + 1}{x^3 + 2x^2 + x}.$$

Consider the fractional part of this expression. Its numerator has lower degree than its denominator, and its denominator may be factored as the product $x(x + 1)^2$. According to the procedure in Sec. 2.7, there are numbers A , B , and C such that

$$\frac{10x^2 + 3x + 1}{x^3 + 2x^2 + x} = \frac{A}{x} + \frac{B}{x + 1} + \frac{C}{(x + 1)^2}.$$

By clearing fractions, we find that

$$\begin{aligned} 10x^2 + 3x + 1 &= A(x + 1)^2 + Bx(x + 1) + Cx \\ &= (A + B)x^2 + (2A + B + C)x + A. \end{aligned}$$

Setting coefficients of like powers of x equal to each other, we obtain the system

$$\begin{aligned} A + B &= 10, \\ 2A + B + C &= 3, \\ A &= 1; \end{aligned}$$

thus $A = 1$, $B = 9$, and $C = -8$. Combining all our results, we have

$$\frac{x^4 - 2x^3 + 3x^2 - x + 1}{x^3 + 2x^2 + x} = x - 4 + \frac{1}{x} + \frac{9}{x + 1} - \frac{8}{(x + 1)^2}.$$

2.9 DEFINITION

The operations of addition, subtraction, multiplication, division, raising to an integral power, and extraction of a root are called **algebraic operations**.

2.10 DEFINITION

If x and y designate unspecified numbers, then an expression which results from the combination of x , y , and a finite number of real numbers by means of a finite number of algebraic operations is called an **algebraic expression** in x and y . $\mathbf{A}(x, y)$ designates an unspecified algebraic expression in x and y .

2.11 DEFINITION

If $\mathbf{A}(x, y)$ is an algebraic expression in x and y , then the equation $\mathbf{A}(x, y) = 0$ may define implicitly one or more functions \mathbf{f} such that $y = \mathbf{f}(x)$. A function \mathbf{f} defined by such an equation is called an **algebraic function**.

2.12 EXAMPLES

The expressions $x^2 + y^2 - 1$, $\frac{y\sqrt{x} - x^2\sqrt[3]{y-2}}{\sqrt{x^2 + y^2}}$, and $xy - 1$ are examples of algebraic expressions. The equations

$$\begin{aligned}x^2 + y^2 - 1 &= 0, \\ \frac{y\sqrt{x} - x^2\sqrt[3]{y-2}}{\sqrt{x^2 + y^2}} &= 0, \\ xy - 1 &= 0\end{aligned}$$

each define implicitly at least one function \mathbf{f} for which $y = \mathbf{f}(x)$. The function defined by the third equation may also be defined explicitly by the equation

$$y = \frac{1}{x}.$$

The equation $x^2 + y^2 - 1 = 0$ defines two functions implicitly, namely, those which are defined explicitly by the equations

$$y = \sqrt{1 - x^2}$$

and

$$y = -\sqrt{1 - x^2}.$$

It is not practical to express explicitly the functions determined by the second equation.

2.13 DEFINITION

Let \mathbf{f} be a function, and let y be a number in the range of \mathbf{f} . Then the rule which assigns to each y those numbers x in the domain of \mathbf{f} for which $\mathbf{f}(x) = y$ is called the **inverse** of \mathbf{f} . If for every y in the range of \mathbf{f} there is exactly one number x in the domain of \mathbf{f} such that $\mathbf{f}(x) = y$, then the inverse of \mathbf{f} is a function called the **inverse function** of \mathbf{f} and denoted \mathbf{f}^{-1} . Thus the composition of \mathbf{f} and \mathbf{f}^{-1} is always the identity function: $\mathbf{f}^{-1}(\mathbf{f}(x)) = \mathbf{f}(\mathbf{f}^{-1}(x)) = x$ for every x common to the domains of both \mathbf{f} and \mathbf{f}^{-1} .

2.14 EXAMPLE

If $\mathbf{f}(x) = x + 1$ and $\mathbf{g}(x) = x - 1$, then

$$\mathbf{f}(\mathbf{g}(x)) = (x - 1) + 1 = x$$

and

$$\mathbf{g}(\mathbf{f}(x)) = (x + 1) - 1 = x,$$

so that \mathbf{f} and \mathbf{g} are inverses of each other.

3

FUNCTION-FUNCTION OPERATIONS

A NUMBER OF LAWS of correspondence, or mappings, whose domains and ranges are sets of functions rather than sets of numbers arise in the calculus. Those used in this handbook are defined below.

3.1 DEFINITION

Given a function f , if there is a function g such that

$$g(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h},$$

then g is called the **first derivative** of f and is denoted by Df . The domain of Df is that part of the domain of f for which values can be assigned to Df .

The **second derivative** of f , denoted by D^2f , is the first derivative of Df ; thus $D^2f = D(Df)$.

The **third derivative** of f , denoted by D^3f , is the first derivative of D^2f ; thus $D^3f = D(D^2f)$.

In general, for any positive integer n , the **n -th derivative** of f , denoted by $D^n f$, is the first derivative of $D^{n-1}f$; thus $D^n f = D(D^{n-1}f)$.

The inverse of the derivative operator D , called the **indefinite integral**, will be considered in Sec. 4.7.

3.2 DEFINITION

Given a function f , the function g for which $g(x) = f(x+1)$ is called the **translate** (or **first translate**) of f and is denoted Ef .

For every x for which $x + 1$ is in the domain of \mathbf{f} , we find that $\mathbf{E}\mathbf{f}(x) = \mathbf{f}(x + 1)$.

The **second translate** of \mathbf{f} , denoted by $\mathbf{E}^2\mathbf{f}$, is the translate of $\mathbf{E}\mathbf{f}$; thus, $\mathbf{E}^2\mathbf{f} = \mathbf{E}(\mathbf{E}\mathbf{f})$. Note that $\mathbf{E}^2\mathbf{f}(x) = \mathbf{f}(x + 2)$.

The **third translate** of \mathbf{f} , denoted by $\mathbf{E}^3\mathbf{f}$, is the translate of $\mathbf{E}^2\mathbf{f}$; thus, $\mathbf{E}^3\mathbf{f} = \mathbf{E}(\mathbf{E}^2\mathbf{f})$. Note that $\mathbf{E}^3\mathbf{f}(x) = \mathbf{f}(x + 3)$.

In general, for any positive integer n , the **n -th translate** of \mathbf{f} , denoted by $\mathbf{E}^n\mathbf{f}$, is the translate of $\mathbf{E}^{n-1}\mathbf{f}$; thus, $\mathbf{E}^n\mathbf{f} = \mathbf{E}(\mathbf{E}^{n-1}\mathbf{f})$. Note that $\mathbf{E}^n\mathbf{f}(x) = \mathbf{f}(x + n)$.

3.3 DEFINITION

Given a function \mathbf{f} , the function \mathbf{g} for which $\mathbf{g}(x) = \mathbf{f}(x + 1) - \mathbf{f}(x)$ is called the **difference** (or **first difference**) of \mathbf{f} and is denoted by $\Delta\mathbf{f}$. For every x for which $x + 1$ is in the domain of $\Delta\mathbf{f}$, we have $\Delta\mathbf{f}(x) = \mathbf{f}(x + 1) - \mathbf{f}(x)$.

The **second difference** of \mathbf{f} , denoted by $\Delta^2\mathbf{f}$, is the difference of $\Delta\mathbf{f}$; thus, $\Delta^2\mathbf{f} = \Delta(\Delta\mathbf{f})$.

The **third difference** of \mathbf{f} , denoted by $\Delta^3\mathbf{f}$, is the difference of $\Delta^2\mathbf{f}$; thus, $\Delta^3\mathbf{f} = \Delta(\Delta^2\mathbf{f})$.

In general, for any positive integer n , the **n -th difference** of \mathbf{f} , denoted by $\Delta^n\mathbf{f}$, is the difference of $\Delta^{n-1}\mathbf{f}$; thus, $\Delta^n\mathbf{f} = \Delta(\Delta^{n-1}\mathbf{f})$.

The inverse of the difference operator Δ , called the **indefinite finite integral**, will be considered in Sec. 4.8.

3.4 OPERATOR POLYNOMIALS

In terms of the operators defined in Secs. 3.1 to 3.3, it is possible to define a large class of operators by using the operations of ordinary arithmetic. For any real number k , we define an operator \mathbf{k} such that the values of $\mathbf{k}\mathbf{f}$ are k times the corresponding values of \mathbf{f} . We can then form polynomials in \mathbf{D} , \mathbf{E} , or Δ . For example, $\mathbf{E}^2 + 2\mathbf{E} - \mathbf{1}$ may be considered as a single operator which for any function \mathbf{f} has the property that

$$(\mathbf{E}^2 + 2\mathbf{E} - \mathbf{1})\mathbf{f}(x) = \mathbf{f}(x + 2) + 2\mathbf{f}(x + 1) - \mathbf{f}(x).$$

If we assume the properties

$$(\mathbf{M} + \mathbf{N})\mathbf{f} = \mathbf{M}\mathbf{f} + \mathbf{N}\mathbf{f},$$

$$\mathbf{M}(\mathbf{k}\mathbf{f}) = \mathbf{k}(\mathbf{M}\mathbf{f}),$$

whenever \mathbf{M} and \mathbf{N} are operators, \mathbf{k} is a constant operator, and \mathbf{f} is a function, then the meaning of the operator polynomial is clear.

3.5 RELATIONS BETWEEN \mathbf{E} AND Δ

The definitions of \mathbf{E} and Δ establish the following relations between them:

$$(1) \quad \Delta = \mathbf{E} - \mathbf{1},$$

$$(2) \quad \mathbf{E} = \Delta + \mathbf{1},$$

$$(3) \quad \Delta^2 = (\mathbf{E} - \mathbf{1})^2 = \mathbf{E}^2 - 2\mathbf{E} + \mathbf{1},$$

$$(4) \quad \mathbf{E}^2 = (\Delta + \mathbf{1})^2 = \Delta^2 + 2\Delta + \mathbf{1}.$$

If n is a positive integer, then

$$(5) \quad \begin{aligned} \Delta^n &= (\mathbf{E} - \mathbf{1})^n \\ &= \mathbf{E}^n - \binom{n}{1} \mathbf{E}^{n-1} + \binom{n}{2} \mathbf{E}^{n-2} - \dots + (-1)^n, \end{aligned}$$

$$(6) \quad \begin{aligned} \mathbf{E}^n &= (\Delta + \mathbf{1})^n \\ &= \Delta^n + \binom{n}{1} \Delta^{n-1} + \binom{n}{2} \Delta^{n-2} + \dots + \mathbf{1}, \end{aligned}$$

where the numbers $\binom{n}{i}$, $1 \leq i \leq n$, are the binomial coefficients given in Table 5.4.

3.6 EXAMPLES

Let $\mathbf{f}(x) = x^2 - 2x + 3$; then

$$\mathbf{E}\mathbf{f}(x) = (x + 1)^2 - 2(x + 1) + 3 = x^2 + 2.$$

$$\mathbf{E}^2\mathbf{f}(x) = (x + 2)^2 - 2(x + 2) + 3 = x^2 + 2x + 3.$$

$$\begin{aligned} \Delta\mathbf{f}(x) &= (\mathbf{E} - \mathbf{1})\mathbf{f}(x) \\ &= [(x + 1)^2 - 2(x + 1) + 3] - [x^2 - 2x + 3] \\ &= 2x - 1. \end{aligned}$$

$$\begin{aligned} \Delta^2\mathbf{f}(x) &= \Delta(\Delta\mathbf{f})(x) = \Delta(2x - 1) \\ &= [2(x + 1) - 1] - [2x - 1] = 2. \end{aligned}$$

$$\Delta^3\mathbf{f}(x) = \Delta(\Delta^2\mathbf{f})(x) = \Delta(2) = 2 - 2 = 0.$$

4

FUNCTIONAL EQUATIONS

4.1 DEFINITION

An equation which sets forth a condition to be satisfied by a function is called a **functional equation**.

4.2 DEFINITION

Any function **f** which satisfies the conditions set forth in a functional equation is called a **particular solution** of the equation.

4.3 DEFINITION

The set of all functions which satisfy a given functional equation is called the **general solution** of the equation. A symbolic representation of the general solution of a functional equation is called a **general solution form** of the equation.

4.4 DEFINITION

A functional equation which involves a finite number of powers of the operator **D** is called a **differential equation**. The highest power of the operator is called the **order** of the differential equation.

4.5 DEFINITION

A functional equation which involves a finite number of powers of the operator \mathbf{E} or the operator Δ is called a **difference equation**. The highest power of the operator involved is called the **order** of the difference equation.

4.6 EXAMPLES

(1) The equation $\mathbf{f}(x_1 + x_2) = \mathbf{f}(x_1) + \mathbf{f}(x_2)$, where x_1 and x_2 are unspecified arguments of \mathbf{f} , is an example of a functional equation. The function \mathbf{f} for which $\mathbf{f}(x) = x$ is a particular solution of this equation, and so is the function \mathbf{f} for which $\mathbf{f}(x) = 3x$. In fact, if C is any constant, then Cx is a solution of the equation, and every continuous solution has this form. However, Cx is not the general solution form, since there are discontinuous solutions not of this form.

(2) The equation $\mathbf{D}\mathbf{f}(x) = 1$ is a simple example of a differential equation. A particular solution of this equation is $\mathbf{f}(x) = x$, and another solution is $\mathbf{f}(x) = x + 3$. In fact, if C is an unspecified constant, then $\mathbf{f}(x) = x + C$ is a general solution form for the equation, since every solution can be written in this form for some value of C .

(3) The equation $(\mathbf{E} - 2)\mathbf{f}(x) = 0$ is an example of a difference equation. A particular solution of this equation is $\mathbf{f}(x) = 2^x$, since $\mathbf{E}(2^x) = 2^{x+1} = 2 \cdot 2^x$, and $(\mathbf{E} - 2)2^x = 0$. Another particular solution is $\mathbf{f}(x) = 3 \cdot 2^x$. In fact, if C is an unspecified constant, then $C \cdot 2^x$ is a general solution form of the equation, since every particular solution can be written in this form for some value of C .

4.7 DEFINITION

Given a function \mathbf{g} , any particular solution of the equation $\mathbf{D}\mathbf{f} = \mathbf{g}$ is called an **antiderivative** of \mathbf{g} . The general solution of the equation $\mathbf{D}\mathbf{f} = \mathbf{g}$ is called the **indefinite integral** of \mathbf{g} and is denoted by $\int \mathbf{g}(x) dx$.

4.8 DEFINITION

Given a function \mathbf{g} , any particular solution of the equation $\Delta \mathbf{f} = \mathbf{g}$ is called an **antidifference** of \mathbf{g} . The general solution of the equation $\Delta \mathbf{f} = \mathbf{g}$ is called the **indefinite finite integral** of \mathbf{g} and is denoted by $\Delta^{-1}\mathbf{g}$.

4.9 ARBITRARY CONSTANTS

In most cases, the representation of a general solution form is made convenient by the fact that the particular solutions of the equation share common features. In particular, the general solution form of a difference or differential equation of order n can be written so that it involves n arbitrary (unspecified) constants, usually designated by $C_1, C_2, \dots; C_n$. A particular solution of such an equation is obtained by replacing each occurrence of C_1 by the same fixed number, each occurrence of C_2 by the same fixed number, and so on. The assignment of a number to any one of the arbitrary constants is independent of the assignments made to the others.

The general solution of a given equation may be written in many ways. Some ways are simpler than others; some may involve more arbitrary constants than necessary.

Example 1. The general solution of the equation of first order $\mathbf{Df}(x) = 1$ can obviously be written $\mathbf{f}(x) = x + C$. (We write C instead of C_1 when only one arbitrary constant appears.) However, by integrating both sides of this equation as follows,

$$\int \mathbf{Df}(x) \, dx = \int dx,$$

we may also obtain the less desirable representations

$$\mathbf{f}(x) + C_1 = x + C_2 \quad \text{or} \quad \mathbf{f}(x) = x + C_2 - C_1.$$

We say that one representation of the general solution of an equation is simpler than another under one or more of the following circumstances:

(1) Fewer different arbitrary constants occur in the first than in the second.

(2) The total number of occurrences of arbitrary constants in the first is smaller than in the second.

(3) Computations with the first representation can be made more conveniently than with the second.

The first step toward satisfying requirement (1) is to reduce the number of different arbitrary constants in a representation of the general solution of an equation to a minimum. This minimum is equal to the order of the equation.

Example 2. If one representation of the general solution of a given equation is $\mathbf{f} = 3C_1x^2 \ln(1 + C_2)$, then a simpler one is $\mathbf{f} = Cx^2$ (see Example 4).

Suppose that an algebraic combination of terms, or a factor of a term, consists entirely of constants at least one of which is arbitrary. The first two conditions for simplicity can be met by giving such a collection of terms or factor of a term a single new name, C_k , provided that C_k does not occur elsewhere in the representation. It will frequently be convenient to make use of some algebraic manipulation before applying this rule. In simplifying an expression which already contains the minimum number of constants, it is often more convenient to express these constants as factors where possible.

Example 3. $\log_a C_1x = \log_a x + \log_a C_1$ may be replaced in any representation by $\log_a x + C$.

Example 4. $3C_1 \ln(1 + C_2)$ may be replaced by C (see Example 2).

Example 5. $C_2 - C_1$ may be replaced by C (see Example 1).

Example 6. Since $3C_1e^{x^2+C_2} = 3C_1e^{C_2}e^{x^2}$, the expression $3C_1e^{x^2+C_2} + 6C_1e^{C_2}$ may be replaced by $Ce^{x^2} + 2C$.

In order to meet the third condition for simplicity, we make use of known identity relations between functions to introduce into a representation of the general solution functions which combine more readily with other parts of the representation than those which already occur. Thus, an expression which is part of a representation, and which involves a nonconstant function, may be replaced by a second expression identically equal to it. Such a replacement often achieves a different distribution of arbitrary constants since it may make possible some simplifications of the type discussed in the preceding paragraph. Of course, simplicity here will depend not so much on the

intrinsic form of the representation as on the particular way in which it is to be used.

Example 7. Since $\sin^2 x = 1 - \cos^2 x$, we have

$$\sin^2 x + C = -\cos^2 x + 1 + C.$$

We may replace $1 + C$ by C_1 , and since the letter C is no longer involved, we may use C instead of C_1 . Thus $\sin^2 x + C$ may be replaced by $-\cos^2 x + C$.

Example 8. $C_1 \sin x + C_2 \cos x$ may be replaced by $C_3 \sin (x + C_4)$, since

$$C_3 \sin (x + C_4) = C_3(\sin x \cos C_4 + \cos x \sin C_4)$$

and we may take

$$C_3 = \sqrt{C_1^2 + C_2^2} \quad \text{and} \quad C_4 = \arctan \frac{C_2}{C_1}.$$

Again, after replacement, C_1 and C_2 are no longer involved, so that we may relabel the new expression to arrive at $C_1 \sin (x + C_2)$.

Example 9. Solve for y in $C_1 x = C_2 e^{2y+C_3}$.

$$C_1 x = C_2 e^{C_3} e^{2y} = C_2 C_4 e^{2y},$$

$$e^{2y} = C_5 x,$$

$$2y = \ln C_5 x = \ln x + C_6,$$

$$y = \frac{1}{2} \ln x + C.$$

4.10 METHODS OF DEFINING FUNCTIONS

Functions can be defined in essentially two different ways: **explicitly** or **implicitly**.

An explicit definition of a function furnishes a procedure by which a value of the function for a given argument may be obtained. Some algebraic functions may be defined explicitly, namely, those which may be represented by an equation of the form $y = f(x)$ which indicates the sequences of algebraic operations that must be applied to an argument of the function to yield a value. The method of defining the trigonometric functions is an explicit one, since it furnishes a direct procedure by which a value of such a function is obtained for a given argument.

An implicit definition of a function furnishes no such procedure.

Instead it sets forth conditions on the function. One way of specifying such conditions is by a functional equation. In general, a differential or a difference equation is satisfied by several functions. In order to specify a single function by means of such an equation, additional conditions must be furnished in order that the particular function to be defined may be distinguished from the other functions in the general solution. These additional conditions are called **boundary conditions**.

4.11 EXAMPLES

(1) The general solution of the equation $\mathbf{D}f(x) = 1/x$ is a class of functions, one of which is the function $\ln x$ (the natural logarithm; see Sec. 6.9). This function is completely defined by the given equation and the additional condition that $f(1) = 0$.

(2) The equation $(\mathbf{E} - \mathbf{1})f(x) = 0$ has $f(x) = C2^x$ as its general solution. This equation along with the condition that $f(0) = 1$ suffices to define the function 2^x for integral values of x .

5

SPECIAL FUNCTIONS DEFINED FOR INTEGERS

THE DOMAINS of the functions defined in this section are sets of integers. They occur frequently in situations from which difference equations arise.

5.1 DEFINITION

$$\begin{aligned}x! &= x(x-1)(x-2) \cdots 2 \cdot 1, \quad \text{if } x > 0, \\ 0! &= 1.\end{aligned}$$

Note that $x!$ is defined only when $x \geq 0$.

Table 5.2 gives values of $x!$ for $0 \leq x \leq 35$. For larger values of x , $x!$ can be approximated by Stirling's formula, which states that

$$x! \approx \sqrt{2x\pi}(x/e)^x$$

More specifically,

$$\begin{aligned}\sqrt{2x\pi}(x/e)^x &< x! \\ &< \sqrt{2x\pi}(x/e)^x [1 + (1/12x - 1)].\end{aligned}$$

The factorial function $x!$ is defined by

5.2 TABLE

VALUES OF $x!$, $0 \leq x \leq 35$

x	$x!$												
0													
1													
2													
3													
4													
5													
6													
7													
8													
9													
10													
11													
12													
13													
14													
15													
16													
17													
18													
19													
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21													
22													
23													
24													
25													
26													
27													
28													
29													
30													
31													
32													
33													
34													
35	10	333	147	966	389	178	681	108	831	996	723	200	000

The following formula makes possible the continuation of Table 5.2:

$$(x + 1)! = (x + 1)x!.$$

5.3 DEFINITION

The **binomial coefficients** $\binom{x}{n}$ are defined for $0 \leq n \leq x$ by the equation

$$\binom{x}{n} = \frac{x!}{n!(x - n)!}.$$

5.4 TABLE

VALUES OF THE BINOMIAL COEFFICIENTS, $0 \leq x \leq 35$

$\begin{matrix} n \\ x \end{matrix}$	0	1	2	3	4	5	6	7	8	9	10
0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66
13	1	13	78	286	715	1287	1716	1716	1287	715	286
14	1	14	91	364	1001	2002	3003	3432	3003	2002	1001
15	1	15	105	455	1365	3003	5005	6435	6435	5005	3003
16	1	16	120	560	1820	4368	8008	11440	12870	11440	8008
17	1	17	136	680	2380	6188	12376	19448	24310	24310	19448
18	1	18	153	816	3060	8568	18564	31824	43758	48620	43758
19	1	19	171	969	3876	11628	27132	50388	75582	92378	92378
20	1	20	190	1140	4845	15504	38760	77520	125970	167960	184756
21	1	21	210	1330	5985	20349	54264	116280	203490	293930	352716
22	1	22	231	1540	7315	26334	74613	170544	319770	497420	646646
23	1	23	253	1771	8855	33649	100947	245157	490314	817190	1144066
24	1	24	276	2024	10626	42504	134596	346104	735471	1307504	1961256
25	1	25	300	2300	12650	53130	177100	480700	1081575	2042975	3268760
26	1	26	325	2600	14950	65780	230230	657800	1562275	3124550	5311735
27	1	27	351	2925	17550	80730	296010	888030	2220075	4686825	8436285
28	1	28	378	3276	20475	98280	376740	1184040	3108105	6906900	13123110
29	1	29	406	3654	23751	118755	475020	1560780	4292145	10015005	20030010
30	1	30	435	4060	27405	142506	593775	2035800	5852925	14307150	30045015
31	1	31	465	4495	31465	169911	736281	2629575	7888725	20160075	44352165
32	1	32	496	4960	35960	201376	906192	3365856	10518300	28048800	64512240
33	1	33	528	5456	40920	237336	1107568	4272048	13884156	38567100	92561040
34	1	34	561	5984	46376	278256	1344904	5379616	18156204	32451256	131128140
35	1	35	595	6545	52360	324632	1623160	6724520	23535820	70607460	183579396

The following formulas make possible the continuation of Table 5.4:

$$\binom{x}{n} = \binom{x}{x-n},$$

$$\binom{x+1}{n} = \binom{x}{n} + \binom{x}{n-1}.$$

5.5 THE FACTORIAL POWERS. DEFINITION

The n -th factorial power of x is given by the formulas

$$x^{(n)} = x(x-1)(x-2) \cdots (x-n+1), \quad n > 0;$$

$$x^{(0)} = 1.$$

(2) For every nonnegative integer n the function $x^{(n)}$ is a polynomial of degree n in x ; that is, there are numbers $s_{n1}, s_{n2}, \dots, s_{nn}$ such that

$$x^{(n)} = s_{n1}x^1 + s_{n2}x^2 + \dots + s_{nn}x^n.$$

The numbers $s_{n1}, s_{n2}, \dots, s_{nn}$ are called **Stirling numbers of the first kind** and are listed in Table 5.7.

(3) For every nonnegative integer n the function x^n is a linear combination of factorial powers of x not higher than the n -th; that is, there are numbers $t_{n1}, t_{n2}, \dots, t_{nn}$ such that

$$x^n = t_{n1}x^{(1)} + t_{n2}x^{(2)} + \dots + t_{nn}x^{(n)}.$$

The numbers $t_{n1}, t_{n2}, \dots, t_{nn}$ are called **Stirling numbers of the second kind** and are listed in Table 5.8.

These three considerations enable us to find differences of polynomials in the following way: First, apply (3) to express each power of x in terms of factorial powers; second, find the required differences of these powers by means of (1); third, by means of (2), write each resulting factorial power as a polynomial.

5.7 TABLE

STIRLING NUMBERS OF THE FIRST KIND

n	s_{n1}	s_{n2}	s_{n3}	s_{n4}	s_{n5}	s_{n6}	s_{n7}	s_{n8}
1	1	0	0	0	0	0	0	0
2	-1	1	0	0	0	0	0	0
3	2	-3	1	0	0	0	0	0
4	-6	11	-6	1	0	0	0	0
5	24	-50	35	-10	1	0	0	0
6	-120	274	-225	85	-15	1	0	0
7	720	-1764	1624	-735	175	-21	1	0
8	-5040	13068	-13132	6769	-1960	322	-28	1

To continue this table to find s_{ni} for $n > 8$, define $s_{n0} = 0$ for all n , and use the formula

$$s_{ni} = s_{n-1,i-1} - (n-1)s_{n-1,i}, \quad i = 1, 2, \dots, n.$$

5.8 TABLE

STIRLING NUMBERS OF THE SECOND KIND

n	t_{n1}	t_{n2}	t_{n3}	t_{n4}	t_{n5}	t_{n6}	t_{n7}	t_{n8}
1	1	0	0	0	0	0	0	0
2	1	1	0	0	0	0	0	0
3	1	3	1	0	0	0	0	0
4	1	7	6	1	0	0	0	0
5	1	15	25	10	1	0	0	0
6	1	31	90	65	15	1	0	0
7	1	63	301	350	140	21	1	0
8	1	127	966	1 701	1 050	266	28	1

To continue this table to find t_{ni} for $n > 8$, define $t_{n0} = 0$ for all n , and use the formula

$$t_{ni} = it_{n-1,i} + t_{n-1,i-1}, \quad i = 1, 2, \dots, n.$$

5.9 EXAMPLE

Express $x^3 + 2x - 1$, using factorial powers. We find from Table 5.8 that

$$\begin{aligned} x^3 &= x^{(1)} + 3x^{(2)} + x^{(3)}, \\ 2x &= 2x^{(1)}, \\ -1 &= -1. \end{aligned}$$

Then, $x^3 + 2x - 1 = x^{(3)} + 3x^{(2)} + 3x^{(1)} - 1$.

5.10 EXAMPLE

Express $x^{(3)} - 2x^{(2)} + 4x^{(1)}$ as a polynomial.
We find from Table 5.7 that

$$\begin{aligned} x^{(3)} &= 2x - 3x^2 + x^3, \\ -2x^{(2)} &= 2x - 2x^2, \\ 4x^{(1)} &= 4x. \end{aligned}$$

Thus, $x^{(3)} - 2x^{(2)} + 4x^{(1)} = x^3 - 5x^2 + 8x$.

6

TRANSCENDENTAL FUNCTIONS

6.1 DEFINITION

Any function which is not an algebraic function is called a **transcendental** function.

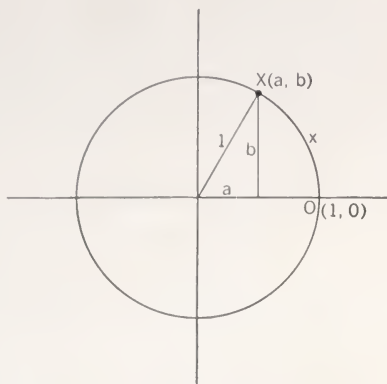
6.2 TRIGONOMETRIC FUNCTIONS

Consider a circle whose center is at the origin of a cartesian coordinate system and whose radius is one unit. Label with the letter O the point $(1, 0)$ at which the circle cuts the positive side of the horizontal axis. Let x be an unspecified real number. We assign to the number x a point X on the circle as follows:

- (1) If $x > 0$, the point X is x units from O , the distance being measured along the circumference of the circle in a counterclockwise sense.
- (2) If $x < 0$, the point X is x units from O , the distance being measured along the circumference of the circle in a clockwise sense.
- (3) If $x = 0$, then X is O .

This assigns a point on the circle to each real number x , but several real numbers are assigned to the same point X on the circle.

For each number x , let the coordinates of the point X be (a, b) . Since the numbers a and b are determined as soon as x is specified, the above procedure determines functions \mathbf{f} and \mathbf{g} such that $\mathbf{f}(x) = a$ and $\mathbf{g}(x) = b$.



6.3 DEFINITIONS

(See the last paragraph of Sec. 6.2.)

- (1) $\sin x = b.$
- (2) $\cos x = a.$
- (3) $\tan x = \frac{\sin x}{\cos x}.$
- (4) $\cot x = \frac{1}{\tan x}.$
- (5) $\sec x = \frac{1}{\cos x}.$
- (6) $\csc x = \frac{1}{\sin x}.$

6.4 PROPERTIES OF TRIGONOMETRIC FUNCTIONS

Pythagorean Identities

- (1) $\sin^2 x + \cos^2 x = 1.$
- (2) $\tan^2 x + 1 = \sec^2 x.$
- (3) $\cot^2 x + 1 = \csc^2 x.$

Negative Argument Identities

$$(4) \quad \sin(-x) = -\sin x.$$

$$(5) \quad \cos(-x) = \cos x.$$

$$(6) \quad \tan(-x) = -\tan x.$$

Reduction Identities

The identities below show that values of these functions for any real number can always be expressed in terms of their values for some arc length between 0 and $\pi/2$.

Let $0 \leq x \leq \pi/2$ and k be any integer. Then

$$(7) \quad \sin\left(x + \frac{n\pi}{2}\right) = \begin{cases} \sin x & \text{if } n = 4k, \\ \cos x & \text{if } n = 4k + 1, \\ -\sin x & \text{if } n = 4k + 2, \\ -\cos x & \text{if } n = 4k + 3. \end{cases}$$

$$(8) \quad \cos\left(x + \frac{n\pi}{2}\right) = \begin{cases} \cos x & \text{if } n = 4k, \\ -\sin x & \text{if } n = 4k + 1, \\ -\cos x & \text{if } n = 4k + 2, \\ \sin x & \text{if } n = 4k + 3. \end{cases}$$

$$(9) \quad \tan\left(x + \frac{n\pi}{2}\right) = \begin{cases} \tan x & \text{if } n = 4k, \\ -\cot x & \text{if } n = 4k + 1, \\ \tan x & \text{if } n = 4k + 2, \\ -\cot x & \text{if } n = 4k + 3. \end{cases}$$

Let x_1 and x_2 be any real numbers.

Argument Sum Identities

$$(10) \quad \sin(x_1 + x_2) = \sin x_1 \cos x_2 + \cos x_1 \sin x_2.$$

$$(11) \quad \cos(x_1 + x_2) = \cos x_1 \cos x_2 - \sin x_1 \sin x_2.$$

$$(12) \quad \tan(x_1 + x_2) = \frac{\tan x_1 + \tan x_2}{1 - \tan x_1 \tan x_2}.$$

Argument Difference Identities

$$(13) \quad \sin(x_1 - x_2) = \sin x_1 \cos x_2 - \cos x_1 \sin x_2.$$

$$(14) \quad \cos(x_1 - x_2) = \cos x_1 \cos x_2 + \sin x_1 \sin x_2.$$

$$(15) \quad \tan(x_1 - x_2) = \frac{\tan x_1 - \tan x_2}{1 + \tan x_1 \tan x_2}.$$

Product to Sum Identities

$$(16) \quad \sin x_1 \cos x_2 = \frac{1}{2} [\sin (x_1 + x_2) + \sin (x_1 - x_2)].$$

$$(17) \quad \cos x_1 \cos x_2 = \frac{1}{2} [\cos (x_1 + x_2) + \cos (x_1 - x_2)].$$

$$(18) \quad \sin x_1 \sin x_2 = \frac{1}{2} [\cos (x_1 - x_2) - \cos (x_1 + x_2)].$$

Trigonometric Functions Involving $2x$

$$(19) \quad \sin 2x = 2 \sin x \cos x.$$

$$(20) \quad \cos 2x = \cos^2 x - \sin^2 x,$$

$$(20a) \quad \cos 2x = 2 \cos^2 x - 1,$$

$$(20b) \quad \cos 2x = 1 - 2 \sin^2 x.$$

$$(21) \quad \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}.$$

Trigonometric Functions Involving $\frac{x}{2}$

$$(22) \quad \sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}.$$

$$(23) \quad \cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}.$$

6.5 TABLES OF VALUES OF THE TRIGONOMETRIC FUNCTIONS

Table 6.5(a)

NATURAL TRIGONOMETRIC FUNCTIONS

x	Sin	Tan	Ctn	Cos	x	Sin	Tan	Ctn	Cos
.00	.0000	.0000	∞	1.0000	.50	.4794	.5463	1.830	.8776
.01	.0100	.0100	99.997	1.0000	.51	.4882	.5594	1.788	.8727
.02	.0200	.0200	49.993	.9998	.52	.4969	.5726	1.747	.8678
.03	.0300	.0300	33.323	.9996	.53	.5055	.5859	1.707	.8628
.04	.0400	.0400	24.987	.9992	.54	.5141	.5994	1.668	.8577
.05	.0500	.0500	19.983	.9988	.55	.5227	.6131	1.631	.8525
.06	.0600	.0601	16.647	.9982	.56	.5312	.6269	1.595	.8473
.07	.0699	.0701	14.262	.9976	.57	.5396	.6410	1.560	.8419
.08	.0799	.0802	12.473	.9968	.58	.5480	.6552	1.526	.8365
.09	.0899	.0902	11.081	.9960	.59	.5564	.6696	1.494	.8309
.10	.0998	.1003	9.967	.9950	.60	.5646	.6841	1.462	.8253
.11	.1098	.1104	9.054	.9940	.61	.5729	.6989	1.431	.8196
.12	.1197	.1206	8.293	.9928	.62	.5810	.7139	1.401	.8139
.13	.1296	.1307	7.649	.9916	.63	.5891	.7291	1.372	.8080
.14	.1395	.1409	7.096	.9902	.64	.5972	.7445	1.343	.8021
.15	.1494	.1511	6.617	.9888	.65	.6052	.7602	1.315	.7961
.16	.1593	.1614	6.197	.9872	.66	.6131	.7761	1.288	.7900
.17	.1692	.1717	5.826	.9856	.67	.6210	.7923	1.262	.7838
.18	.1790	.1820	5.495	.9838	.68	.6288	.8087	1.237	.7776
.19	.1889	.1923	5.200	.9820	.69	.6365	.8253	1.212	.7712
.20	.1987	.2027	4.933	.9801	.70	.6442	.8423	1.187	.7648
.21	.2085	.2131	4.692	.9780	.71	.6518	.8595	1.163	.7584
.22	.2182	.2236	4.472	.9759	.72	.6594	.8771	1.140	.7518
.23	.2280	.2341	4.271	.9737	.73	.6669	.8949	1.117	.7452
.24	.2377	.2447	4.086	.9713	.74	.6743	.9131	1.095	.7385
.25	.2474	.2553	3.916	.9689	.75	.6816	.9316	1.073	.7317
.26	.2571	.2660	3.759	.9664	.76	.6889	.9505	1.052	.7248
.27	.2667	.2768	3.613	.9638	.77	.6961	.9697	1.031	.7179
.28	.2764	.2876	3.478	.9611	.78	.7033	.9893	1.011	.7109
.29	.2860	.2984	3.351	.9582	.79	.7104	1.009	.9908	.7038
.30	.2955	.3093	3.233	.9553	.80	.7174	1.030	.9712	.6967
.31	.3051	.3203	3.122	.9523	.81	.7243	1.050	.9520	.6895
.32	.3146	.3314	3.018	.9492	.82	.7311	1.072	.9331	.6822
.33	.3240	.3425	2.920	.9460	.83	.7379	1.093	.9146	.6749
.34	.3335	.3537	2.827	.9428	.84	.7446	1.116	.8964	.6675
.35	.3429	.3650	2.740	.9394	.85	.7513	1.138	.8785	.6600
.36	.3523	.3764	2.657	.9359	.86	.7578	1.162	.8609	.6524
.37	.3616	.3879	2.578	.9323	.87	.7643	1.185	.8437	.6448
.38	.3709	.3994	2.504	.9287	.88	.7707	1.210	.8267	.6372
.39	.3802	.4111	2.433	.9249	.89	.7771	1.235	.8100	.6294
.40	.3894	.4228	2.365	.9211	.90	.7833	1.260	.7936	.6216
.41	.3986	.4346	2.301	.9171	.91	.7895	1.286	.7774	.6137
.42	.4078	.4466	2.239	.9131	.92	.7956	1.313	.7615	.6058
.43	.4169	.4586	2.180	.9090	.93	.8016	1.341	.7458	.5978
.44	.4259	.4708	2.124	.9048	.94	.8076	1.369	.7303	.5898
.45	.4350	.4831	2.070	.9004	.95	.8134	1.398	.7151	.5817
.46	.4439	.4954	2.018	.8961	.96	.8192	1.428	.7001	.5735
.47	.4529	.5080	1.969	.8916	.97	.8249	1.459	.6853	.5653
.48	.4618	.5206	1.921	.8870	.98	.8305	1.491	.6707	.5570
.49	.4706	.5334	1.875	.8823	.99	.8360	1.524	.6563	.5487
.50	.4794	.5463	1.830	.8776	1.00	.8415	1.557	.6421	.5403

Table 6.5(a) (Cont'd)
NATURAL TRIGONOMETRIC FUNCTIONS

X'	Sin	Tan	Ctn	Cos	X'	Sin	Tan	Ctn	Cos
1.00	.8415	1.557	.6421	.5403	1.30	.9636	3.602	.2776	.2675
1.01	.8468	1.592	.6281	.5319	1.31	.9662	3.747	.2669	.2579
1.02	.8521	1.628	.6142	.5234	1.32	.9687	3.903	.2562	.2482
1.03	.8573	1.665	.6005	.5148	1.33	.9711	4.072	.2456	.2385
1.04	.8624	1.704	.5870	.5062	1.34	.9735	4.256	.2350	.2288
1.05	.8674	1.743	.5736	.4976	1.35	.9757	4.455	.2245	.2190
1.06	.8724	1.784	.5604	.4889	1.36	.9779	4.673	.2140	.2092
1.07	.8772	1.827	.5473	.4801	1.37	.9799	4.913	.2035	.1994
1.08	.8820	1.871	.5344	.4713	1.38	.9819	5.177	.1931	.1896
1.09	.8866	1.917	.5216	.4625	1.39	.9837	5.471	.1828	.1798
1.10	.8912	1.965	.5090	.4536	1.40	.9854	5.798	.1725	.1700
1.11	.8957	2.014	.4964	.4447	1.41	.9871	6.165	.1622	.1601
1.12	.9001	2.066	.4840	.4357	1.42	.9887	6.581	.1519	.1502
1.13	.9044	2.120	.4718	.4267	1.43	.9901	7.055	.1417	.1403
1.14	.9086	2.176	.4596	.4176	1.44	.9915	7.602	.1315	.1304
1.15	.9128	2.234	.4475	.4085	1.45	.9927	8.238	.1214	.1205
1.16	.9168	2.296	.4356	.3993	1.46	.9939	8.989	.1113	.1106
1.17	.9208	2.360	.4237	.3902	1.47	.9949	9.887	.1011	.1006
1.18	.9246	2.427	.4120	.3809	1.48	.9959	10.983	.0910	.0907
1.19	.9284	2.498	.4003	.3717	1.49	.9967	12.350	.0810	.0807
1.20	.9320	2.572	.3888	.3624	1.50	.9975	14.101	.0709	.0707
1.21	.9356	2.650	.3773	.3530	1.51	.9982	16.428	.0609	.0608
1.22	.9391	2.733	.3659	.3436	1.52	.9987	19.670	.0508	.0508
1.23	.9425	2.820	.3546	.3342	1.53	.9992	24.498	.0408	.0408
1.24	.9458	2.912	.3434	.3248	1.54	.9995	32.461	.0308	.0308
1.25	.9490	3.010	.3323	.3153	1.55	.9998	48.078	.0208	.0208
1.26	.9521	3.113	.3212	.3058	1.56	.9999	92.620	.0108	.0108
1.27	.9551	3.224	.3102	.2963	1.57	1.0000	1255.8	.0008	.0008
1.28	.9580	3.341	.2993	.2867	1.58	1.0000	-108.65	-.0092	-.0092
1.29	.9608	3.467	.2884	.2771					
1.30	.9636	3.602	.2776	.2675					

In certain applications, it is convenient to allow trigonometric functions to have angular measurements as arguments. The natural measure of an angle, called its radian measure, is the arclength it subtends from the center of a unit circle. Angles also may be measured in degrees. The entire circumference of the unit circle is subtended by a central angle of 2π radians, or 360° . Each degree is subdivided into 60 minutes; each minute is subdivided into 60 seconds.

Table 6.5(b)

RADIANS TO DEGREES, MINUTES AND SECONDS

Rad		Rad		Rad		Rad		Rad	
1	57°17'44".8	.1	5°43'46".5	.01	0°34'22".6	.001	0° 3'26".3	.0001	0°0'20".6
2	114°35'29".6	.2	11°27'33".0	.02	1° 8'45".3	.002	0° 6'52".5	.0002	0°0'41".3
3	171°53'14".4	.3	17°11'19".4	.03	1°43'07".9	.003	0°10'18".8	.0003	0°1'01".9
4	229°10'59".2	.4	22°55'05".9	.04	2°17'30".6	.004	0°13'45".1	.0004	0°1'22".5
5	286°28'44".0	.5	28°38'52".4	.05	2°51'53".2	.005	0°17'11".3	.0005	0°1'43".1
6	343°46'28".8	.6	34°22'38".9	.06	3°26'15".9	.006	0°20'37".6	.0006	0°2'03".8
7	401° 4'13".6	.7	40° 6'25".4	.07	4° 0'38".5	.007	0°24'03".9	.0007	0°2'24".4
8	458°21'58".4	.8	45°50'11".8	.08	4°35'01".2	.008	0°27'30".1	.0008	0°2'45".0
9	515°39'43".3	.9	51°33'58".3	.09	5° 9'23".8	.009	0°30'56".4	.0009	0°3'05".6

DEGREES, MINUTES AND SECONDS TO RADIAN

Deg.	Rad	Min	Rad	Sec	Rad
1	0.01745 33	1	0.00029 09	1	0.00000 48
2	0.03490 66	2	0.00058 18	2	0.00000 97
3	0.05235 99	3	0.00087 27	3	0.00001 45
4	0.06981 32	4	0.00116 36	4	0.00001 94
5	0.08726 65	5	0.00145 44	5	0.00002 42
6	0.10471 98	6	0.00174 53	6	0.00002 91
7	0.12217 30	7	0.00203 62	7	0.00003 39
8	0.13962 63	8	0.00232 71	8	0.00003 88
9	0.15707 96	9	0.00261 80	9	0.00004 36
10	0.17453 29	10	0.00290 89	10	0.00004 85
20	0.34906 59	20	0.00581 78	20	0.00009 70
30	0.52359 88	30	0.00872 66	30	0.00014 54
40	0.69813 17	40	0.01163 55	40	0.00019 39
50	0.87266 46	50	0.01454 44	50	0.00024 24
60	1.04719 76	60	0.01745 33	60	0.00029 09
70	1.22173 05				
80	1.39626 34				
90	1.57079 63				

Table 6.5(c)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

0° (180°)					(359°) 179°					1° (181°)					(358°) 178°						
/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos	/
0	.00000	.00000	∞	1.0000	60	0	.01745	.01746	57.290	.99985	60										
1	.00029	.00029	3437.7	1.0000	59	1	.01774	.01775	56.351	.99984	59										
2	.00058	.00058	1718.9	1.0000	58	2	.01803	.01804	55.442	.99984	58										
3	.00087	.00087	1145.9	1.0000	57	3	.01832	.01833	54.561	.99983	57										
4	.00116	.00116	859.44	1.0000	56	4	.01862	.01862	53.709	.99983	56										
5	.00145	.00145	687.55	1.0000	55	5	.01891	.01891	52.882	.99982	55										
6	.00175	.00175	572.96	1.0000	54	6	.01920	.01920	52.081	.99982	54										
7	.00204	.00204	491.11	1.0000	53	7	.01949	.01949	51.303	.99981	53										
8	.00233	.00233	429.72	1.0000	52	8	.01978	.01978	50.549	.99980	52										
9	.00262	.00262	381.97	1.0000	51	9	.02007	.02007	49.816	.99980	51										
10	.00291	.00291	343.77	1.0000	50	10	.02036	.02036	49.104	.99979	50										
11	.00320	.00320	312.52	.99999	49	11	.02065	.02066	48.412	.99979	49										
12	.00349	.00349	286.48	.99999	48	12	.02094	.02095	47.740	.99978	48										
13	.00378	.00378	264.44	.99999	47	13	.02123	.02124	47.085	.99977	47										
14	.00407	.00407	245.55	.99999	46	14	.02152	.02153	46.449	.99977	46										
15	.00436	.00436	229.18	.99999	45	15	.02181	.02182	45.829	.99976	45										
16	.00465	.00465	214.86	.99999	44	16	.02211	.02211	45.226	.99976	44										
17	.00495	.00495	202.22	.99999	43	17	.02240	.02240	44.639	.99975	43										
18	.00524	.00524	190.98	.99999	42	18	.02269	.02269	44.066	.99974	42										
19	.00553	.00553	180.93	.99998	41	19	.02298	.02298	43.508	.99974	41										
20	.00582	.00582	171.89	.99998	40	20	.02327	.02328	42.964	.99973	40										
21	.00611	.00611	163.70	.99998	39	21	.02356	.02357	42.433	.99972	39										
22	.00640	.00640	156.26	.99998	38	22	.02385	.02386	41.916	.99972	38										
23	.00669	.00669	149.47	.99998	37	23	.02414	.02415	41.411	.99971	37										
24	.00698	.00698	143.24	.99998	36	24	.02443	.02444	40.917	.99970	36										
25	.00727	.00727	137.51	.99997	35	25	.02472	.02473	40.436	.99969	35										
26	.00756	.00756	132.22	.99997	34	26	.02501	.02502	39.965	.99969	34										
27	.00785	.00785	127.32	.99997	33	27	.02530	.02531	39.506	.99968	33										
28	.00814	.00815	122.77	.99997	32	28	.02560	.02560	39.057	.99967	32										
29	.00844	.00844	118.54	.99996	31	29	.02589	.02589	38.618	.99966	31										
30	.00873	.00873	114.59	.99996	30	30	.02618	.02619	38.188	.99966	30										
31	.00902	.00902	110.89	.99996	29	31	.02647	.02648	37.769	.99965	29										
32	.00931	.00931	107.43	.99996	28	32	.02676	.02677	37.358	.99964	28										
33	.00960	.00960	104.17	.99995	27	33	.02705	.02706	36.956	.99963	27										
34	.00989	.00989	101.11	.99995	26	34	.02734	.02735	36.563	.99963	26										
35	.01018	.01018	98.218	.99995	25	35	.02763	.02764	36.178	.99962	25										
36	.01047	.01047	95.489	.99995	24	36	.02792	.02793	35.801	.99961	24										
37	.01076	.01076	92.908	.99994	23	37	.02821	.02822	35.431	.99960	23										
38	.01105	.01105	90.463	.99994	22	38	.02850	.02851	35.070	.99959	22										
39	.01134	.01135	88.144	.99994	21	39	.02879	.02881	34.715	.99959	21										
40	.01164	.01164	85.940	.99993	20	40	.02908	.02910	34.368	.99958	20										
41	.01193	.01193	83.844	.99993	19	41	.02938	.02939	34.027	.99957	19										
42	.01222	.01222	81.847	.99993	18	42	.02967	.02968	33.694	.99956	18										
43	.01251	.01251	79.943	.99992	17	43	.02996	.02997	33.366	.99955	17										
44	.01280	.01280	78.126	.99992	16	44	.03025	.03026	33.045	.99954	16										
45	.01309	.01309	76.390	.99991	15	45	.03054	.03055	32.730	.99953	15										
46	.01338	.01338	74.729	.99991	14	46	.03083	.03084	32.421	.99952	14										
47	.01367	.01367	73.139	.99991	13	47	.03112	.03114	32.118	.99952	13										
48	.01396	.01396	71.615	.99990	12	48	.03141	.03143	31.821	.99951	12										
49	.01425	.01425	70.153	.99990	11	49	.03170	.03172	31.528	.99950	11										
50	.01454	.01455	68.750	.99989	10	50	.03199	.03201	31.242	.99949	10										
51	.01483	.01484	67.402	.99989	9	51	.03228	.03230	30.960	.99948	9										
52	.01513	.01513	66.105	.99989	8	52	.03257	.03259	30.683	.99947	8										
53	.01542	.01542	64.858	.99988	7	53	.03286	.03288	30.412	.99946	7										
54	.01571	.01571	63.657	.99988	6	54	.03316	.03317	30.145	.99945	6										
55	.01600	.01600	62.499	.99987	5	55	.03345	.03346	29.882	.99944	5										
56	.01629	.01629	61.383	.99987	4	56	.03374	.03376	29.624	.99943	4										
57	.01658	.01658	60.306	.99986	3	57	.03403	.03405	29.371	.99942	3										
58	.01687	.01687	59.266	.99986	2	58	.03432	.03434	29.122	.99941	2										
59	.01716	.01716	58.261	.99985	1	59	.03461	.03463	28.877	.99940	1										
60	.01745	.01746	57.290	.99985	0	60	.03490	.03492	28.636	.99939	0										
/	Cos	Ctn	Tan	Sin	/	/	Cos	Ctn	Tan	Sin	/										

90° (270°)

(269°) 89°

91° (271°)

(268°) 88°

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

2° (182°)					(357°) 177°					3° (183°)					(356°) 176°						
/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos	/
0	.03490	.03492	28.636	.99939	60	0	.05234	.05241	19.081	.99863	60	.05234	.05241	19.081	.99863	60	.05234	.05241	19.081	.99863	60
1	.03519	.03521	28.399	.99938	59	1	.05263	.05270	18.976	.99861	59	.05263	.05270	18.976	.99861	59	.05263	.05270	18.976	.99861	59
2	.03548	.03550	28.166	.99937	58	2	.05292	.05299	18.871	.99860	58	.05292	.05299	18.871	.99860	58	.05292	.05299	18.871	.99860	58
3	.03577	.03579	27.937	.99936	57	3	.05321	.05328	18.768	.99858	57	.05321	.05328	18.768	.99858	57	.05321	.05328	18.768	.99858	57
4	.03606	.03609	27.712	.99935	56	4	.05350	.05357	18.666	.99857	56	.05350	.05357	18.666	.99857	56	.05350	.05357	18.666	.99857	56
5	.03635	.03638	27.490	.99934	55	5	.05379	.05387	18.564	.99855	55	.05379	.05387	18.564	.99855	55	.05379	.05387	18.564	.99855	55
6	.03664	.03667	27.271	.99933	54	6	.05408	.05416	18.464	.99854	54	.05408	.05416	18.464	.99854	54	.05408	.05416	18.464	.99854	54
7	.03693	.03696	27.057	.99932	53	7	.05437	.05445	18.366	.99852	53	.05437	.05445	18.366	.99852	53	.05437	.05445	18.366	.99852	53
8	.03723	.03725	26.845	.99931	52	8	.05466	.05474	18.268	.99851	52	.05466	.05474	18.268	.99851	52	.05466	.05474	18.268	.99851	52
9	.03752	.03754	26.637	.99930	51	9	.05495	.05503	18.171	.99849	51	.05495	.05503	18.171	.99849	51	.05495	.05503	18.171	.99849	51
10	.03781	.03783	26.432	.99929	50	10	.05524	.05533	18.075	.99847	50	.05524	.05533	18.075	.99847	50	.05524	.05533	18.075	.99847	50
11	.03810	.03812	26.230	.99927	49	11	.05553	.05562	17.980	.99846	49	.05553	.05562	17.980	.99846	49	.05553	.05562	17.980	.99846	49
12	.03839	.03842	26.031	.99926	48	12	.05582	.05591	17.886	.99844	48	.05582	.05591	17.886	.99844	48	.05582	.05591	17.886	.99844	48
13	.03868	.03871	25.835	.99925	47	13	.05611	.05620	17.793	.99842	47	.05611	.05620	17.793	.99842	47	.05611	.05620	17.793	.99842	47
14	.03897	.03900	25.642	.99924	46	14	.05640	.05649	17.702	.99841	46	.05640	.05649	17.702	.99841	46	.05640	.05649	17.702	.99841	46
15	.03926	.03929	25.452	.99923	45	15	.05669	.05678	17.611	.99839	45	.05669	.05678	17.611	.99839	45	.05669	.05678	17.611	.99839	45
16	.03955	.03958	25.264	.99922	44	16	.05698	.05708	17.521	.99838	44	.05698	.05708	17.521	.99838	44	.05698	.05708	17.521	.99838	44
17	.03984	.03987	25.080	.99921	43	17	.05727	.05737	17.431	.99836	43	.05727	.05737	17.431	.99836	43	.05727	.05737	17.431	.99836	43
18	.04013	.04016	24.898	.99919	42	18	.05756	.05766	17.343	.99834	42	.05756	.05766	17.343	.99834	42	.05756	.05766	17.343	.99834	42
19	.04042	.04046	24.719	.99918	41	19	.05785	.05795	17.256	.99833	41	.05785	.05795	17.256	.99833	41	.05785	.05795	17.256	.99833	41
20	.04071	.04075	24.542	.99917	40	20	.05814	.05824	17.169	.99831	40	.05814	.05824	17.169	.99831	40	.05814	.05824	17.169	.99831	40
21	.04100	.04104	24.368	.99916	39	21	.05844	.05854	17.084	.99829	39	.05844	.05854	17.084	.99829	39	.05844	.05854	17.084	.99829	39
22	.04129	.04133	24.196	.99915	38	22	.05873	.05883	16.999	.99827	38	.05873	.05883	16.999	.99827	38	.05873	.05883	16.999	.99827	38
23	.04159	.04162	24.026	.99913	37	23	.05902	.05912	16.915	.99826	37	.05902	.05912	16.915	.99826	37	.05902	.05912	16.915	.99826	37
24	.04188	.04191	23.859	.99912	36	24	.05931	.05941	16.832	.99824	36	.05931	.05941	16.832	.99824	36	.05931	.05941	16.832	.99824	36
25	.04217	.04220	23.695	.99911	35	25	.05960	.05970	16.750	.99822	35	.05960	.05970	16.750	.99822	35	.05960	.05970	16.750	.99822	35
26	.04246	.04250	23.532	.99910	34	26	.05989	.05999	16.668	.99821	34	.05989	.05999	16.668	.99821	34	.05989	.05999	16.668	.99821	34
27	.04275	.04279	23.372	.99909	33	27	.06018	.06029	16.587	.99819	33	.06018	.06029	16.587	.99819	33	.06018	.06029	16.587	.99819	33
28	.04304	.04308	23.214	.99907	32	28	.06047	.06058	16.507	.99817	32	.06047	.06058	16.507	.99817	32	.06047	.06058	16.507	.99817	32
29	.04333	.04337	23.058	.99906	31	29	.06076	.06087	16.428	.99815	31	.06076	.06087	16.428	.99815	31	.06076	.06087	16.428	.99815	31
30	.04362	.04366	22.904	.99905	30	30	.06105	.06116	16.350	.99813	30	.06105	.06116	16.350	.99813	30	.06105	.06116	16.350	.99813	30
31	.04391	.04395	22.752	.99904	29	31	.06134	.06145	16.272	.99812	29	.06134	.06145	16.272	.99812	29	.06134	.06145	16.272	.99812	29
32	.04420	.04424	22.602	.99902	28	32	.06163	.06175	16.195	.99810	28	.06163	.06175	16.195	.99810	28	.06163	.06175	16.195	.99810	28
33	.04449	.04454	22.454	.99901	27	33	.06192	.06204	16.119	.99808	27	.06192	.06204	16.119	.99808	27	.06192	.06204	16.119	.99808	27
34	.04478	.04483	22.308	.99900	26	34	.06221	.06233	16.043	.99806	26	.06221	.06233	16.043	.99806	26	.06221	.06233	16.043	.99806	26
35	.04507	.04512	22.164	.99898	25	35	.06250	.06262	15.969	.99804	25	.06250	.06262	15.969	.99804	25	.06250	.06262	15.969	.99804	25
36	.04536	.04541	22.022	.99897	24	36	.06279	.06291	15.895	.99803	24	.06279	.06291	15.895	.99803	24	.06279	.06291	15.895	.99803	24
37	.04565	.04570	21.881	.99896	23	37	.06308	.06321	15.821	.99801	23	.06308	.06321	15.821	.99801	23	.06308	.06321	15.821	.99801	23
38	.04594	.04599	21.743	.99894	22	38	.06337	.06350	15.748	.99799	22	.06337	.06350	15.748	.99799	22	.06337	.06350	15.748	.99799	22
39	.04623	.04628	21.606	.99893	21	39	.06366	.06379	15.676	.99797	21	.06366	.06379	15.676	.99797	21	.06366	.06379	15.676	.99797	21
40	.04653	.04658	21.470	.99892	20	40	.06395	.06408	15.605	.99795	20	.06395	.06408	15.605	.99795	20	.06395	.06408	15.605	.99795	20
41	.04682	.04687	21.337	.99890	19	41	.06424	.06438	15.534	.99793	19	.06424	.06438	15.534	.99793	19	.06424	.06438	15.534	.99793	19
42	.04711	.04716	21.205	.99889	18	42	.06453	.06467	15.464	.99792	18	.06453	.06467	15.464	.99792	18	.06453	.06467	15.464	.99792	18
43	.04740	.04745	21.075	.99888	17	43	.06482	.06496	15.394	.99790	17	.06482	.06496	15.394	.99790	17	.06482	.06496	15.394	.99790	17
44	.04769	.04774	20.946	.99886	16	44	.06511	.06525	15.325	.99788	16	.06511	.06525	15.325	.99788	16	.06511	.06525	15.325	.99788	16
45	.04798	.04803	20.819	.99885	15	45	.06540	.06554	15.257	.99786	15	.06540	.06554	15.257	.99786	15	.06540	.06554	15.257	.99786	15
46	.04827	.04833	20.693	.99883	14	46	.06569	.06584	15.189	.99784	14	.06569	.06584	15.189	.99784	14	.06569	.06584	15.189	.99784	14
47	.04856	.04862	20.569	.99882	13	47	.06598	.06613	15.122	.99782	13	.06598	.06613	15.122	.99782	13	.06598	.06613	15.122	.99782	13
48	.04885	.04891	20.446	.99881	12	48	.06627	.06642	15.056	.99780	12	.06627	.06642	15.056	.99780	12	.06627	.06642	15.056	.99780	12
49	.04914	.04920	20.325	.99879	11	49	.06656	.06671	14.990	.99778	11	.06656	.06671	14.990	.99778	11	.06656	.06671	14.990	.99778	11
50	.04943	.04949	20.206	.99878	10	50	.06685	.06700	14.924	.99776	10	.06685	.06700	14.924	.99776	10	.06685	.06700	14.924	.99776	10
51	.04972	.04978	20.087	.99876	9	51	.06714	.06730	14.860	.99774	9	.06714	.06730	14.860	.99774	9	.06714	.06730	14.860	.99774	9
52	.05001	.05007	19.970	.99875	8	52	.06743	.06759	14.795	.99772	8	.06743	.06759	14.795	.99772	8	.06743	.06759	14.795	.99772	8
53	.05030	.05037	19.855	.99873	7	53	.06773	.06788	14.732	.99770	7	.06773	.06788	14.732	.99770	7	.06773	.06788	14.732	.99770	7
54	.05059	.05066	19.740	.99872	6	54	.06802	.06817	14.669	.99768	6	.06802	.06817	14.669	.99768	6	.06802	.06817	14.669	.99768	6
55	.05088	.05095	19.627	.99870	5	55	.06831	.06847	14.606	.99766	5	.06831	.06847	14.606	.99766	5	.06831	.06847	14.606	.99766	5
56	.05117	.05124	19.516	.99869	4	56	.06860	.06876	14.544	.99764	4	.06860	.06876	14.							

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

4° (184°)					(355°) 175°					5° (185°)					(354°) 174°				
/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/		
0	.06976	.06993	14.301	.99756	60	0	.08716	.08749	11.430	.99619	60	0	.08716	.08749	11.430	.99619	60		
1	.07005	.07022	14.241	.99754	59	1	.08745	.08778	11.392	.99617	59	1	.08745	.08778	11.392	.99617	59		
2	.07034	.07051	14.182	.99752	58	2	.08774	.08807	11.354	.99614	58	2	.08774	.08807	11.354	.99614	58		
3	.07063	.07080	14.124	.99750	57	3	.08803	.08837	11.316	.99612	57	3	.08803	.08837	11.316	.99612	57		
4	.07092	.07110	14.065	.99748	56	4	.08831	.08866	11.279	.99609	56	4	.08831	.08866	11.279	.99609	56		
5	.07121	.07139	14.008	.99746	55	5	.08860	.08895	11.242	.99607	55	5	.08860	.08895	11.242	.99607	55		
6	.07150	.07168	13.951	.99744	54	6	.08889	.08925	11.205	.99604	54	6	.08889	.08925	11.205	.99604	54		
7	.07179	.07197	13.894	.99742	53	7	.08918	.08954	11.168	.99602	53	7	.08918	.08954	11.168	.99602	53		
8	.07208	.07227	13.838	.99740	52	8	.08947	.08983	11.132	.99599	52	8	.08947	.08983	11.132	.99599	52		
9	.07237	.07256	13.782	.99738	51	9	.08976	.09013	11.095	.99596	51	9	.08976	.09013	11.095	.99596	51		
10	.07266	.07285	13.727	.99736	50	10	.09005	.09042	11.059	.99594	50	10	.09005	.09042	11.059	.99594	50		
11	.07295	.07314	13.672	.99734	49	11	.09034	.09071	11.024	.99591	49	11	.09034	.09071	11.024	.99591	49		
12	.07324	.07344	13.617	.99731	48	12	.09063	.09101	10.988	.99588	48	12	.09063	.09101	10.988	.99588	48		
13	.07353	.07373	13.563	.99729	47	13	.09092	.09130	10.953	.99586	47	13	.09092	.09130	10.953	.99586	47		
14	.07382	.07402	13.510	.99727	46	14	.09121	.09159	10.918	.99583	46	14	.09121	.09159	10.918	.99583	46		
15	.07411	.07431	13.457	.99725	45	15	.09150	.09189	10.883	.99580	45	15	.09150	.09189	10.883	.99580	45		
16	.07440	.07461	13.404	.99723	44	16	.09179	.09218	10.848	.99578	44	16	.09179	.09218	10.848	.99578	44		
17	.07469	.07490	13.352	.99721	43	17	.09208	.09247	10.814	.99575	43	17	.09208	.09247	10.814	.99575	43		
18	.07498	.07519	13.300	.99719	42	18	.09237	.09277	10.780	.99572	42	18	.09237	.09277	10.780	.99572	42		
19	.07527	.07548	13.248	.99716	41	19	.09266	.09306	10.746	.99570	41	19	.09266	.09306	10.746	.99570	41		
20	.07556	.07578	13.197	.99714	40	20	.09295	.09335	10.712	.99567	40	20	.09295	.09335	10.712	.99567	40		
21	.07585	.07607	13.146	.99712	39	21	.09324	.09365	10.678	.99564	39	21	.09324	.09365	10.678	.99564	39		
22	.07614	.07636	13.096	.99710	38	22	.09353	.09394	10.645	.99562	38	22	.09353	.09394	10.645	.99562	38		
23	.07643	.07665	13.046	.99708	37	23	.09382	.09423	10.612	.99559	37	23	.09382	.09423	10.612	.99559	37		
24	.07672	.07695	12.996	.99705	36	24	.09411	.09453	10.579	.99556	36	24	.09411	.09453	10.579	.99556	36		
25	.07701	.07724	12.947	.99703	35	25	.09440	.09482	10.546	.99553	35	25	.09440	.09482	10.546	.99553	35		
26	.07730	.07753	12.898	.99701	34	26	.09469	.09511	10.514	.99551	34	26	.09469	.09511	10.514	.99551	34		
27	.07759	.07782	12.850	.99699	33	27	.09498	.09541	10.481	.99548	33	27	.09498	.09541	10.481	.99548	33		
28	.07788	.07812	12.801	.99696	32	28	.09527	.09570	10.449	.99545	32	28	.09527	.09570	10.449	.99545	32		
29	.07817	.07841	12.754	.99694	31	29	.09556	.09600	10.417	.99542	31	29	.09556	.09600	10.417	.99542	31		
30	.07846	.07870	12.706	.99692	30	30	.09585	.09629	10.385	.99540	30	30	.09585	.09629	10.385	.99540	30		
31	.07875	.07899	12.659	.99689	29	31	.09614	.09658	10.354	.99537	29	31	.09614	.09658	10.354	.99537	29		
32	.07904	.07929	12.612	.99687	28	32	.09643	.09688	10.322	.99534	28	32	.09643	.09688	10.322	.99534	28		
33	.07933	.07958	12.566	.99685	27	33	.09671	.09717	10.291	.99531	27	33	.09671	.09717	10.291	.99531	27		
34	.07962	.07987	12.520	.99683	26	34	.09700	.09746	10.260	.99528	26	34	.09700	.09746	10.260	.99528	26		
35	.07991	.08017	12.474	.99680	25	35	.09729	.09776	10.229	.99526	25	35	.09729	.09776	10.229	.99526	25		
36	.08020	.08046	12.429	.99678	24	36	.09758	.09805	10.199	.99523	24	36	.09758	.09805	10.199	.99523	24		
37	.08049	.08075	12.384	.99676	23	37	.09787	.09834	10.168	.99520	23	37	.09787	.09834	10.168	.99520	23		
38	.08078	.08104	12.339	.99673	22	38	.09816	.09864	10.138	.99517	22	38	.09816	.09864	10.138	.99517	22		
39	.08107	.08134	12.295	.99671	21	39	.09845	.09893	10.108	.99514	21	39	.09845	.09893	10.108	.99514	21		
40	.08136	.08163	12.251	.99668	20	40	.09874	.09922	10.078	.99511	20	40	.09874	.09922	10.078	.99511	20		
41	.08165	.08192	12.207	.99666	19	41	.09903	.09952	10.048	.99508	19	41	.09903	.09952	10.048	.99508	19		
42	.08194	.08221	12.163	.99664	18	42	.09932	.09981	10.019	.99505	18	42	.09932	.09981	10.019	.99505	18		
43	.08223	.08251	12.120	.99661	17	43	.09961	.10011	9.9893	.99503	17	43	.09961	.10011	9.9893	.99503	17		
44	.08252	.08280	12.077	.99659	16	44	.09990	.10040	9.9601	.99500	16	44	.09990	.10040	9.9601	.99500	16		
45	.08281	.08309	12.035	.99657	15	45	.10019	.10069	9.9310	.99497	15	45	.10019	.10069	9.9310	.99497	15		
46	.08310	.08339	11.992	.99654	14	46	.10048	.10099	9.9021	.99494	14	46	.10048	.10099	9.9021	.99494	14		
47	.08339	.08368	11.950	.99652	13	47	.10077	.10128	9.8734	.99491	13	47	.10077	.10128	9.8734	.99491	13		
48	.08368	.08397	11.909	.99649	12	48	.10106	.10158	9.8448	.99488	12	48	.10106	.10158	9.8448	.99488	12		
49	.08397	.08427	11.867	.99647	11	49	.10135	.10187	9.8164	.99485	11	49	.10135	.10187	9.8164	.99485	11		
50	.08426	.08456	11.826	.99644	10	50	.10164	.10216	9.7882	.99482	10	50	.10164	.10216	9.7882	.99482	10		
51	.08455	.08485	11.785	.99642	9	51	.10192	.10246	9.7601	.99479	9	51	.10192	.10246	9.7601	.99479	9		
52	.08484	.08514	11.745	.99639	8	52	.10221	.10275	9.7322	.99476	8	52	.10221	.10275	9.7322	.99476	8		
53	.08513	.08544	11.705	.99637	7	53	.10250	.10305	9.7044	.99473	7	53	.10250	.10305	9.7044	.99473	7		
54	.08542	.08573	11.664	.99635	6	54	.10279	.10334	9.6768	.99470	6	54	.10279	.10334	9.6768	.99470	6		
55	.08571	.08602	11.625	.99632	5	55	.10308	.10363	9.6493	.99467	5	55	.10308	.10363	9.6493	.99467	5		
56	.08600	.08632	11.585	.99630	4	56	.10337	.10393	9.6220	.99464	4	56	.10337	.10393	9.6220	.99464	4		
57	.08629	.08661	11.546	.99627	3	57	.10366	.10422	9.5949	.99461	3	57	.10366	.10422	9.5949	.99461	3		
58	.08658	.08690	11.507	.99625	2	58	.10395	.10452	9.5679	.99458	2	58	.10395	.10452	9.5679	.99458	2		
59	.08687	.08720	11.468	.99622	1	59	.10424	.10481	9.5411	.99455	1	59	.10424	.10481	9.5411	.99455	1		
60	.08716	.08749	11.430	.99619	0	60	.10453	.10510	9.5144	.99452	0	60	.10453	.10510	9.5144	.99452	0		
/	Cos	Ctn	Tan	Sin	/	/	Cos	Ctn	Tan	Sin	/	/	Cos	Ctn	Tan	Sin	/		

94° (274°)

(265°) 85°

95° (275°)

(264°) 84°

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

6° (186°)					(353°) 173°					7° (187°)					(352°) 172°				
/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos
0	.10453	.10510	9.5144	.99452	60	.12187	.12278	8.1443	.99255	60	.12187	.12278	8.1443	.99255	60	.12187	.12278	8.1443	.99255
1	.10482	.10540	9.4878	.99449	59	.12216	.12308	8.1248	.99251	59	.12216	.12308	8.1248	.99251	59	.12216	.12308	8.1248	.99251
2	.10511	.10569	9.4614	.99446	58	.12245	.12338	8.1054	.99248	58	.12245	.12338	8.1054	.99248	58	.12245	.12338	8.1054	.99248
3	.10540	.10599	9.4352	.99443	57	.12274	.12367	8.0860	.99244	57	.12274	.12367	8.0860	.99244	57	.12274	.12367	8.0860	.99244
4	.10569	.10628	9.4090	.99440	56	.12302	.12397	8.0667	.99240	56	.12302	.12397	8.0667	.99240	56	.12302	.12397	8.0667	.99240
5	.10597	.10657	9.3831	.99437	55	.12331	.12426	8.0476	.99237	55	.12331	.12426	8.0476	.99237	55	.12331	.12426	8.0476	.99237
6	.10626	.10687	9.3572	.99434	54	.12360	.12456	8.0285	.99233	54	.12360	.12456	8.0285	.99233	54	.12360	.12456	8.0285	.99233
7	.10655	.10716	9.3315	.99431	53	.12389	.12485	8.0095	.99230	53	.12389	.12485	8.0095	.99230	53	.12389	.12485	8.0095	.99230
8	.10684	.10746	9.3060	.99428	52	.12418	.12515	7.9906	.99226	52	.12418	.12515	7.9906	.99226	52	.12418	.12515	7.9906	.99226
9	.10713	.10775	9.2806	.99424	51	.12447	.12544	7.9718	.99222	51	.12447	.12544	7.9718	.99222	51	.12447	.12544	7.9718	.99222
10	.10742	.10805	9.2553	.99421	50	.12476	.12574	7.9530	.99219	50	.12476	.12574	7.9530	.99219	50	.12476	.12574	7.9530	.99219
11	.10771	.10834	9.2302	.99418	49	.12504	.12603	7.9344	.99215	49	.12504	.12603	7.9344	.99215	49	.12504	.12603	7.9344	.99215
12	.10800	.10863	9.2052	.99415	48	.12533	.12633	7.9158	.99211	48	.12533	.12633	7.9158	.99211	48	.12533	.12633	7.9158	.99211
13	.10829	.10893	9.1803	.99412	47	.12562	.12662	7.8973	.99208	47	.12562	.12662	7.8973	.99208	47	.12562	.12662	7.8973	.99208
14	.10858	.10922	9.1555	.99409	46	.12591	.12692	7.8789	.99204	46	.12591	.12692	7.8789	.99204	46	.12591	.12692	7.8789	.99204
15	.10887	.10952	9.1309	.99406	45	.12620	.12722	7.8606	.99200	45	.12620	.12722	7.8606	.99200	45	.12620	.12722	7.8606	.99200
16	.10916	.10981	9.1065	.99402	44	.12649	.12751	7.8424	.99197	44	.12649	.12751	7.8424	.99197	44	.12649	.12751	7.8424	.99197
17	.10945	.11011	9.0821	.99399	43	.12678	.12781	7.8243	.99193	43	.12678	.12781	7.8243	.99193	43	.12678	.12781	7.8243	.99193
18	.10973	.11040	9.0579	.99396	42	.12706	.12810	7.8062	.99189	42	.12706	.12810	7.8062	.99189	42	.12706	.12810	7.8062	.99189
19	.11002	.11070	9.0338	.99393	41	.12735	.12840	7.7882	.99186	41	.12735	.12840	7.7882	.99186	41	.12735	.12840	7.7882	.99186
20	.11031	.11099	9.0098	.99390	40	.12764	.12869	7.7704	.99182	40	.12764	.12869	7.7704	.99182	40	.12764	.12869	7.7704	.99182
21	.11060	.11128	8.9860	.99386	39	.12793	.12899	7.7525	.99178	39	.12793	.12899	7.7525	.99178	39	.12793	.12899	7.7525	.99178
22	.11089	.11158	8.9623	.99383	38	.12822	.12929	7.7348	.99175	38	.12822	.12929	7.7348	.99175	38	.12822	.12929	7.7348	.99175
23	.11118	.11187	8.9387	.99380	37	.12851	.12958	7.7171	.99171	37	.12851	.12958	7.7171	.99171	37	.12851	.12958	7.7171	.99171
24	.11147	.11217	8.9152	.99377	36	.12880	.12988	7.6996	.99167	36	.12880	.12988	7.6996	.99167	36	.12880	.12988	7.6996	.99167
25	.11176	.11246	8.8919	.99374	35	.12908	.13017	7.6821	.99163	35	.12908	.13017	7.6821	.99163	35	.12908	.13017	7.6821	.99163
26	.11205	.11276	8.8686	.99370	34	.12937	.13047	7.6647	.99160	34	.12937	.13047	7.6647	.99160	34	.12937	.13047	7.6647	.99160
27	.11234	.11305	8.8455	.99367	33	.12966	.13076	7.6473	.99156	33	.12966	.13076	7.6473	.99156	33	.12966	.13076	7.6473	.99156
28	.11263	.11335	8.8225	.99364	32	.12995	.13106	7.6301	.99152	32	.12995	.13106	7.6301	.99152	32	.12995	.13106	7.6301	.99152
29	.11291	.11364	8.7996	.99360	31	.13024	.13136	7.6129	.99148	31	.13024	.13136	7.6129	.99148	31	.13024	.13136	7.6129	.99148
30	.11320	.11394	8.7769	.99357	30	.13053	.13165	7.5958	.99144	30	.13053	.13165	7.5958	.99144	30	.13053	.13165	7.5958	.99144
31	.11349	.11423	8.7542	.99354	29	.13081	.13195	7.5787	.99141	29	.13081	.13195	7.5787	.99141	29	.13081	.13195	7.5787	.99141
32	.11378	.11452	8.7317	.99351	28	.13110	.13224	7.5618	.99137	28	.13110	.13224	7.5618	.99137	28	.13110	.13224	7.5618	.99137
33	.11407	.11482	8.7093	.99347	27	.13139	.13254	7.5449	.99133	27	.13139	.13254	7.5449	.99133	27	.13139	.13254	7.5449	.99133
34	.11436	.11511	8.6870	.99344	26	.13168	.13284	7.5281	.99129	26	.13168	.13284	7.5281	.99129	26	.13168	.13284	7.5281	.99129
35	.11465	.11541	8.6648	.99341	25	.13197	.13313	7.5113	.99125	25	.13197	.13313	7.5113	.99125	25	.13197	.13313	7.5113	.99125
36	.11494	.11570	8.6427	.99337	24	.13226	.13343	7.4947	.99122	24	.13226	.13343	7.4947	.99122	24	.13226	.13343	7.4947	.99122
37	.11523	.11600	8.6208	.99334	23	.13254	.13372	7.4781	.99118	23	.13254	.13372	7.4781	.99118	23	.13254	.13372	7.4781	.99118
38	.11552	.11629	8.5989	.99331	22	.13283	.13402	7.4615	.99114	22	.13283	.13402	7.4615	.99114	22	.13283	.13402	7.4615	.99114
39	.11580	.11659	8.5772	.99327	21	.13312	.13432	7.4451	.99110	21	.13312	.13432	7.4451	.99110	21	.13312	.13432	7.4451	.99110
40	.11609	.11688	8.5555	.99324	20	.13341	.13461	7.4287	.99106	20	.13341	.13461	7.4287	.99106	20	.13341	.13461	7.4287	.99106
41	.11638	.11718	8.5340	.99320	19	.13370	.13491	7.4124	.99102	19	.13370	.13491	7.4124	.99102	19	.13370	.13491	7.4124	.99102
42	.11667	.11747	8.5126	.99317	18	.13399	.13521	7.3962	.99098	18	.13399	.13521	7.3962	.99098	18	.13399	.13521	7.3962	.99098
43	.11696	.11777	8.4913	.99314	17	.13427	.13550	7.3800	.99094	17	.13427	.13550	7.3800	.99094	17	.13427	.13550	7.3800	.99094
44	.11725	.11806	8.4701	.99310	16	.13456	.13580	7.3639	.99091	16	.13456	.13580	7.3639	.99091	16	.13456	.13580	7.3639	.99091
45	.11754	.11836	8.4490	.99307	15	.13485	.13609	7.3479	.99087	15	.13485	.13609	7.3479	.99087	15	.13485	.13609	7.3479	.99087
46	.11783	.11865	8.4280	.99303	14	.13514	.13639	7.3319	.99083	14	.13514	.13639	7.3319	.99083	14	.13514	.13639	7.3319	.99083
47	.11812	.11895	8.4071	.99300	13	.13543	.13669	7.3160	.99079	13	.13543	.13669	7.3160	.99079	13	.13543	.13669	7.3160	.99079
48	.11840	.11924	8.3863	.99297	12	.13572	.13698	7.3002	.99075	12	.13572	.13698	7.3002	.99075	12	.13572	.13698	7.3002	.99075
49	.11869	.11954	8.3656	.99293	11	.13600	.13728	7.2844	.99071	11	.13600	.13728	7.2844	.99071	11	.13600	.13728	7.2844	.99071
50	.11898	.11983	8.3450	.99290	10	.13629	.13758	7.2687	.99067	10	.13629	.13758	7.2687	.99067	10	.13629	.13758	7.2687	.99067
51	.11927	.12013	8.3245	.99286	9	.13658	.13787	7.2531	.99063	9	.13658	.13787	7.2531	.99063	9	.13658	.13787	7.2531	.99063
52	.11956	.12042	8.3041	.99283	8	.13687	.13817	7.2375	.99059	8	.13687	.13817	7.2375	.99059	8	.13687	.13817	7.2375	.99059
53	.11985	.12072	8.2838	.99279	7	.13716	.13846	7.2220	.99055	7	.13716	.13846	7.2220	.99055	7	.13716	.13846	7.2220	.99055
54	.12014	.12101	8.2636	.99276	6	.13744	.13876	7.2066	.99051	6	.13744	.13876	7.2066	.99051	6	.13744	.13876	7.2066	.99051
55	.12043	.12131	8.2434	.99272	5	.13773	.13906	7.1912	.99047	5	.13773	.13906	7.1912	.99047	5	.13773	.13906	7.1912	.99047
56	.12071	.12160	8.2234	.99269	4	.13802	.13935	7.1759	.99043	4	.13802	.13935	7.1759	.99043	4	.13802	.13935	7.1759	.99043
57	.12100	.12190	8.2035	.99265	3	.13831	.13965	7.1607	.99039	3	.13831	.13965	7.1607	.99039	3	.13831	.13965	7.1607	.99039
58	.12129	.12219	8.1837	.99262	2	.13860	.13995	7.1455	.99035	2	.13860	.13995	7.1455	.99035	2	.13860	.13995	7.1455	.99035
59	.12158	.12249	8.1640	.99258	1	.13889	.14024	7.1304	.99031	1	.13889	.14024	7.1304	.99031	1	.13889	.14024	7.1304	.99031
60	.12187	.12278	8.1443	.99255	0	.13917	.14054	7.1154	.99027	0	.13917	.14054	7.1154	.99					

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

8° (188°)					(351°) 171°					9° (189°)					(350°) 170°				
/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos
0	.13917	.14054	7.1154	.99027	60	.15643	.15838	6.3138	.98769	60	.15643	.15838	6.3138	.98769	60	.15643	.15838	6.3138	.98769
1	.13946	.14084	7.1004	.99023	59	.15672	.15868	6.3019	.98764	59	.15672	.15868	6.3019	.98764	59	.15672	.15868	6.3019	.98764
2	.13975	.14113	7.0855	.99019	58	.15701	.15898	6.2901	.98760	58	.15701	.15898	6.2901	.98760	58	.15701	.15898	6.2901	.98760
3	.14004	.14143	7.0706	.99015	57	.15730	.15928	6.2783	.98755	57	.15730	.15928	6.2783	.98755	57	.15730	.15928	6.2783	.98755
4	.14033	.14173	7.0558	.99011	56	.15758	.15958	6.2666	.98751	56	.15758	.15958	6.2666	.98751	56	.15758	.15958	6.2666	.98751
5	.14061	.14202	7.0410	.99006	55	.15787	.15988	6.2549	.98746	55	.15787	.15988	6.2549	.98746	55	.15787	.15988	6.2549	.98746
6	.14090	.14232	7.0264	.99002	54	.15816	.16017	6.2432	.98741	54	.15816	.16017	6.2432	.98741	54	.15816	.16017	6.2432	.98741
7	.14119	.14262	7.0117	.98998	53	.15845	.16047	6.2316	.98737	53	.15845	.16047	6.2316	.98737	53	.15845	.16047	6.2316	.98737
8	.14148	.14291	6.9972	.98994	52	.15873	.16077	6.2200	.98732	52	.15873	.16077	6.2200	.98732	52	.15873	.16077	6.2200	.98732
9	.14177	.14321	6.9827	.98990	51	.15902	.16107	6.2085	.98728	51	.15902	.16107	6.2085	.98728	51	.15902	.16107	6.2085	.98728
10	.14205	.14351	6.9682	.98986	50	.15931	.16137	6.1970	.98723	50	.15931	.16137	6.1970	.98723	50	.15931	.16137	6.1970	.98723
11	.14234	.14381	6.9538	.98982	49	.15959	.16167	6.1856	.98718	49	.15959	.16167	6.1856	.98718	49	.15959	.16167	6.1856	.98718
12	.14263	.14410	6.9395	.98978	48	.15988	.16196	6.1742	.98714	48	.15988	.16196	6.1742	.98714	48	.15988	.16196	6.1742	.98714
13	.14292	.14440	6.9252	.98973	47	.16017	.16226	6.1628	.98709	47	.16017	.16226	6.1628	.98709	47	.16017	.16226	6.1628	.98709
14	.14320	.14470	6.9110	.98969	46	.16046	.16256	6.1515	.98704	46	.16046	.16256	6.1515	.98704	46	.16046	.16256	6.1515	.98704
15	.14349	.14499	6.8969	.98965	45	.16074	.16286	6.1402	.98700	45	.16074	.16286	6.1402	.98700	45	.16074	.16286	6.1402	.98700
16	.14378	.14529	6.8828	.98961	44	.16103	.16316	6.1290	.98695	44	.16103	.16316	6.1290	.98695	44	.16103	.16316	6.1290	.98695
17	.14407	.14559	6.8687	.98957	43	.16132	.16346	6.1178	.98690	43	.16132	.16346	6.1178	.98690	43	.16132	.16346	6.1178	.98690
18	.14436	.14588	6.8548	.98953	42	.16160	.16376	6.1066	.98686	42	.16160	.16376	6.1066	.98686	42	.16160	.16376	6.1066	.98686
19	.14464	.14618	6.8408	.98948	41	.16189	.16405	6.0955	.98681	41	.16189	.16405	6.0955	.98681	41	.16189	.16405	6.0955	.98681
20	.14493	.14648	6.8269	.98944	40	.16218	.16435	6.0844	.98676	40	.16218	.16435	6.0844	.98676	40	.16218	.16435	6.0844	.98676
21	.14522	.14678	6.8131	.98940	39	.16246	.16465	6.0734	.98671	39	.16246	.16465	6.0734	.98671	39	.16246	.16465	6.0734	.98671
22	.14551	.14707	6.7994	.98936	38	.16275	.16495	6.0624	.98667	38	.16275	.16495	6.0624	.98667	38	.16275	.16495	6.0624	.98667
23	.14580	.14737	6.7856	.98931	37	.16304	.16525	6.0514	.98662	37	.16304	.16525	6.0514	.98662	37	.16304	.16525	6.0514	.98662
24	.14608	.14767	6.7720	.98927	36	.16333	.16555	6.0405	.98657	36	.16333	.16555	6.0405	.98657	36	.16333	.16555	6.0405	.98657
25	.14637	.14796	6.7584	.98923	35	.16361	.16585	6.0296	.98652	35	.16361	.16585	6.0296	.98652	35	.16361	.16585	6.0296	.98652
26	.14666	.14826	6.7448	.98919	34	.16390	.16615	6.0188	.98648	34	.16390	.16615	6.0188	.98648	34	.16390	.16615	6.0188	.98648
27	.14695	.14856	6.7313	.98914	33	.16419	.16645	6.0080	.98643	33	.16419	.16645	6.0080	.98643	33	.16419	.16645	6.0080	.98643
28	.14723	.14886	6.7179	.98910	32	.16447	.16674	5.9972	.98638	32	.16447	.16674	5.9972	.98638	32	.16447	.16674	5.9972	.98638
29	.14752	.14915	6.7045	.98906	31	.16476	.16704	5.9865	.98633	31	.16476	.16704	5.9865	.98633	31	.16476	.16704	5.9865	.98633
30	.14781	.14945	6.6912	.98902	30	.16505	.16734	5.9758	.98629	30	.16505	.16734	5.9758	.98629	30	.16505	.16734	5.9758	.98629
31	.14810	.14975	6.6779	.98897	29	.16533	.16764	5.9651	.98624	29	.16533	.16764	5.9651	.98624	29	.16533	.16764	5.9651	.98624
32	.14838	.15005	6.6646	.98893	28	.16562	.16794	5.9545	.98619	28	.16562	.16794	5.9545	.98619	28	.16562	.16794	5.9545	.98619
33	.14867	.15034	6.6514	.98889	27	.16591	.16824	5.9439	.98614	27	.16591	.16824	5.9439	.98614	27	.16591	.16824	5.9439	.98614
34	.14896	.15064	6.6383	.98884	26	.16620	.16854	5.9333	.98609	26	.16620	.16854	5.9333	.98609	26	.16620	.16854	5.9333	.98609
35	.14925	.15094	6.6252	.98880	25	.16648	.16884	5.9228	.98604	25	.16648	.16884	5.9228	.98604	25	.16648	.16884	5.9228	.98604
36	.14954	.15124	6.6122	.98876	24	.16677	.16914	5.9124	.98600	24	.16677	.16914	5.9124	.98600	24	.16677	.16914	5.9124	.98600
37	.14982	.15153	6.5992	.98871	23	.16706	.16944	5.9019	.98595	23	.16706	.16944	5.9019	.98595	23	.16706	.16944	5.9019	.98595
38	.15011	.15183	6.5863	.98867	22	.16734	.16974	5.8915	.98590	22	.16734	.16974	5.8915	.98590	22	.16734	.16974	5.8915	.98590
39	.15040	.15213	6.5734	.98863	21	.16763	.17004	5.8811	.98585	21	.16763	.17004	5.8811	.98585	21	.16763	.17004	5.8811	.98585
40	.15069	.15243	6.5606	.98858	20	.16792	.17033	5.8708	.98580	20	.16792	.17033	5.8708	.98580	20	.16792	.17033	5.8708	.98580
41	.15097	.15272	6.5478	.98854	19	.16820	.17063	5.8605	.98575	19	.16820	.17063	5.8605	.98575	19	.16820	.17063	5.8605	.98575
42	.15126	.15302	6.5350	.98849	18	.16849	.17093	5.8502	.98570	18	.16849	.17093	5.8502	.98570	18	.16849	.17093	5.8502	.98570
43	.15155	.15332	6.5223	.98845	17	.16878	.17123	5.8400	.98565	17	.16878	.17123	5.8400	.98565	17	.16878	.17123	5.8400	.98565
44	.15184	.15362	6.5097	.98841	16	.16906	.17153	5.8298	.98561	16	.16906	.17153	5.8298	.98561	16	.16906	.17153	5.8298	.98561
45	.15212	.15391	6.4971	.98836	15	.16935	.17183	5.8197	.98556	15	.16935	.17183	5.8197	.98556	15	.16935	.17183	5.8197	.98556
46	.15241	.15421	6.4846	.98832	14	.16964	.17213	5.8095	.98551	14	.16964	.17213	5.8095	.98551	14	.16964	.17213	5.8095	.98551
47	.15270	.15451	6.4721	.98827	13	.16992	.17243	5.7994	.98546	13	.16992	.17243	5.7994	.98546	13	.16992	.17243	5.7994	.98546
48	.15299	.15481	6.4596	.98823	12	.17021	.17273	5.7894	.98541	12	.17021	.17273	5.7894	.98541	12	.17021	.17273	5.7894	.98541
49	.15327	.15511	6.4472	.98818	11	.17050	.17303	5.7794	.98536	11	.17050	.17303	5.7794	.98536	11	.17050	.17303	5.7794	.98536
50	.15356	.15540	6.4348	.98814	10	.17078	.17333	5.7694	.98531	10	.17078	.17333	5.7694	.98531	10	.17078	.17333	5.7694	.98531
51	.15385	.15570	6.4225	.98809	9	.17107	.17363	5.7594	.98526	9	.17107	.17363	5.7594	.98526	9	.17107	.17363	5.7594	.98526
52	.15414	.15600	6.4103	.98805	8	.17136	.17393	5.7495	.98521	8	.17136	.17393	5.7495	.98521	8	.17136	.17393	5.7495	.98521
53	.15442	.15630	6.3980	.98800	7	.17164	.17423	5.7396	.98516	7	.17164	.17423	5.7396	.98516	7	.17164	.17423	5.7396	.98516
54	.15471	.15660	6.3859	.98796	6	.17193	.17453	5.7297	.98511	6	.17193	.17453	5.7297	.98511	6	.17193	.17453	5.7297	.98511
55	.15500	.15689	6.3737	.98791	5	.17222	.17483	5.7199	.98506	5	.17222	.17483	5.7199	.98506	5	.17222	.17483	5.7199	.98506
56	.15529	.15719	6.3617	.98787	4	.17250	.17513	5.7101	.98501	4	.17250	.17513	5.7101	.98501	4	.17250	.17513	5.7101	.98501
57	.15557	.15749	6.3496	.98782	3	.17279	.17543	5.7004	.98496	3	.17279	.17543	5.7004	.98496	3	.17279	.17543	5.7004	.98496
58	.15586	.15779	6.3376	.98778	2	.17308	.17573	5.6906	.98491	2	.17308	.17573	5.6906	.98491	2	.17308	.17573	5.6906	.98491
59	.15615	.15809	6.3257	.98773	1	.17336	.17603	5.6809	.98486	1	.17336	.17603	5.6809	.98486	1	.17336	.17603	5.6809	.98486
60	.15643	.15838	6.3138	.98769	0	.17365	.17633	5.6713	.98481	0	.17365	.17633	5.6713	.98					

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

10° (190°)					(349°) 169°					11° (191°)					(348°) 168°				
/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/		
0	.17365	.17633	5.6713	.98481	60	0	.19081	.19438	5.1446	.98163	60	0	.19081	.19438	5.1446	.98163	60		
1	.17393	.17663	5.6617	.98476	59	1	.19109	.19468	5.1366	.98157	59	1	.19109	.19468	5.1366	.98157	59		
2	.17422	.17693	5.6521	.98471	58	2	.19138	.19498	5.1286	.98152	58	2	.19138	.19498	5.1286	.98152	58		
3	.17451	.17723	5.6425	.98466	57	3	.19167	.19529	5.1207	.98146	57	3	.19167	.19529	5.1207	.98146	57		
4	.17479	.17753	5.6329	.98461	56	4	.19195	.19559	5.1128	.98140	56	4	.19195	.19559	5.1128	.98140	56		
5	.17508	.17783	5.6234	.98455	55	5	.19224	.19589	5.1049	.98135	55	5	.19224	.19589	5.1049	.98135	55		
6	.17537	.17813	5.6140	.98450	54	6	.19252	.19619	5.0970	.98129	54	6	.19252	.19619	5.0970	.98129	54		
7	.17565	.17843	5.6045	.98445	53	7	.19281	.19649	5.0892	.98124	53	7	.19281	.19649	5.0892	.98124	53		
8	.17594	.17873	5.5951	.98440	52	8	.19309	.19680	5.0814	.98118	52	8	.19309	.19680	5.0814	.98118	52		
9	.17623	.17903	5.5857	.98435	51	9	.19338	.19710	5.0736	.98112	51	9	.19338	.19710	5.0736	.98112	51		
10	.17651	.17933	5.5764	.98430	50	10	.19366	.19740	5.0658	.98107	50	10	.19366	.19740	5.0658	.98107	50		
11	.17680	.17963	5.5671	.98425	49	11	.19395	.19770	5.0581	.98101	49	11	.19395	.19770	5.0581	.98101	49		
12	.17708	.17993	5.5578	.98420	48	12	.19423	.19801	5.0504	.98096	48	12	.19423	.19801	5.0504	.98096	48		
13	.17737	.18023	5.5485	.98414	47	13	.19452	.19831	5.0427	.98090	47	13	.19452	.19831	5.0427	.98090	47		
14	.17766	.18053	5.5393	.98409	46	14	.19481	.19861	5.0350	.98084	46	14	.19481	.19861	5.0350	.98084	46		
15	.17794	.18083	5.5301	.98404	45	15	.19509	.19891	5.0273	.98079	45	15	.19509	.19891	5.0273	.98079	45		
16	.17823	.18113	5.5209	.98399	44	16	.19538	.19921	5.0197	.98073	44	16	.19538	.19921	5.0197	.98073	44		
17	.17852	.18143	5.5118	.98394	43	17	.19566	.19952	5.0121	.98067	43	17	.19566	.19952	5.0121	.98067	43		
18	.17880	.18173	5.5026	.98389	42	18	.19595	.19982	5.0045	.98061	42	18	.19595	.19982	5.0045	.98061	42		
19	.17909	.18203	5.4936	.98383	41	19	.19623	.20012	4.9969	.98056	41	19	.19623	.20012	4.9969	.98056	41		
20	.17937	.18233	5.4845	.98378	40	20	.19652	.20042	4.9894	.98050	40	20	.19652	.20042	4.9894	.98050	40		
21	.17966	.18263	5.4755	.98373	39	21	.19680	.20073	4.9819	.98044	39	21	.19680	.20073	4.9819	.98044	39		
22	.17995	.18293	5.4665	.98368	38	22	.19709	.20103	4.9744	.98038	38	22	.19709	.20103	4.9744	.98038	38		
23	.18023	.18323	5.4575	.98362	37	23	.19737	.20133	4.9669	.98033	37	23	.19737	.20133	4.9669	.98033	37		
24	.18052	.18353	5.4486	.98357	36	24	.19766	.20164	4.9594	.98027	36	24	.19766	.20164	4.9594	.98027	36		
25	.18081	.18384	5.4397	.98352	35	25	.19794	.20194	4.9520	.98021	35	25	.19794	.20194	4.9520	.98021	35		
26	.18109	.18414	5.4308	.98347	34	26	.19823	.20224	4.9446	.98016	34	26	.19823	.20224	4.9446	.98016	34		
27	.18138	.18444	5.4219	.98341	33	27	.19851	.20254	4.9372	.98010	33	27	.19851	.20254	4.9372	.98010	33		
28	.18166	.18474	5.4131	.98336	32	28	.19880	.20285	4.9298	.98004	32	28	.19880	.20285	4.9298	.98004	32		
29	.18195	.18504	5.4043	.98331	31	29	.19908	.20315	4.9225	.97998	31	29	.19908	.20315	4.9225	.97998	31		
30	.18224	.18534	5.3955	.98325	30	30	.19937	.20345	4.9152	.97992	30	30	.19937	.20345	4.9152	.97992	30		
31	.18252	.18564	5.3868	.98320	29	31	.19965	.20376	4.9078	.97987	29	31	.19965	.20376	4.9078	.97987	29		
32	.18281	.18594	5.3781	.98315	28	32	.19994	.20406	4.9006	.97981	28	32	.19994	.20406	4.9006	.97981	28		
33	.18309	.18624	5.3694	.98310	27	33	.20022	.20436	4.8933	.97975	27	33	.20022	.20436	4.8933	.97975	27		
34	.18338	.18654	5.3607	.98304	26	34	.20051	.20466	4.8860	.97969	26	34	.20051	.20466	4.8860	.97969	26		
35	.18367	.18684	5.3521	.98299	25	35	.20079	.20497	4.8788	.97963	25	35	.20079	.20497	4.8788	.97963	25		
36	.18395	.18714	5.3435	.98294	24	36	.20108	.20527	4.8716	.97958	24	36	.20108	.20527	4.8716	.97958	24		
37	.18424	.18745	5.3349	.98288	23	37	.20136	.20557	4.8644	.97952	23	37	.20136	.20557	4.8644	.97952	23		
38	.18452	.18775	5.3263	.98283	22	38	.20165	.20588	4.8573	.97946	22	38	.20165	.20588	4.8573	.97946	22		
39	.18481	.18805	5.3178	.98277	21	39	.20193	.20618	4.8501	.97940	21	39	.20193	.20618	4.8501	.97940	21		
40	.18509	.18835	5.3093	.98272	20	40	.20222	.20648	4.8430	.97934	20	40	.20222	.20648	4.8430	.97934	20		
41	.18538	.18865	5.3008	.98267	19	41	.20250	.20679	4.8359	.97928	19	41	.20250	.20679	4.8359	.97928	19		
42	.18567	.18895	5.2924	.98261	18	42	.20279	.20709	4.8288	.97922	18	42	.20279	.20709	4.8288	.97922	18		
43	.18595	.18925	5.2839	.98256	17	43	.20307	.20739	4.8218	.97916	17	43	.20307	.20739	4.8218	.97916	17		
44	.18624	.18955	5.2755	.98250	16	44	.20336	.20770	4.8147	.97910	16	44	.20336	.20770	4.8147	.97910	16		
45	.18652	.18986	5.2672	.98245	15	45	.20364	.20800	4.8077	.97905	15	45	.20364	.20800	4.8077	.97905	15		
46	.18681	.19016	5.2588	.98240	14	46	.20393	.20830	4.8007	.97899	14	46	.20393	.20830	4.8007	.97899	14		
47	.18710	.19046	5.2505	.98234	13	47	.20421	.20861	4.7937	.97893	13	47	.20421	.20861	4.7937	.97893	13		
48	.18738	.19076	5.2422	.98229	12	48	.20450	.20891	4.7867	.97887	12	48	.20450	.20891	4.7867	.97887	12		
49	.18767	.19106	5.2339	.98223	11	49	.20478	.20921	4.7798	.97881	11	49	.20478	.20921	4.7798	.97881	11		
50	.18795	.19136	5.2257	.98218	10	50	.20507	.20952	4.7729	.97875	10	50	.20507	.20952	4.7729	.97875	10		
51	.18824	.19166	5.2174	.98212	9	51	.20535	.20982	4.7659	.97869	9	51	.20535	.20982	4.7659	.97869	9		
52	.18852	.19197	5.2092	.98207	8	52	.20563	.21013	4.7591	.97863	8	52	.20563	.21013	4.7591	.97863	8		
53	.18881	.19227	5.2011	.98201	7	53	.20592	.21043	4.7522	.97857	7	53	.20592	.21043	4.7522	.97857	7		
54	.18910	.19257	5.1929	.98196	6	54	.20620	.21073	4.7453	.97851	6	54	.20620	.21073	4.7453	.97851	6		
55	.18938	.19287	5.1848	.98190	5	55	.20649	.21104	4.7385	.97845	5	55	.20649	.21104	4.7385	.97845	5		
56	.18967	.19317	5.1767	.98185	4	56	.20677	.21134	4.7317	.97839	4	56	.20677	.21134	4.7317	.97839	4		
57	.18995	.19347	5.1686	.98179	3	57	.20706	.21164	4.7249	.97833	3	57	.20706	.21164	4.7249	.97833	3		
58	.19024	.19378	5.1606	.98174	2	58	.20734	.21195	4.7181	.97827	2	58	.20734	.21195	4.7181	.97827	2		
59	.19052	.19408	5.1526	.98168	1	59	.20763	.21225	4.7114	.97821	1	59	.20763	.21225	4.7114	.97821	1		
60	.19081	.19438	5.1446	.98163	0	60	.20791	.21256	4.7046	.97815	0	60	.20791	.21256	4.7046	.97815	0		
/	Cos	Ctn	Tan	Sin	/	/	Cos	Ctn	Tan	Sin	/	/	Cos	Ctn	Tan	Sin	/		
100° (280°)					(259°) 79°					101° (281°)					(258°) 78°				

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

12° (192°)					(347°) 167°					13° (193°)					(346°) 166°						
/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos	/
0	.20791	.21256	4.7046	.97815	60	0	.22495	.23087	4.3315	.97437	60	.22495	.23087	4.3315	.97437	60	.22495	.23087	4.3315	.97437	60
1	.20820	.21286	4.6979	.97809	59	1	.22523	.23117	4.3257	.97430	59	.22523	.23117	4.3257	.97430	59	.22523	.23117	4.3257	.97430	59
2	.20848	.21316	4.6912	.97803	58	2	.22552	.23148	4.3200	.97424	58	.22552	.23148	4.3200	.97424	58	.22552	.23148	4.3200	.97424	58
3	.20877	.21347	4.6845	.97797	57	3	.22580	.23179	4.3143	.97417	57	.22580	.23179	4.3143	.97417	57	.22580	.23179	4.3143	.97417	57
4	.20905	.21377	4.6779	.97791	56	4	.22608	.23209	4.3086	.97411	56	.22608	.23209	4.3086	.97411	56	.22608	.23209	4.3086	.97411	56
5	.20933	.21408	4.6712	.97784	55	5	.22637	.23240	4.3029	.97404	55	.22637	.23240	4.3029	.97404	55	.22637	.23240	4.3029	.97404	55
6	.20962	.21438	4.6646	.97778	54	6	.22665	.23271	4.2972	.97398	54	.22665	.23271	4.2972	.97398	54	.22665	.23271	4.2972	.97398	54
7	.20990	.21469	4.6580	.97772	53	7	.22693	.23301	4.2916	.97391	53	.22693	.23301	4.2916	.97391	53	.22693	.23301	4.2916	.97391	53
8	.21019	.21499	4.6514	.97766	52	8	.22722	.23332	4.2859	.97384	52	.22722	.23332	4.2859	.97384	52	.22722	.23332	4.2859	.97384	52
9	.21047	.21529	4.6448	.97760	51	9	.22750	.23363	4.2803	.97378	51	.22750	.23363	4.2803	.97378	51	.22750	.23363	4.2803	.97378	51
10	.21076	.21560	4.6382	.97754	50	10	.22778	.23393	4.2747	.97371	50	.22778	.23393	4.2747	.97371	50	.22778	.23393	4.2747	.97371	50
11	.21104	.21590	4.6317	.97748	49	11	.22807	.23424	4.2691	.97365	49	.22807	.23424	4.2691	.97365	49	.22807	.23424	4.2691	.97365	49
12	.21132	.21621	4.6252	.97742	48	12	.22835	.23455	4.2635	.97358	48	.22835	.23455	4.2635	.97358	48	.22835	.23455	4.2635	.97358	48
13	.21161	.21651	4.6187	.97735	47	13	.22863	.23485	4.2580	.97351	47	.22863	.23485	4.2580	.97351	47	.22863	.23485	4.2580	.97351	47
14	.21189	.21682	4.6122	.97729	46	14	.22892	.23516	4.2524	.97345	46	.22892	.23516	4.2524	.97345	46	.22892	.23516	4.2524	.97345	46
15	.21218	.21712	4.6057	.97723	45	15	.22920	.23547	4.2468	.97338	45	.22920	.23547	4.2468	.97338	45	.22920	.23547	4.2468	.97338	45
16	.21246	.21743	4.5993	.97717	44	16	.22948	.23578	4.2413	.97331	44	.22948	.23578	4.2413	.97331	44	.22948	.23578	4.2413	.97331	44
17	.21275	.21773	4.5928	.97711	43	17	.22977	.23608	4.2358	.97325	43	.22977	.23608	4.2358	.97325	43	.22977	.23608	4.2358	.97325	43
18	.21303	.21804	4.5864	.97705	42	18	.23005	.23639	4.2303	.97318	42	.23005	.23639	4.2303	.97318	42	.23005	.23639	4.2303	.97318	42
19	.21331	.21834	4.5800	.97698	41	19	.23033	.23670	4.2248	.97311	41	.23033	.23670	4.2248	.97311	41	.23033	.23670	4.2248	.97311	41
20	.21360	.21864	4.5736	.97692	40	20	.23062	.23700	4.2193	.97304	40	.23062	.23700	4.2193	.97304	40	.23062	.23700	4.2193	.97304	40
21	.21388	.21895	4.5673	.97686	39	21	.23090	.23731	4.2139	.97298	39	.23090	.23731	4.2139	.97298	39	.23090	.23731	4.2139	.97298	39
22	.21417	.21925	4.5609	.97680	38	22	.23118	.23762	4.2084	.97291	38	.23118	.23762	4.2084	.97291	38	.23118	.23762	4.2084	.97291	38
23	.21445	.21956	4.5546	.97673	37	23	.23146	.23793	4.2030	.97284	37	.23146	.23793	4.2030	.97284	37	.23146	.23793	4.2030	.97284	37
24	.21474	.21986	4.5483	.97667	36	24	.23175	.23823	4.1976	.97278	36	.23175	.23823	4.1976	.97278	36	.23175	.23823	4.1976	.97278	36
25	.21502	.22017	4.5420	.97661	35	25	.23203	.23854	4.1922	.97271	35	.23203	.23854	4.1922	.97271	35	.23203	.23854	4.1922	.97271	35
26	.21530	.22047	4.5357	.97655	34	26	.23231	.23885	4.1868	.97264	34	.23231	.23885	4.1868	.97264	34	.23231	.23885	4.1868	.97264	34
27	.21559	.22078	4.5294	.97648	33	27	.23260	.23916	4.1814	.97257	33	.23260	.23916	4.1814	.97257	33	.23260	.23916	4.1814	.97257	33
28	.21587	.22108	4.5232	.97642	32	28	.23288	.23946	4.1760	.97251	32	.23288	.23946	4.1760	.97251	32	.23288	.23946	4.1760	.97251	32
29	.21616	.22139	4.5169	.97636	31	29	.23316	.23977	4.1706	.97244	31	.23316	.23977	4.1706	.97244	31	.23316	.23977	4.1706	.97244	31
30	.21644	.22169	4.5107	.97630	30	30	.23345	.24008	4.1653	.97237	30	.23345	.24008	4.1653	.97237	30	.23345	.24008	4.1653	.97237	30
31	.21672	.22200	4.5045	.97623	29	31	.23373	.24039	4.1600	.97230	29	.23373	.24039	4.1600	.97230	29	.23373	.24039	4.1600	.97230	29
32	.21701	.22231	4.4983	.97617	28	32	.23401	.24069	4.1547	.97223	28	.23401	.24069	4.1547	.97223	28	.23401	.24069	4.1547	.97223	28
33	.21729	.22261	4.4922	.97611	27	33	.23429	.24100	4.1493	.97217	27	.23429	.24100	4.1493	.97217	27	.23429	.24100	4.1493	.97217	27
34	.21758	.22292	4.4860	.97604	26	34	.23458	.24131	4.1441	.97210	26	.23458	.24131	4.1441	.97210	26	.23458	.24131	4.1441	.97210	26
35	.21786	.22322	4.4799	.97598	25	35	.23486	.24162	4.1388	.97203	25	.23486	.24162	4.1388	.97203	25	.23486	.24162	4.1388	.97203	25
36	.21814	.22353	4.4737	.97592	24	36	.23514	.24193	4.1335	.97196	24	.23514	.24193	4.1335	.97196	24	.23514	.24193	4.1335	.97196	24
37	.21843	.22383	4.4676	.97585	23	37	.23542	.24223	4.1282	.97189	23	.23542	.24223	4.1282	.97189	23	.23542	.24223	4.1282	.97189	23
38	.21871	.22414	4.4615	.97579	22	38	.23571	.24254	4.1230	.97182	22	.23571	.24254	4.1230	.97182	22	.23571	.24254	4.1230	.97182	22
39	.21899	.22444	4.4555	.97573	21	39	.23599	.24285	4.1178	.97176	21	.23599	.24285	4.1178	.97176	21	.23599	.24285	4.1178	.97176	21
40	.21928	.22475	4.4494	.97566	20	40	.23627	.24316	4.1126	.97169	20	.23627	.24316	4.1126	.97169	20	.23627	.24316	4.1126	.97169	20
41	.21956	.22505	4.4434	.97560	19	41	.23656	.24347	4.1074	.97162	19	.23656	.24347	4.1074	.97162	19	.23656	.24347	4.1074	.97162	19
42	.21985	.22536	4.4373	.97553	18	42	.23684	.24377	4.1022	.97155	18	.23684	.24377	4.1022	.97155	18	.23684	.24377	4.1022	.97155	18
43	.22013	.22567	4.4313	.97547	17	43	.23712	.24408	4.0970	.97148	17	.23712	.24408	4.0970	.97148	17	.23712	.24408	4.0970	.97148	17
44	.22041	.22597	4.4253	.97541	16	44	.23740	.24439	4.0918	.97141	16	.23740	.24439	4.0918	.97141	16	.23740	.24439	4.0918	.97141	16
45	.22070	.22628	4.4194	.97534	15	45	.23769	.24470	4.0867	.97134	15	.23769	.24470	4.0867	.97134	15	.23769	.24470	4.0867	.97134	15
46	.22098	.22658	4.4134	.97528	14	46	.23797	.24501	4.0815	.97127	14	.23797	.24501	4.0815	.97127	14	.23797	.24501	4.0815	.97127	14
47	.22126	.22689	4.4075	.97521	13	47	.23825	.24532	4.0764	.97120	13	.23825	.24532	4.0764	.97120	13	.23825	.24532	4.0764	.97120	13
48	.22155	.22719	4.4015	.97515	12	48	.23853	.24562	4.0713	.97113	12	.23853	.24562	4.0713	.97113	12	.23853	.24562	4.0713	.97113	12
49	.22183	.22750	4.3956	.97508	11	49	.23882	.24593	4.0662	.97106	11	.23882	.24593	4.0662	.97106	11	.23882	.24593	4.0662	.97106	11
50	.22212	.22781	4.3897	.97502	10	50	.23910	.24624	4.0611	.97100	10	.23910	.24624	4.0611	.97100	10	.23910	.24624	4.0611	.97100	10
51	.22240	.22811	4.3838	.97496	9	51	.23938	.24655	4.0560	.97093	9	.23938	.24655	4.0560	.97093	9	.23938	.24655	4.0560	.97093	9
52	.22268	.22842	4.3779	.97489	8	52	.23966	.24686	4.0509	.97086	8	.23966	.24686	4.0509	.97086	8	.23966	.24686	4.0509	.97086	8
53	.22297	.22872	4.3721	.97483	7	53	.23995	.24717	4.0459	.97079	7	.23995	.24717	4.0459	.97079	7	.23995	.24717	4.0459	.97079	7
54	.22325	.22903	4.3662	.97476	6	54	.24023	.24747	4.0408	.97072	6	.24023	.24747	4.0408	.97072	6	.24023	.24747	4.0408	.97072	6
55	.22353	.22934	4.3604	.97470	5	55	.24051	.24778	4.0358	.97065	5	.24051	.24778	4.0358	.97065	5	.24051	.24778	4.0358	.97065	5
56	.22382	.22964	4.3546	.97463	4	56	.24079	.24809	4.0308	.97058	4	.24079	.24809	4.0							

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

14° (194°)					(345°) 165°					15° (195°)					(344°) 164°						
/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos	/
0	.24192	.24933	4.0108	.97030	60	0	.25882	.26795	3.7321	.96593	60	.25882	.26795	3.7321	.96593	60	.25882	.26795	3.7321	.96593	60
1	.24220	.24964	4.0058	.97023	59	1	.25910	.26826	3.7277	.96585	59	.25910	.26826	3.7277	.96585	59	.25910	.26826	3.7277	.96585	59
2	.24249	.24995	4.0009	.97015	58	2	.25938	.26857	3.7234	.96578	58	.25938	.26857	3.7234	.96578	58	.25938	.26857	3.7234	.96578	58
3	.24277	.25026	3.9959	.97008	57	3	.25966	.26888	3.7191	.96570	57	.25966	.26888	3.7191	.96570	57	.25966	.26888	3.7191	.96570	57
4	.24305	.25056	3.9910	.97001	56	4	.25994	.26920	3.7148	.96562	56	.25994	.26920	3.7148	.96562	56	.25994	.26920	3.7148	.96562	56
5	.24333	.25087	3.9861	.96994	55	5	.26022	.26951	3.7105	.96555	55	.26022	.26951	3.7105	.96555	55	.26022	.26951	3.7105	.96555	55
6	.24362	.25118	3.9812	.96987	54	6	.26050	.26982	3.7062	.96547	54	.26050	.26982	3.7062	.96547	54	.26050	.26982	3.7062	.96547	54
7	.24390	.25149	3.9763	.96980	53	7	.26079	.27013	3.7019	.96540	53	.26079	.27013	3.7019	.96540	53	.26079	.27013	3.7019	.96540	53
8	.24418	.25180	3.9714	.96973	52	8	.26107	.27044	3.6976	.96532	52	.26107	.27044	3.6976	.96532	52	.26107	.27044	3.6976	.96532	52
9	.24446	.25211	3.9665	.96966	51	9	.26135	.27076	3.6933	.96524	51	.26135	.27076	3.6933	.96524	51	.26135	.27076	3.6933	.96524	51
10	.24474	.25242	3.9617	.96959	50	10	.26163	.27107	3.6891	.96517	50	.26163	.27107	3.6891	.96517	50	.26163	.27107	3.6891	.96517	50
11	.24503	.25273	3.9568	.96952	49	11	.26191	.27138	3.6848	.96509	49	.26191	.27138	3.6848	.96509	49	.26191	.27138	3.6848	.96509	49
12	.24531	.25304	3.9520	.96945	48	12	.26219	.27169	3.6806	.96502	48	.26219	.27169	3.6806	.96502	48	.26219	.27169	3.6806	.96502	48
13	.24559	.25335	3.9471	.96937	47	13	.26247	.27201	3.6764	.96494	47	.26247	.27201	3.6764	.96494	47	.26247	.27201	3.6764	.96494	47
14	.24587	.25366	3.9423	.96930	46	14	.26275	.27232	3.6722	.96486	46	.26275	.27232	3.6722	.96486	46	.26275	.27232	3.6722	.96486	46
15	.24615	.25397	3.9375	.96923	45	15	.26303	.27263	3.6680	.96479	45	.26303	.27263	3.6680	.96479	45	.26303	.27263	3.6680	.96479	45
16	.24644	.25428	3.9327	.96916	44	16	.26331	.27294	3.6638	.96471	44	.26331	.27294	3.6638	.96471	44	.26331	.27294	3.6638	.96471	44
17	.24672	.25459	3.9279	.96909	43	17	.26359	.27326	3.6596	.96463	43	.26359	.27326	3.6596	.96463	43	.26359	.27326	3.6596	.96463	43
18	.24700	.25490	3.9232	.96902	42	18	.26387	.27357	3.6554	.96456	42	.26387	.27357	3.6554	.96456	42	.26387	.27357	3.6554	.96456	42
19	.24728	.25521	3.9184	.96894	41	19	.26415	.27388	3.6512	.96448	41	.26415	.27388	3.6512	.96448	41	.26415	.27388	3.6512	.96448	41
20	.24756	.25552	3.9136	.96887	40	20	.26443	.27419	3.6470	.96440	40	.26443	.27419	3.6470	.96440	40	.26443	.27419	3.6470	.96440	40
21	.24784	.25583	3.9089	.96880	39	21	.26471	.27451	3.6429	.96433	39	.26471	.27451	3.6429	.96433	39	.26471	.27451	3.6429	.96433	39
22	.24813	.25614	3.9042	.96873	38	22	.26500	.27482	3.6387	.96425	38	.26500	.27482	3.6387	.96425	38	.26500	.27482	3.6387	.96425	38
23	.24841	.25645	3.8995	.96866	37	23	.26528	.27513	3.6346	.96417	37	.26528	.27513	3.6346	.96417	37	.26528	.27513	3.6346	.96417	37
24	.24869	.25676	3.8947	.96858	36	24	.26556	.27545	3.6305	.96410	36	.26556	.27545	3.6305	.96410	36	.26556	.27545	3.6305	.96410	36
25	.24897	.25707	3.8900	.96851	35	25	.26584	.27576	3.6264	.96402	35	.26584	.27576	3.6264	.96402	35	.26584	.27576	3.6264	.96402	35
26	.24925	.25738	3.8854	.96844	34	26	.26612	.27607	3.6222	.96394	34	.26612	.27607	3.6222	.96394	34	.26612	.27607	3.6222	.96394	34
27	.24954	.25769	3.8807	.96837	33	27	.26640	.27638	3.6181	.96386	33	.26640	.27638	3.6181	.96386	33	.26640	.27638	3.6181	.96386	33
28	.24982	.25800	3.8760	.96829	32	28	.26668	.27670	3.6140	.96379	32	.26668	.27670	3.6140	.96379	32	.26668	.27670	3.6140	.96379	32
29	.25010	.25831	3.8714	.96822	31	29	.26696	.27701	3.6100	.96371	31	.26696	.27701	3.6100	.96371	31	.26696	.27701	3.6100	.96371	31
30	.25038	.25862	3.8667	.96815	30	30	.26724	.27732	3.6059	.96363	30	.26724	.27732	3.6059	.96363	30	.26724	.27732	3.6059	.96363	30
31	.25066	.25893	3.8621	.96807	29	31	.26752	.27764	3.6018	.96355	29	.26752	.27764	3.6018	.96355	29	.26752	.27764	3.6018	.96355	29
32	.25094	.25924	3.8575	.96800	28	32	.26780	.27795	3.5978	.96347	28	.26780	.27795	3.5978	.96347	28	.26780	.27795	3.5978	.96347	28
33	.25122	.25955	3.8528	.96793	27	33	.26808	.27826	3.5937	.96340	27	.26808	.27826	3.5937	.96340	27	.26808	.27826	3.5937	.96340	27
34	.25151	.25986	3.8482	.96786	26	34	.26836	.27858	3.5897	.96332	26	.26836	.27858	3.5897	.96332	26	.26836	.27858	3.5897	.96332	26
35	.25179	.26017	3.8436	.96778	25	35	.26864	.27889	3.5856	.96324	25	.26864	.27889	3.5856	.96324	25	.26864	.27889	3.5856	.96324	25
36	.25207	.26048	3.8391	.96771	24	36	.26892	.27921	3.5816	.96316	24	.26892	.27921	3.5816	.96316	24	.26892	.27921	3.5816	.96316	24
37	.25235	.26079	3.8345	.96764	23	37	.26920	.27952	3.5776	.96308	23	.26920	.27952	3.5776	.96308	23	.26920	.27952	3.5776	.96308	23
38	.25263	.26110	3.8299	.96756	22	38	.26948	.27983	3.5736	.96301	22	.26948	.27983	3.5736	.96301	22	.26948	.27983	3.5736	.96301	22
39	.25291	.26141	3.8254	.96749	21	39	.26976	.28015	3.5696	.96293	21	.26976	.28015	3.5696	.96293	21	.26976	.28015	3.5696	.96293	21
40	.25320	.26172	3.8208	.96742	20	40	.27004	.28046	3.5656	.96285	20	.27004	.28046	3.5656	.96285	20	.27004	.28046	3.5656	.96285	20
41	.25348	.26203	3.8163	.96734	19	41	.27032	.28077	3.5616	.96277	19	.27032	.28077	3.5616	.96277	19	.27032	.28077	3.5616	.96277	19
42	.25376	.26235	3.8118	.96727	18	42	.27060	.28109	3.5576	.96269	18	.27060	.28109	3.5576	.96269	18	.27060	.28109	3.5576	.96269	18
43	.25404	.26266	3.8073	.96719	17	43	.27088	.28140	3.5536	.96261	17	.27088	.28140	3.5536	.96261	17	.27088	.28140	3.5536	.96261	17
44	.25432	.26297	3.8028	.96712	16	44	.27116	.28172	3.5497	.96253	16	.27116	.28172	3.5497	.96253	16	.27116	.28172	3.5497	.96253	16
45	.25460	.26328	3.7983	.96705	15	45	.27144	.28203	3.5457	.96246	15	.27144	.28203	3.5457	.96246	15	.27144	.28203	3.5457	.96246	15
46	.25488	.26359	3.7938	.96697	14	46	.27172	.28234	3.5418	.96238	14	.27172	.28234	3.5418	.96238	14	.27172	.28234	3.5418	.96238	14
47	.25516	.26390	3.7893	.96690	13	47	.27200	.28266	3.5379	.96230	13	.27200	.28266	3.5379	.96230	13	.27200	.28266	3.5379	.96230	13
48	.25545	.26421	3.7848	.96682	12	48	.27228	.28297	3.5339	.96222	12	.27228	.28297	3.5339	.96222	12	.27228	.28297	3.5339	.96222	12
49	.25573	.26452	3.7804	.96675	11	49	.27256	.28329	3.5300	.96214	11	.27256	.28329	3.5300	.96214	11	.27256	.28329	3.5300	.96214	11
50	.25601	.26483	3.7760	.96667	10	50	.27284	.28360	3.5261	.96206	10	.27284	.28360	3.5261	.96206	10	.27284	.28360	3.5261	.96206	10
51	.25629	.26515	3.7715	.96660	9	51	.27312	.28391	3.5222	.96198	9	.27312	.28391	3.5222	.96198	9	.27312	.28391	3.5222	.96198	9
52	.25657	.26546	3.7671	.96653	8	52	.27340	.28423	3.5183	.96190	8	.27340	.28423	3.5183	.96190	8	.27340	.28423	3.5183	.96190	8
53	.25685	.26577	3.7627	.96645	7	53	.27368	.28454	3.5144	.96182	7	.27368	.28454	3.5144	.96182	7	.27368	.28454	3.5144	.96182	7
54	.25713	.26608	3.7583	.96638	6	54	.27396	.28486	3.5105	.96174	6	.27396	.28486	3.5105	.96174	6	.27396	.28486	3.5105	.96174	6
55	.25741	.26639	3.7539	.96630	5	55	.27424	.28517	3.5067	.96166	5	.27424	.28517	3.5067	.96166	5	.27424	.28517	3.5067	.96166	5
56	.25769	.26670	3.7495	.96623	4	56	.27452	.28549	3.5028	.96158	4	.27452	.28549	3							

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

16° (196°)					(343°) 163°					17° (197°)					(342°) 162°						
/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos	/
0	.27564	.28675	3.4874	.96126	60	0	.29237	.30573	3.2709	.95630	60	.29237	.30573	3.2709	.95630	60	.29237	.30573	3.2709	.95630	60
1	.27592	.28706	3.4836	.96118	59	1	.29265	.30605	3.2675	.95622	59	.29265	.30605	3.2675	.95622	59	.29265	.30605	3.2675	.95622	59
2	.27620	.28738	3.4798	.96110	58	2	.29293	.30637	3.2641	.95613	58	.29293	.30637	3.2641	.95613	58	.29293	.30637	3.2641	.95613	58
3	.27648	.28769	3.4760	.96102	57	3	.29321	.30669	3.2607	.95605	57	.29321	.30669	3.2607	.95605	57	.29321	.30669	3.2607	.95605	57
4	.27676	.28801	3.4722	.96094	56	4	.29348	.30700	3.2573	.95596	56	.29348	.30700	3.2573	.95596	56	.29348	.30700	3.2573	.95596	56
5	.27704	.28832	3.4684	.96086	55	5	.29376	.30732	3.2539	.95588	55	.29376	.30732	3.2539	.95588	55	.29376	.30732	3.2539	.95588	55
6	.27731	.28864	3.4646	.96078	54	6	.29404	.30764	3.2506	.95579	54	.29404	.30764	3.2506	.95579	54	.29404	.30764	3.2506	.95579	54
7	.27759	.28895	3.4608	.96070	53	7	.29432	.30796	3.2472	.95571	53	.29432	.30796	3.2472	.95571	53	.29432	.30796	3.2472	.95571	53
8	.27787	.28927	3.4570	.96062	52	8	.29460	.30828	3.2438	.95562	52	.29460	.30828	3.2438	.95562	52	.29460	.30828	3.2438	.95562	52
9	.27815	.28958	3.4533	.96054	51	9	.29487	.30860	3.2405	.95554	51	.29487	.30860	3.2405	.95554	51	.29487	.30860	3.2405	.95554	51
10	.27843	.28990	3.4495	.96046	50	10	.29515	.30891	3.2371	.95545	50	.29515	.30891	3.2371	.95545	50	.29515	.30891	3.2371	.95545	50
11	.27871	.29021	3.4458	.96037	49	11	.29543	.30923	3.2338	.95536	49	.29543	.30923	3.2338	.95536	49	.29543	.30923	3.2338	.95536	49
12	.27899	.29053	3.4420	.96029	48	12	.29571	.30955	3.2305	.95528	48	.29571	.30955	3.2305	.95528	48	.29571	.30955	3.2305	.95528	48
13	.27927	.29084	3.4383	.96021	47	13	.29599	.30987	3.2272	.95519	47	.29599	.30987	3.2272	.95519	47	.29599	.30987	3.2272	.95519	47
14	.27955	.29116	3.4346	.96013	46	14	.29626	.31019	3.2238	.95511	46	.29626	.31019	3.2238	.95511	46	.29626	.31019	3.2238	.95511	46
15	.27983	.29147	3.4308	.96005	45	15	.29654	.31051	3.2205	.95502	45	.29654	.31051	3.2205	.95502	45	.29654	.31051	3.2205	.95502	45
16	.28011	.29179	3.4271	.95997	44	16	.29682	.31083	3.2172	.95493	44	.29682	.31083	3.2172	.95493	44	.29682	.31083	3.2172	.95493	44
17	.28039	.29210	3.4234	.95989	43	17	.29710	.31115	3.2139	.95485	43	.29710	.31115	3.2139	.95485	43	.29710	.31115	3.2139	.95485	43
18	.28067	.29242	3.4197	.95981	42	18	.29737	.31147	3.2106	.95476	42	.29737	.31147	3.2106	.95476	42	.29737	.31147	3.2106	.95476	42
19	.28095	.29274	3.4160	.95972	41	19	.29765	.31178	3.2073	.95467	41	.29765	.31178	3.2073	.95467	41	.29765	.31178	3.2073	.95467	41
20	.28123	.29305	3.4124	.95964	40	20	.29793	.31210	3.2041	.95459	40	.29793	.31210	3.2041	.95459	40	.29793	.31210	3.2041	.95459	40
21	.28150	.29337	3.4087	.95956	39	21	.29821	.31242	3.2008	.95450	39	.29821	.31242	3.2008	.95450	39	.29821	.31242	3.2008	.95450	39
22	.28178	.29368	3.4050	.95948	38	22	.29849	.31274	3.1975	.95441	38	.29849	.31274	3.1975	.95441	38	.29849	.31274	3.1975	.95441	38
23	.28206	.29400	3.4014	.95940	37	23	.29876	.31306	3.1943	.95433	37	.29876	.31306	3.1943	.95433	37	.29876	.31306	3.1943	.95433	37
24	.28234	.29432	3.3977	.95931	36	24	.29904	.31338	3.1910	.95424	36	.29904	.31338	3.1910	.95424	36	.29904	.31338	3.1910	.95424	36
25	.28262	.29463	3.3941	.95923	35	25	.29932	.31370	3.1878	.95415	35	.29932	.31370	3.1878	.95415	35	.29932	.31370	3.1878	.95415	35
26	.28290	.29495	3.3904	.95915	34	26	.29960	.31402	3.1845	.95407	34	.29960	.31402	3.1845	.95407	34	.29960	.31402	3.1845	.95407	34
27	.28318	.29526	3.3868	.95907	33	27	.29987	.31434	3.1813	.95398	33	.29987	.31434	3.1813	.95398	33	.29987	.31434	3.1813	.95398	33
28	.28346	.29558	3.3832	.95898	32	28	.30015	.31466	3.1780	.95389	32	.30015	.31466	3.1780	.95389	32	.30015	.31466	3.1780	.95389	32
29	.28374	.29590	3.3796	.95890	31	29	.30043	.31498	3.1748	.95380	31	.30043	.31498	3.1748	.95380	31	.30043	.31498	3.1748	.95380	31
30	.28402	.29621	3.3759	.95882	30	30	.30071	.31530	3.1716	.95372	30	.30071	.31530	3.1716	.95372	30	.30071	.31530	3.1716	.95372	30
31	.28429	.29653	3.3723	.95874	29	31	.30098	.31562	3.1684	.95363	29	.30098	.31562	3.1684	.95363	29	.30098	.31562	3.1684	.95363	29
32	.28457	.29685	3.3687	.95865	28	32	.30126	.31594	3.1652	.95354	28	.30126	.31594	3.1652	.95354	28	.30126	.31594	3.1652	.95354	28
33	.28485	.29716	3.3652	.95857	27	33	.30154	.31626	3.1620	.95345	27	.30154	.31626	3.1620	.95345	27	.30154	.31626	3.1620	.95345	27
34	.28513	.29748	3.3616	.95849	26	34	.30182	.31658	3.1588	.95337	26	.30182	.31658	3.1588	.95337	26	.30182	.31658	3.1588	.95337	26
35	.28541	.29780	3.3580	.95841	25	35	.30209	.31690	3.1556	.95328	25	.30209	.31690	3.1556	.95328	25	.30209	.31690	3.1556	.95328	25
36	.28569	.29811	3.3544	.95832	24	36	.30237	.31722	3.1524	.95319	24	.30237	.31722	3.1524	.95319	24	.30237	.31722	3.1524	.95319	24
37	.28597	.29843	3.3509	.95824	23	37	.30265	.31754	3.1492	.95310	23	.30265	.31754	3.1492	.95310	23	.30265	.31754	3.1492	.95310	23
38	.28625	.29875	3.3473	.95816	22	38	.30292	.31786	3.1460	.95301	22	.30292	.31786	3.1460	.95301	22	.30292	.31786	3.1460	.95301	22
39	.28652	.29906	3.3438	.95807	21	39	.30320	.31818	3.1429	.95293	21	.30320	.31818	3.1429	.95293	21	.30320	.31818	3.1429	.95293	21
40	.28680	.29938	3.3402	.95799	20	40	.30348	.31850	3.1397	.95284	20	.30348	.31850	3.1397	.95284	20	.30348	.31850	3.1397	.95284	20
41	.28708	.29970	3.3367	.95791	19	41	.30376	.31882	3.1366	.95275	19	.30376	.31882	3.1366	.95275	19	.30376	.31882	3.1366	.95275	19
42	.28736	.30001	3.3332	.95782	18	42	.30403	.31914	3.1334	.95266	18	.30403	.31914	3.1334	.95266	18	.30403	.31914	3.1334	.95266	18
43	.28764	.30033	3.3297	.95774	17	43	.30431	.31946	3.1303	.95257	17	.30431	.31946	3.1303	.95257	17	.30431	.31946	3.1303	.95257	17
44	.28792	.30065	3.3261	.95766	16	44	.30459	.31978	3.1271	.95248	16	.30459	.31978	3.1271	.95248	16	.30459	.31978	3.1271	.95248	16
45	.28820	.30097	3.3226	.95757	15	45	.30486	.32010	3.1240	.95240	15	.30486	.32010	3.1240	.95240	15	.30486	.32010	3.1240	.95240	15
46	.28847	.30128	3.3191	.95749	14	46	.30514	.32042	3.1209	.95231	14	.30514	.32042	3.1209	.95231	14	.30514	.32042	3.1209	.95231	14
47	.28875	.30160	3.3156	.95740	13	47	.30542	.32074	3.1178	.95222	13	.30542	.32074	3.1178	.95222	13	.30542	.32074	3.1178	.95222	13
48	.28903	.30192	3.3122	.95732	12	48	.30570	.32106	3.1146	.95213	12	.30570	.32106	3.1146	.95213	12	.30570	.32106	3.1146	.95213	12
49	.28931	.30224	3.3087	.95724	11	49	.30597	.32139	3.1115	.95204	11	.30597	.32139	3.1115	.95204	11	.30597	.32139	3.1115	.95204	11
50	.28959	.30255	3.3052	.95715	10	50	.30625	.32171	3.1084	.95195	10	.30625	.32171	3.1084	.95195	10	.30625	.32171	3.1084	.95195	10
51	.28987	.30287	3.3017	.95707	9	51	.30653	.32203	3.1053	.95186	9	.30653	.32203	3.1053	.95186	9	.30653	.32203	3.1053	.95186	9
52	.29015	.30319	3.2983	.95698	8	52	.30680	.32235	3.1022	.95177	8	.30680	.32235	3.1022	.95177	8	.30680	.32235	3.1022	.95177	8
53	.29042	.30351	3.2948	.95690	7	53	.30708	.32267	3.0991	.95168	7	.30708	.32267	3.0991	.95168	7	.30708	.32267	3.0991	.95168	7
54	.29070	.30382	3.2914	.95681	6	54	.30736	.32299	3.0961	.95159	6	.30736	.32299	3.0961	.95159	6	.30736	.32299	3.0961	.95159	6
55	.29098	.30414	3.2879	.95673	5	55	.30763	.32331	3.0930	.95150	5	.30763	.32331	3.0930	.95150	5	.30763	.32331	3.0930	.95150	5
56	.29126	.30446	3.2845	.95664	4	56	.30791	.32363	3.0899	.95142	4	.30791	.32363	3							

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

18° (198°)						(341°) 161°						19° (199°)						(340°) 160°					
/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/
0	.30902	.32492	3.0777	.95106	60	0	.32557	.34433	2.9042	.94552	60	0	.32557	.34433	2.9042	.94552	60	0	.32557	.34433	2.9042	.94552	60
1	.30929	.32524	3.0746	.95097	59	1	.32584	.34465	2.9015	.94542	59	1	.32584	.34465	2.9015	.94542	59	1	.32584	.34465	2.9015	.94542	59
2	.30957	.32556	3.0716	.95088	58	2	.32612	.34498	2.8987	.94533	58	2	.32612	.34498	2.8987	.94533	58	2	.32612	.34498	2.8987	.94533	58
3	.30985	.32588	3.0686	.95079	57	3	.32639	.34530	2.8960	.94523	57	3	.32639	.34530	2.8960	.94523	57	3	.32639	.34530	2.8960	.94523	57
4	.31012	.32621	3.0655	.95070	56	4	.32667	.34563	2.8933	.94514	56	4	.32667	.34563	2.8933	.94514	56	4	.32667	.34563	2.8933	.94514	56
5	.31040	.32653	3.0625	.95061	55	5	.32694	.34596	2.8905	.94504	55	5	.32694	.34596	2.8905	.94504	55	5	.32694	.34596	2.8905	.94504	55
6	.31068	.32685	3.0595	.95052	54	6	.32722	.34628	2.8878	.94495	54	6	.32722	.34628	2.8878	.94495	54	6	.32722	.34628	2.8878	.94495	54
7	.31095	.32717	3.0565	.95043	53	7	.32749	.34661	2.8851	.94485	53	7	.32749	.34661	2.8851	.94485	53	7	.32749	.34661	2.8851	.94485	53
8	.31123	.32749	3.0535	.95033	52	8	.32777	.34693	2.8824	.94476	52	8	.32777	.34693	2.8824	.94476	52	8	.32777	.34693	2.8824	.94476	52
9	.31151	.32782	3.0505	.95024	51	9	.32804	.34726	2.8797	.94466	51	9	.32804	.34726	2.8797	.94466	51	9	.32804	.34726	2.8797	.94466	51
10	.31178	.32814	3.0475	.95015	50	10	.32832	.34758	2.8770	.94457	50	10	.32832	.34758	2.8770	.94457	50	10	.32832	.34758	2.8770	.94457	50
11	.31206	.32846	3.0445	.95006	49	11	.32859	.34791	2.8743	.94447	49	11	.32859	.34791	2.8743	.94447	49	11	.32859	.34791	2.8743	.94447	49
12	.31233	.32878	3.0415	.94997	48	12	.32887	.34824	2.8716	.94438	48	12	.32887	.34824	2.8716	.94438	48	12	.32887	.34824	2.8716	.94438	48
13	.31261	.32911	3.0385	.94988	47	13	.32914	.34856	2.8689	.94428	47	13	.32914	.34856	2.8689	.94428	47	13	.32914	.34856	2.8689	.94428	47
14	.31289	.32943	3.0356	.94979	46	14	.32942	.34889	2.8662	.94418	46	14	.32942	.34889	2.8662	.94418	46	14	.32942	.34889	2.8662	.94418	46
15	.31316	.32975	3.0326	.94970	45	15	.32969	.34922	2.8636	.94409	45	15	.32969	.34922	2.8636	.94409	45	15	.32969	.34922	2.8636	.94409	45
16	.31344	.33007	3.0296	.94961	44	16	.32997	.34954	2.8609	.94399	44	16	.32997	.34954	2.8609	.94399	44	16	.32997	.34954	2.8609	.94399	44
17	.31372	.33040	3.0267	.94952	43	17	.33024	.34987	2.8582	.94390	43	17	.33024	.34987	2.8582	.94390	43	17	.33024	.34987	2.8582	.94390	43
18	.31399	.33072	3.0237	.94943	42	18	.33051	.35020	2.8556	.94380	42	18	.33051	.35020	2.8556	.94380	42	18	.33051	.35020	2.8556	.94380	42
19	.31427	.33104	3.0208	.94933	41	19	.33079	.35052	2.8529	.94370	41	19	.33079	.35052	2.8529	.94370	41	19	.33079	.35052	2.8529	.94370	41
20	.31454	.33136	3.0178	.94924	40	20	.33106	.35085	2.8502	.94361	40	20	.33106	.35085	2.8502	.94361	40	20	.33106	.35085	2.8502	.94361	40
21	.31482	.33169	3.0149	.94915	39	21	.33134	.35118	2.8476	.94351	39	21	.33134	.35118	2.8476	.94351	39	21	.33134	.35118	2.8476	.94351	39
22	.31510	.33201	3.0120	.94906	38	22	.33161	.35150	2.8449	.94342	38	22	.33161	.35150	2.8449	.94342	38	22	.33161	.35150	2.8449	.94342	38
23	.31537	.33233	3.0090	.94897	37	23	.33189	.35183	2.8423	.94332	37	23	.33189	.35183	2.8423	.94332	37	23	.33189	.35183	2.8423	.94332	37
24	.31565	.33266	3.0061	.94888	36	24	.33216	.35216	2.8397	.94322	36	24	.33216	.35216	2.8397	.94322	36	24	.33216	.35216	2.8397	.94322	36
25	.31593	.33298	3.0032	.94878	35	25	.33244	.35248	2.8370	.94313	35	25	.33244	.35248	2.8370	.94313	35	25	.33244	.35248	2.8370	.94313	35
26	.31620	.33330	3.0003	.94869	34	26	.33271	.35281	2.8344	.94303	34	26	.33271	.35281	2.8344	.94303	34	26	.33271	.35281	2.8344	.94303	34
27	.31648	.33363	2.9974	.94860	33	27	.33298	.35314	2.8318	.94293	33	27	.33298	.35314	2.8318	.94293	33	27	.33298	.35314	2.8318	.94293	33
28	.31675	.33395	2.9945	.94851	32	28	.33326	.35346	2.8291	.94284	32	28	.33326	.35346	2.8291	.94284	32	28	.33326	.35346	2.8291	.94284	32
29	.31703	.33427	2.9916	.94842	31	29	.33353	.35379	2.8265	.94274	31	29	.33353	.35379	2.8265	.94274	31	29	.33353	.35379	2.8265	.94274	31
30	.31730	.33460	2.9887	.94832	30	30	.33381	.35412	2.8239	.94264	30	30	.33381	.35412	2.8239	.94264	30	30	.33381	.35412	2.8239	.94264	30
31	.31758	.33492	2.9858	.94823	29	31	.33408	.35445	2.8213	.94254	29	31	.33408	.35445	2.8213	.94254	29	31	.33408	.35445	2.8213	.94254	29
32	.31786	.33524	2.9829	.94814	28	32	.33436	.35477	2.8187	.94245	28	32	.33436	.35477	2.8187	.94245	28	32	.33436	.35477	2.8187	.94245	28
33	.31813	.33557	2.9800	.94805	27	33	.33463	.35510	2.8161	.94235	27	33	.33463	.35510	2.8161	.94235	27	33	.33463	.35510	2.8161	.94235	27
34	.31841	.33589	2.9772	.94795	26	34	.33490	.35543	2.8135	.94225	26	34	.33490	.35543	2.8135	.94225	26	34	.33490	.35543	2.8135	.94225	26
35	.31868	.33621	2.9743	.94786	25	35	.33518	.35576	2.8109	.94215	25	35	.33518	.35576	2.8109	.94215	25	35	.33518	.35576	2.8109	.94215	25
36	.31896	.33654	2.9714	.94777	24	36	.33545	.35608	2.8083	.94206	24	36	.33545	.35608	2.8083	.94206	24	36	.33545	.35608	2.8083	.94206	24
37	.31923	.33686	2.9686	.94768	23	37	.33573	.35641	2.8057	.94196	23	37	.33573	.35641	2.8057	.94196	23	37	.33573	.35641	2.8057	.94196	23
38	.31951	.33718	2.9657	.94758	22	38	.33600	.35674	2.8032	.94186	22	38	.33600	.35674	2.8032	.94186	22	38	.33600	.35674	2.8032	.94186	22
39	.31979	.33751	2.9629	.94749	21	39	.33627	.35707	2.8006	.94176	21	39	.33627	.35707	2.8006	.94176	21	39	.33627	.35707	2.8006	.94176	21
40	.32006	.33783	2.9600	.94740	20	40	.33655	.35740	2.7980	.94167	20	40	.33655	.35740	2.7980	.94167	20	40	.33655	.35740	2.7980	.94167	20
41	.32034	.33816	2.9572	.94730	19	41	.33682	.35772	2.7955	.94157	19	41	.33682	.35772	2.7955	.94157	19	41	.33682	.35772	2.7955	.94157	19
42	.32061	.33848	2.9544	.94721	18	42	.33710	.35805	2.7929	.94147	18	42	.33710	.35805	2.7929	.94147	18	42	.33710	.35805	2.7929	.94147	18
43	.32089	.33881	2.9515	.94712	17	43	.33737	.35838	2.7903	.94137	17	43	.33737	.35838	2.7903	.94137	17	43	.33737	.35838	2.7903	.94137	17
44	.32116	.33913	2.9487	.94702	16	44	.33764	.35871	2.7878	.94127	16	44	.33764	.35871	2.7878	.94127	16	44	.33764	.35871	2.7878	.94127	16
45	.32144	.33945	2.9459	.94693	15	45	.33792	.35904	2.7852	.94118	15	45	.33792	.35904	2.7852	.94118	15	45	.33792	.35904	2.7852	.94118	15
46	.32171	.33978	2.9431	.94684	14	46	.33819	.35937	2.7827	.94108	14	46	.33819	.35937	2.7827	.94108	14	46	.33819	.35937	2.7827	.94108	14
47	.32199	.34010	2.9403	.94674	13	47	.33846	.35969	2.7801	.94098	13	47	.33846	.35969	2.7801	.94098	13	47	.33846	.35969	2.7801	.94098	13
48	.32227	.34043	2.9375	.94665	12	48	.33874	.36002	2.7776	.94088	12	48	.33874	.36002	2.7776	.94088	12	48	.33874	.36002	2.7776	.94088	12
49	.32254	.34075	2.9347	.94656	11	49	.33901	.36035	2.7751	.94078	11	49	.33901	.36035	2.7751	.94078	11	49	.33901	.36035	2.7751	.94078	11
50	.32282	.34108	2.9319	.94646	10	50	.33929	.36068	2.7725	.94068	10	50	.33929	.36068	2.7725	.94068	10	50	.33929	.36068	2.7725	.94068	10
51	.32309	.34140	2.9291	.94637	9	51	.33956	.36101	2.7700	.94058	9	51	.33956	.36101	2.7700	.94058	9	51	.33956	.36101	2.7700	.94058	9
52	.32337	.34173	2.9263	.94627	8	52	.33983	.36134	2.7675	.94049	8	52	.33983	.36134	2.7675	.94049	8	52	.33983	.36134	2.7675	.94049	

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

20° (200°)					(339°) 159°					21° (201°)					(338°) 158°						
/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos	/
0	.34202	.36397	2.7475	.93969	60	0	.35837	.38386	2.6051	.93358	60	.35837	.38386	2.6051	.93358	60	.35837	.38386	2.6051	.93358	60
1	.34229	.36430	2.7450	.93959	59	1	.35864	.38420	2.6028	.93348	59	.35864	.38420	2.6028	.93348	59	.35864	.38420	2.6028	.93348	59
2	.34257	.36463	2.7425	.93949	58	2	.35891	.38453	2.6006	.93337	58	.35891	.38453	2.6006	.93337	58	.35891	.38453	2.6006	.93337	58
3	.34284	.36496	2.7400	.93939	57	3	.35918	.38487	2.5983	.93327	57	.35918	.38487	2.5983	.93327	57	.35918	.38487	2.5983	.93327	57
4	.34311	.36529	2.7376	.93929	56	4	.35945	.38520	2.5961	.93316	56	.35945	.38520	2.5961	.93316	56	.35945	.38520	2.5961	.93316	56
5	.34339	.36562	2.7351	.93919	55	5	.35973	.38553	2.5938	.93306	55	.35973	.38553	2.5938	.93306	55	.35973	.38553	2.5938	.93306	55
6	.34366	.36595	2.7326	.93909	54	6	.36000	.38587	2.5916	.93295	54	.36000	.38587	2.5916	.93295	54	.36000	.38587	2.5916	.93295	54
7	.34393	.36628	2.7302	.93899	53	7	.36027	.38620	2.5893	.93285	53	.36027	.38620	2.5893	.93285	53	.36027	.38620	2.5893	.93285	53
8	.34421	.36661	2.7277	.93889	52	8	.36054	.38654	2.5871	.93274	52	.36054	.38654	2.5871	.93274	52	.36054	.38654	2.5871	.93274	52
9	.34448	.36694	2.7253	.93879	51	9	.36081	.38687	2.5848	.93264	51	.36081	.38687	2.5848	.93264	51	.36081	.38687	2.5848	.93264	51
10	.34475	.36727	2.7228	.93869	50	10	.36108	.38721	2.5826	.93253	50	.36108	.38721	2.5826	.93253	50	.36108	.38721	2.5826	.93253	50
11	.34503	.36760	2.7204	.93859	49	11	.36135	.38754	2.5804	.93243	49	.36135	.38754	2.5804	.93243	49	.36135	.38754	2.5804	.93243	49
12	.34530	.36793	2.7179	.93849	48	12	.36162	.38787	2.5782	.93232	48	.36162	.38787	2.5782	.93232	48	.36162	.38787	2.5782	.93232	48
13	.34557	.36826	2.7155	.93839	47	13	.36190	.38821	2.5759	.93222	47	.36190	.38821	2.5759	.93222	47	.36190	.38821	2.5759	.93222	47
14	.34584	.36859	2.7130	.93829	46	14	.36217	.38854	2.5737	.93211	46	.36217	.38854	2.5737	.93211	46	.36217	.38854	2.5737	.93211	46
15	.34612	.36892	2.7106	.93819	45	15	.36244	.38888	2.5715	.93201	45	.36244	.38888	2.5715	.93201	45	.36244	.38888	2.5715	.93201	45
16	.34639	.36925	2.7082	.93809	44	16	.36271	.38921	2.5693	.93190	44	.36271	.38921	2.5693	.93190	44	.36271	.38921	2.5693	.93190	44
17	.34666	.36958	2.7058	.93799	43	17	.36298	.38955	2.5671	.93180	43	.36298	.38955	2.5671	.93180	43	.36298	.38955	2.5671	.93180	43
18	.34694	.36991	2.7034	.93789	42	18	.36325	.38988	2.5649	.93169	42	.36325	.38988	2.5649	.93169	42	.36325	.38988	2.5649	.93169	42
19	.34721	.37024	2.7009	.93779	41	19	.36352	.39022	2.5627	.93159	41	.36352	.39022	2.5627	.93159	41	.36352	.39022	2.5627	.93159	41
20	.34748	.37057	2.6985	.93769	40	20	.36379	.39055	2.5605	.93148	40	.36379	.39055	2.5605	.93148	40	.36379	.39055	2.5605	.93148	40
21	.34775	.37090	2.6961	.93759	39	21	.36406	.39089	2.5583	.93137	39	.36406	.39089	2.5583	.93137	39	.36406	.39089	2.5583	.93137	39
22	.34803	.37123	2.6937	.93748	38	22	.36434	.39122	2.5561	.93127	38	.36434	.39122	2.5561	.93127	38	.36434	.39122	2.5561	.93127	38
23	.34830	.37157	2.6913	.93738	37	23	.36461	.39156	2.5539	.93116	37	.36461	.39156	2.5539	.93116	37	.36461	.39156	2.5539	.93116	37
24	.34857	.37190	2.6889	.93728	36	24	.36488	.39190	2.5517	.93106	36	.36488	.39190	2.5517	.93106	36	.36488	.39190	2.5517	.93106	36
25	.34884	.37223	2.6865	.93718	35	25	.36515	.39223	2.5495	.93095	35	.36515	.39223	2.5495	.93095	35	.36515	.39223	2.5495	.93095	35
26	.34912	.37256	2.6841	.93708	34	26	.36542	.39257	2.5473	.93084	34	.36542	.39257	2.5473	.93084	34	.36542	.39257	2.5473	.93084	34
27	.34939	.37289	2.6818	.93698	33	27	.36569	.39290	2.5452	.93074	33	.36569	.39290	2.5452	.93074	33	.36569	.39290	2.5452	.93074	33
28	.34966	.37322	2.6794	.93688	32	28	.36596	.39324	2.5430	.93063	32	.36596	.39324	2.5430	.93063	32	.36596	.39324	2.5430	.93063	32
29	.34993	.37355	2.6770	.93677	31	29	.36623	.39357	2.5408	.93052	31	.36623	.39357	2.5408	.93052	31	.36623	.39357	2.5408	.93052	31
30	.35021	.37388	2.6746	.93667	30	30	.36650	.39391	2.5386	.93042	30	.36650	.39391	2.5386	.93042	30	.36650	.39391	2.5386	.93042	30
31	.35048	.37422	2.6723	.93657	29	31	.36677	.39425	2.5365	.93031	29	.36677	.39425	2.5365	.93031	29	.36677	.39425	2.5365	.93031	29
32	.35075	.37455	2.6699	.93647	28	32	.36704	.39458	2.5343	.93021	28	.36704	.39458	2.5343	.93021	28	.36704	.39458	2.5343	.93021	28
33	.35102	.37488	2.6675	.93637	27	33	.36731	.39492	2.5322	.93010	27	.36731	.39492	2.5322	.93010	27	.36731	.39492	2.5322	.93010	27
34	.35130	.37521	2.6652	.93626	26	34	.36758	.39526	2.5300	.92999	26	.36758	.39526	2.5300	.92999	26	.36758	.39526	2.5300	.92999	26
35	.35157	.37554	2.6628	.93616	25	35	.36785	.39559	2.5279	.92988	25	.36785	.39559	2.5279	.92988	25	.36785	.39559	2.5279	.92988	25
36	.35184	.37588	2.6605	.93606	24	36	.36812	.39593	2.5257	.92978	24	.36812	.39593	2.5257	.92978	24	.36812	.39593	2.5257	.92978	24
37	.35211	.37621	2.6581	.93596	23	37	.36839	.39626	2.5236	.92967	23	.36839	.39626	2.5236	.92967	23	.36839	.39626	2.5236	.92967	23
38	.35239	.37654	2.6558	.93585	22	38	.36867	.39660	2.5214	.92956	22	.36867	.39660	2.5214	.92956	22	.36867	.39660	2.5214	.92956	22
39	.35266	.37687	2.6534	.93575	21	39	.36894	.39694	2.5193	.92945	21	.36894	.39694	2.5193	.92945	21	.36894	.39694	2.5193	.92945	21
40	.35293	.37720	2.6511	.93565	20	40	.36921	.39727	2.5172	.92935	20	.36921	.39727	2.5172	.92935	20	.36921	.39727	2.5172	.92935	20
41	.35320	.37754	2.6488	.93555	19	41	.36948	.39761	2.5150	.92924	19	.36948	.39761	2.5150	.92924	19	.36948	.39761	2.5150	.92924	19
42	.35347	.37787	2.6464	.93544	18	42	.36975	.39795	2.5129	.92913	18	.36975	.39795	2.5129	.92913	18	.36975	.39795	2.5129	.92913	18
43	.35375	.37820	2.6441	.93534	17	43	.37002	.39829	2.5108	.92902	17	.37002	.39829	2.5108	.92902	17	.37002	.39829	2.5108	.92902	17
44	.35402	.37853	2.6418	.93524	16	44	.37029	.39862	2.5086	.92892	16	.37029	.39862	2.5086	.92892	16	.37029	.39862	2.5086	.92892	16
45	.35429	.37887	2.6395	.93514	15	45	.37056	.39896	2.5065	.92881	15	.37056	.39896	2.5065	.92881	15	.37056	.39896	2.5065	.92881	15
46	.35456	.37920	2.6371	.93503	14	46	.37083	.39930	2.5044	.92870	14	.37083	.39930	2.5044	.92870	14	.37083	.39930	2.5044	.92870	14
47	.35484	.37953	2.6348	.93493	13	47	.37110	.39963	2.5023	.92859	13	.37110	.39963	2.5023	.92859	13	.37110	.39963	2.5023	.92859	13
48	.35511	.37986	2.6325	.93483	12	48	.37137	.39997	2.5002	.92849	12	.37137	.39997	2.5002	.92849	12	.37137	.39997	2.5002	.92849	12
49	.35538	.38020	2.6302	.93472	11	49	.37164	.40031	2.4981	.92838	11	.37164	.40031	2.4981	.92838	11	.37164	.40031	2.4981	.92838	11
50	.35565	.38053	2.6279	.93462	10	50	.37191	.40065	2.4960	.92827	10	.37191	.40065	2.4960	.92827	10	.37191	.40065	2.4960	.92827	10
51	.35592	.38086	2.6256	.93452	9	51	.37218	.40098	2.4939	.92816	9	.37218	.40098	2.4939	.92816	9	.37218	.40098	2.4939	.92816	9
52	.35619	.38120	2.6233	.93441	8	52	.37245	.40132	2.4918	.92805	8	.37245	.40132	2.4918	.92805	8	.37245	.40132	2.4918	.92805	8
53	.35647	.38153	2.6210	.93431	7	53	.37272	.40166	2.4897	.92794	7	.37272	.40166	2.4897	.92794	7	.37272	.40166	2.4897	.92794	7
54	.35674	.38186	2.6187	.93420	6	54	.37299	.40200	2.4876	.92783	6	.37299	.40200	2.4876	.92783	6	.37299	.40200	2.4876	.92783	6
55	.35701	.38220	2.6165	.93410	5	55	.37326	.40234	2.4855	.92773	5	.37326	.40234	2.4855	.92773	5	.37326	.40234	2.4855	.92773	5
56	.35728	.38253	2.6142	.93400	4	56	.37353	.40267	2.4834	.92762	4	.37353	.40267	2.4							

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

22° (202°)					(337°) 157°					23° (203°)					(336°) 156°						
/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos	/
0	.37461	.40403	2.4751	.92718	60	0	.39073	.42447	2.3559	.92050	0	.39073	.42447	2.3559	.92050	60	.39073	.42447	2.3559	.92050	60
1	.37488	.40436	2.4730	.92707	59	1	.39100	.42482	2.3539	.92039	59	.39100	.42482	2.3539	.92039	59	.39100	.42482	2.3539	.92039	59
2	.37515	.40470	2.4709	.92697	58	2	.39127	.42516	2.3520	.92028	58	.39127	.42516	2.3520	.92028	58	.39127	.42516	2.3520	.92028	58
3	.37542	.40504	2.4689	.92686	57	3	.39153	.42551	2.3501	.92016	57	.39153	.42551	2.3501	.92016	57	.39153	.42551	2.3501	.92016	57
4	.37569	.40538	2.4668	.92675	56	4	.39180	.42585	2.3483	.92005	56	.39180	.42585	2.3483	.92005	56	.39180	.42585	2.3483	.92005	56
5	.37595	.40572	2.4648	.92664	55	5	.39207	.42619	2.3464	.91994	55	.39207	.42619	2.3464	.91994	55	.39207	.42619	2.3464	.91994	55
6	.37622	.40606	2.4627	.92653	54	6	.39234	.42654	2.3445	.91982	54	.39234	.42654	2.3445	.91982	54	.39234	.42654	2.3445	.91982	54
7	.37649	.40640	2.4606	.92642	53	7	.39260	.42688	2.3426	.91971	53	.39260	.42688	2.3426	.91971	53	.39260	.42688	2.3426	.91971	53
8	.37676	.40674	2.4586	.92631	52	8	.39287	.42722	2.3407	.91959	52	.39287	.42722	2.3407	.91959	52	.39287	.42722	2.3407	.91959	52
9	.37703	.40707	2.4566	.92620	51	9	.39314	.42757	2.3388	.91948	51	.39314	.42757	2.3388	.91948	51	.39314	.42757	2.3388	.91948	51
10	.37730	.40741	2.4545	.92609	50	10	.39341	.42791	2.3369	.91936	50	.39341	.42791	2.3369	.91936	50	.39341	.42791	2.3369	.91936	50
11	.37757	.40775	2.4525	.92598	49	11	.39367	.42826	2.3351	.91925	49	.39367	.42826	2.3351	.91925	49	.39367	.42826	2.3351	.91925	49
12	.37784	.40809	2.4504	.92587	48	12	.39394	.42860	2.3332	.91914	48	.39394	.42860	2.3332	.91914	48	.39394	.42860	2.3332	.91914	48
13	.37811	.40843	2.4484	.92576	47	13	.39421	.42894	2.3313	.91902	47	.39421	.42894	2.3313	.91902	47	.39421	.42894	2.3313	.91902	47
14	.37838	.40877	2.4464	.92565	46	14	.39448	.42929	2.3294	.91891	46	.39448	.42929	2.3294	.91891	46	.39448	.42929	2.3294	.91891	46
15	.37865	.40911	2.4443	.92554	45	15	.39474	.42963	2.3276	.91879	45	.39474	.42963	2.3276	.91879	45	.39474	.42963	2.3276	.91879	45
16	.37892	.40945	2.4423	.92543	44	16	.39501	.42998	2.3257	.91868	44	.39501	.42998	2.3257	.91868	44	.39501	.42998	2.3257	.91868	44
17	.37919	.40979	2.4403	.92532	43	17	.39528	.43032	2.3238	.91856	43	.39528	.43032	2.3238	.91856	43	.39528	.43032	2.3238	.91856	43
18	.37946	.41013	2.4383	.92521	42	18	.39555	.43067	2.3220	.91845	42	.39555	.43067	2.3220	.91845	42	.39555	.43067	2.3220	.91845	42
19	.37973	.41047	2.4362	.92510	41	19	.39581	.43101	2.3201	.91833	41	.39581	.43101	2.3201	.91833	41	.39581	.43101	2.3201	.91833	41
20	.37999	.41081	2.4342	.92499	40	20	.39608	.43136	2.3183	.91822	40	.39608	.43136	2.3183	.91822	40	.39608	.43136	2.3183	.91822	40
21	.38026	.41115	2.4322	.92488	39	21	.39635	.43170	2.3164	.91810	39	.39635	.43170	2.3164	.91810	39	.39635	.43170	2.3164	.91810	39
22	.38053	.41149	2.4302	.92477	38	22	.39661	.43205	2.3146	.91799	38	.39661	.43205	2.3146	.91799	38	.39661	.43205	2.3146	.91799	38
23	.38080	.41183	2.4282	.92466	37	23	.39688	.43239	2.3127	.91787	37	.39688	.43239	2.3127	.91787	37	.39688	.43239	2.3127	.91787	37
24	.38107	.41217	2.4262	.92455	36	24	.39715	.43274	2.3109	.91775	36	.39715	.43274	2.3109	.91775	36	.39715	.43274	2.3109	.91775	36
25	.38134	.41251	2.4242	.92444	35	25	.39741	.43308	2.3090	.91764	35	.39741	.43308	2.3090	.91764	35	.39741	.43308	2.3090	.91764	35
26	.38161	.41285	2.4222	.92432	34	26	.39768	.43343	2.3072	.91752	34	.39768	.43343	2.3072	.91752	34	.39768	.43343	2.3072	.91752	34
27	.38188	.41319	2.4202	.92421	33	27	.39795	.43378	2.3053	.91741	33	.39795	.43378	2.3053	.91741	33	.39795	.43378	2.3053	.91741	33
28	.38215	.41353	2.4182	.92410	32	28	.39822	.43412	2.3035	.91729	32	.39822	.43412	2.3035	.91729	32	.39822	.43412	2.3035	.91729	32
29	.38241	.41387	2.4162	.92399	31	29	.39848	.43447	2.3017	.91718	31	.39848	.43447	2.3017	.91718	31	.39848	.43447	2.3017	.91718	31
30	.38268	.41421	2.4142	.92388	30	30	.39875	.43481	2.2998	.91706	30	.39875	.43481	2.2998	.91706	30	.39875	.43481	2.2998	.91706	30
31	.38295	.41455	2.4122	.92377	29	31	.39902	.43516	2.2980	.91694	29	.39902	.43516	2.2980	.91694	29	.39902	.43516	2.2980	.91694	29
32	.38322	.41490	2.4102	.92366	28	32	.39928	.43550	2.2962	.91683	28	.39928	.43550	2.2962	.91683	28	.39928	.43550	2.2962	.91683	28
33	.38349	.41524	2.4083	.92355	27	33	.39955	.43585	2.2944	.91671	27	.39955	.43585	2.2944	.91671	27	.39955	.43585	2.2944	.91671	27
34	.38376	.41558	2.4063	.92343	26	34	.39982	.43620	2.2925	.91660	26	.39982	.43620	2.2925	.91660	26	.39982	.43620	2.2925	.91660	26
35	.38403	.41592	2.4043	.92332	25	35	.40008	.43654	2.2907	.91648	25	.40008	.43654	2.2907	.91648	25	.40008	.43654	2.2907	.91648	25
36	.38430	.41626	2.4023	.92321	24	36	.40035	.43689	2.2889	.91636	24	.40035	.43689	2.2889	.91636	24	.40035	.43689	2.2889	.91636	24
37	.38456	.41660	2.4004	.92310	23	37	.40062	.43724	2.2871	.91625	23	.40062	.43724	2.2871	.91625	23	.40062	.43724	2.2871	.91625	23
38	.38483	.41694	2.3984	.92299	22	38	.40088	.43758	2.2853	.91613	22	.40088	.43758	2.2853	.91613	22	.40088	.43758	2.2853	.91613	22
39	.38510	.41728	2.3964	.92287	21	39	.40115	.43793	2.2835	.91601	21	.40115	.43793	2.2835	.91601	21	.40115	.43793	2.2835	.91601	21
40	.38537	.41763	2.3945	.92276	20	40	.40141	.43828	2.2817	.91590	20	.40141	.43828	2.2817	.91590	20	.40141	.43828	2.2817	.91590	20
41	.38564	.41797	2.3925	.92265	19	41	.40168	.43862	2.2799	.91578	19	.40168	.43862	2.2799	.91578	19	.40168	.43862	2.2799	.91578	19
42	.38591	.41831	2.3906	.92254	18	42	.40195	.43897	2.2781	.91566	18	.40195	.43897	2.2781	.91566	18	.40195	.43897	2.2781	.91566	18
43	.38617	.41865	2.3886	.92243	17	43	.40221	.43932	2.2763	.91555	17	.40221	.43932	2.2763	.91555	17	.40221	.43932	2.2763	.91555	17
44	.38644	.41899	2.3867	.92231	16	44	.40248	.43966	2.2745	.91543	16	.40248	.43966	2.2745	.91543	16	.40248	.43966	2.2745	.91543	16
45	.38671	.41933	2.3847	.92220	15	45	.40275	.44001	2.2727	.91531	15	.40275	.44001	2.2727	.91531	15	.40275	.44001	2.2727	.91531	15
46	.38698	.41968	2.3828	.92209	14	46	.40301	.44036	2.2709	.91519	14	.40301	.44036	2.2709	.91519	14	.40301	.44036	2.2709	.91519	14
47	.38725	.42002	2.3808	.92198	13	47	.40328	.44071	2.2691	.91508	13	.40328	.44071	2.2691	.91508	13	.40328	.44071	2.2691	.91508	13
48	.38752	.42036	2.3789	.92186	12	48	.40355	.44105	2.2673	.91496	12	.40355	.44105	2.2673	.91496	12	.40355	.44105	2.2673	.91496	12
49	.38778	.42070	2.3770	.92175	11	49	.40381	.44140	2.2655	.91484	11	.40381	.44140	2.2655	.91484	11	.40381	.44140	2.2655	.91484	11
50	.38805	.42105	2.3750	.92164	10	50	.40408	.44175	2.2637	.91472	10	.40408	.44175	2.2637	.91472	10	.40408	.44175	2.2637	.91472	10
51	.38832	.42139	2.3731	.92152	9	51	.40434	.44210	2.2620	.91460	9	.40434	.44210	2.2620	.91460	9	.40434	.44210	2.2620	.91460	9
52	.38859	.42173	2.3712	.92141	8	52	.40461	.44244	2.2602	.91449	8	.40461	.44244	2.2602	.91449	8	.40461	.44244	2.2602	.91449	8
53	.38886	.42207	2.3693	.92130	7	53	.40488	.44279	2.2584	.91437	7	.40488	.44279	2.2584	.91437	7	.40488	.44279	2.2584	.91437	7
54	.38912	.42242	2.3673	.92119	6	54	.40514	.44314	2.2566	.91425	6	.40514	.44314	2.2566	.91425	6	.40514	.44314	2.2566	.91425	6
55	.38939	.42276	2.3654	.92107	5	55	.40541	.44349	2.2549	.91414	5	.40541	.44349	2.2549	.91414	5	.40541	.44349	2.2549	.91414	5
56	.38966	.42310	2.3635	.92096	4	56	.40567	.44384	2.2531	.91402	4	.40567	.44384	2.							

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

24° (204°)					(335°) 155°					25° (205°)					(334°) 154°				
/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/		
0	.40674	.44523	2 2460	.91355	60	0	.42262	.46631	2 1445	.90631	60	0	.42262	.46631	2 1445	.90631	60		
1	.40700	.44558	2 2440	.91343	59	1	.42288	.46666	2 1429	.90618	59	1	.42288	.46666	2 1429	.90618	59		
2	.40727	.44593	2 2425	.91331	58	2	.42315	.46702	2 1413	.90606	58	2	.42315	.46702	2 1413	.90606	58		
3	.40753	.44627	2 2408	.91319	57	3	.42341	.46737	2 1396	.90594	57	3	.42341	.46737	2 1396	.90594	57		
4	.40780	.44662	2 2390	.91307	56	4	.42367	.46772	2 1380	.90582	56	4	.42367	.46772	2 1380	.90582	56		
5	.40806	.44697	2 2373	.91295	55	5	.42394	.46808	2 1364	.90569	55	5	.42394	.46808	2 1364	.90569	55		
6	.40833	.44732	2 2355	.91283	54	6	.42420	.46843	2 1348	.90557	54	6	.42420	.46843	2 1348	.90557	54		
7	.40860	.44767	2 2338	.91272	53	7	.42446	.46879	2 1332	.90545	53	7	.42446	.46879	2 1332	.90545	53		
8	.40886	.44802	2 2320	.91260	52	8	.42473	.46914	2 1315	.90532	52	8	.42473	.46914	2 1315	.90532	52		
9	.40913	.44837	2 2303	.91248	51	9	.42499	.46950	2 1299	.90520	51	9	.42499	.46950	2 1299	.90520	51		
10	.40939	.44872	2 2286	.91236	50	10	.42525	.46985	2 1283	.90507	50	10	.42525	.46985	2 1283	.90507	50		
11	.40966	.44907	2 2268	.91224	49	11	.42552	.47021	2 1267	.90495	49	11	.42552	.47021	2 1267	.90495	49		
12	.40992	.44942	2 2251	.91212	48	12	.42578	.47056	2 1251	.90483	48	12	.42578	.47056	2 1251	.90483	48		
13	.41019	.44977	2 2234	.91200	47	13	.42604	.47092	2 1235	.90470	47	13	.42604	.47092	2 1235	.90470	47		
14	.41045	.45012	2 2216	.91188	46	14	.42631	.47128	2 1219	.90458	46	14	.42631	.47128	2 1219	.90458	46		
15	.41072	.45047	2 2199	.91176	45	15	.42657	.47163	2 1203	.90446	45	15	.42657	.47163	2 1203	.90446	45		
16	.41098	.45082	2 2182	.91164	44	16	.42683	.47199	2 1187	.90433	44	16	.42683	.47199	2 1187	.90433	44		
17	.41125	.45117	2 2165	.91152	43	17	.42709	.47234	2 1171	.90421	43	17	.42709	.47234	2 1171	.90421	43		
18	.41151	.45152	2 2148	.91140	42	18	.42736	.47270	2 1155	.90408	42	18	.42736	.47270	2 1155	.90408	42		
19	.41178	.45187	2 2130	.91128	41	19	.42762	.47305	2 1139	.90396	41	19	.42762	.47305	2 1139	.90396	41		
20	.41204	.45222	2 2113	.91116	40	20	.42788	.47341	2 1123	.90383	40	20	.42788	.47341	2 1123	.90383	40		
21	.41231	.45257	2 2096	.91104	39	21	.42815	.47377	2 1107	.90371	39	21	.42815	.47377	2 1107	.90371	39		
22	.41257	.45292	2 2079	.91092	38	22	.42841	.47412	2 1092	.90358	38	22	.42841	.47412	2 1092	.90358	38		
23	.41284	.45327	2 2062	.91080	37	23	.42867	.47448	2 1076	.90346	37	23	.42867	.47448	2 1076	.90346	37		
24	.41310	.45362	2 2045	.91068	36	24	.42894	.47483	2 1060	.90334	36	24	.42894	.47483	2 1060	.90334	36		
25	.41337	.45397	2 2028	.91056	35	25	.42920	.47519	2 1044	.90321	35	25	.42920	.47519	2 1044	.90321	35		
26	.41363	.45432	2 2011	.91044	34	26	.42946	.47555	2 1028	.90309	34	26	.42946	.47555	2 1028	.90309	34		
27	.41390	.45467	2 1994	.91032	33	27	.42972	.47590	2 1013	.90296	33	27	.42972	.47590	2 1013	.90296	33		
28	.41416	.45502	2 1977	.91020	32	28	.42999	.47626	2 0997	.90284	32	28	.42999	.47626	2 0997	.90284	32		
29	.41443	.45538	2 1960	.91008	31	29	.43025	.47662	2 0981	.90271	31	29	.43025	.47662	2 0981	.90271	31		
30	.41469	.45573	2 1943	.90996	30	30	.43051	.47698	2 0965	.90259	30	30	.43051	.47698	2 0965	.90259	30		
31	.41496	.45608	2 1926	.90984	29	31	.43077	.47733	2 0950	.90246	29	31	.43077	.47733	2 0950	.90246	29		
32	.41522	.45643	2 1909	.90972	28	32	.43104	.47769	2 0934	.90233	28	32	.43104	.47769	2 0934	.90233	28		
33	.41549	.45678	2 1892	.90960	27	33	.43130	.47805	2 0918	.90221	27	33	.43130	.47805	2 0918	.90221	27		
34	.41575	.45713	2 1876	.90948	26	34	.43156	.47840	2 0903	.90208	26	34	.43156	.47840	2 0903	.90208	26		
35	.41602	.45748	2 1859	.90936	25	35	.43182	.47876	2 0887	.90196	25	35	.43182	.47876	2 0887	.90196	25		
36	.41628	.45784	2 1842	.90924	24	36	.43209	.47912	2 0872	.90183	24	36	.43209	.47912	2 0872	.90183	24		
37	.41655	.45819	2 1825	.90911	23	37	.43235	.47948	2 0856	.90171	23	37	.43235	.47948	2 0856	.90171	23		
38	.41681	.45854	2 1808	.90899	22	38	.43261	.47984	2 0840	.90158	22	38	.43261	.47984	2 0840	.90158	22		
39	.41707	.45889	2 1792	.90887	21	39	.43287	.48019	2 0825	.90146	21	39	.43287	.48019	2 0825	.90146	21		
40	.41734	.45924	2 1775	.90875	20	40	.43313	.48055	2 0809	.90133	20	40	.43313	.48055	2 0809	.90133	20		
41	.41760	.45960	2 1758	.90863	19	41	.43340	.48091	2 0794	.90120	19	41	.43340	.48091	2 0794	.90120	19		
42	.41787	.45995	2 1742	.90851	18	42	.43366	.48127	2 0778	.90108	18	42	.43366	.48127	2 0778	.90108	18		
43	.41813	.46030	2 1725	.90839	17	43	.43392	.48163	2 0763	.90095	17	43	.43392	.48163	2 0763	.90095	17		
44	.41840	.46065	2 1708	.90826	16	44	.43418	.48198	2 0748	.90082	16	44	.43418	.48198	2 0748	.90082	16		
45	.41866	.46101	2 1692	.90814	15	45	.43445	.48234	2 0732	.90070	15	45	.43445	.48234	2 0732	.90070	15		
46	.41892	.46136	2 1675	.90802	14	46	.43471	.48270	2 0717	.90057	14	46	.43471	.48270	2 0717	.90057	14		
47	.41919	.46171	2 1659	.90790	13	47	.43497	.48306	2 0701	.90045	13	47	.43497	.48306	2 0701	.90045	13		
48	.41945	.46206	2 1642	.90778	12	48	.43523	.48342	2 0686	.90032	12	48	.43523	.48342	2 0686	.90032	12		
49	.41972	.46242	2 1625	.90766	11	49	.43549	.48378	2 0671	.90019	11	49	.43549	.48378	2 0671	.90019	11		
50	.41998	.46277	2 1609	.90753	10	50	.43575	.48414	2 0655	.90007	10	50	.43575	.48414	2 0655	.90007	10		
51	.42024	.46312	2 1592	.90741	9	51	.43602	.48450	2 0640	.89994	9	51	.43602	.48450	2 0640	.89994	9		
52	.42051	.46348	2 1576	.90729	8	52	.43628	.48486	2 0625	.89981	8	52	.43628	.48486	2 0625	.89981	8		
53	.42077	.46383	2 1560	.90717	7	53	.43654	.48521	2 0609	.89968	7	53	.43654	.48521	2 0609	.89968	7		
54	.42104	.46418	2 1543	.90704	6	54	.43680	.48557	2 0594	.89956	6	54	.43680	.48557	2 0594	.89956	6		
55	.42130	.46454	2 1527	.90692	5	55	.43706	.48593	2 0579	.89943	5	55	.43706	.48593	2 0579	.89943	5		
56	.42156	.46489	2 1510	.90680	4	56	.43733	.48629	2 0564	.89930	4	56	.43733	.48629	2 0564	.89930	4		
57	.42183	.46525	2 1494	.90668	3	57	.43759	.48665	2 0549	.89918	3	57	.43759	.48665	2 0549	.89918	3		
58	.42209	.46560	2 1478	.90655	2	58	.43785	.48701	2 0533	.89905	2	58	.43785	.48701	2 0533	.89905	2		
59	.42235	.46595	2 1461	.90643	1	59	.43811	.48737	2 0518	.89892	1	59	.43811	.48737	2 0518	.89892	1		
60	.42262	.46631	2 1445	.90631	0	60	.43837	.48773	2 0503	.89879	0	60	.43837	.48773	2 0503	.89879	0		
/	Cos	Ctn	Tan	Sin	/	/	Cos	Ctn	Tan	Sin	/	/	Cos	Ctn	Tan	Sin	/		

114 (294°)

(245°) 65°

115° (295°)

(244°) 64°

Table 6.5(c) (Cont'd)

26° (206°)						(333°) 153°						27° (207°)						(332°) 152°					
/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/
0	.43837	.48773	2.0503	.89879	60	0	.45399	.50953	1.9626	.89101	60	0	.45399	.50953	1.9626	.89101	60	0	.45399	.50953	1.9626	.89101	60
1	.43863	.48809	2.0488	.89867	59	1	.45425	.50989	1.9612	.89087	59	1	.45425	.50989	1.9612	.89087	59	1	.45425	.50989	1.9612	.89087	59
2	.43889	.48845	2.0473	.89854	58	2	.45451	.51026	1.9598	.89074	58	2	.45451	.51026	1.9598	.89074	58	2	.45451	.51026	1.9598	.89074	58
3	.43916	.48881	2.0458	.89841	57	3	.45477	.51063	1.9584	.89061	57	3	.45477	.51063	1.9584	.89061	57	3	.45477	.51063	1.9584	.89061	57
4	.43942	.48917	2.0443	.89828	56	4	.45503	.51099	1.9570	.89048	56	4	.45503	.51099	1.9570	.89048	56	4	.45503	.51099	1.9570	.89048	56
5	.43968	.48953	2.0428	.89816	55	5	.45529	.51136	1.9556	.89035	55	5	.45529	.51136	1.9556	.89035	55	5	.45529	.51136	1.9556	.89035	55
6	.43994	.48989	2.0413	.89803	54	6	.45554	.51173	1.9542	.89021	54	6	.45554	.51173	1.9542	.89021	54	6	.45554	.51173	1.9542	.89021	54
7	.44020	.49026	2.0398	.89790	53	7	.45580	.51209	1.9528	.89008	53	7	.45580	.51209	1.9528	.89008	53	7	.45580	.51209	1.9528	.89008	53
8	.44046	.49062	2.0383	.89777	52	8	.45606	.51246	1.9514	.88995	52	8	.45606	.51246	1.9514	.88995	52	8	.45606	.51246	1.9514	.88995	52
9	.44072	.49098	2.0368	.89764	51	9	.45632	.51283	1.9500	.88981	51	9	.45632	.51283	1.9500	.88981	51	9	.45632	.51283	1.9500	.88981	51
10	.44098	.49134	2.0353	.89752	50	10	.45658	.51319	1.9486	.88968	50	10	.45658	.51319	1.9486	.88968	50	10	.45658	.51319	1.9486	.88968	50
11	.44124	.49170	2.0338	.89739	49	11	.45684	.51356	1.9472	.88955	49	11	.45684	.51356	1.9472	.88955	49	11	.45684	.51356	1.9472	.88955	49
12	.44151	.49206	2.0323	.89726	48	12	.45710	.51393	1.9458	.88942	48	12	.45710	.51393	1.9458	.88942	48	12	.45710	.51393	1.9458	.88942	48
13	.44177	.49242	2.0308	.89713	47	13	.45736	.51430	1.9444	.88928	47	13	.45736	.51430	1.9444	.88928	47	13	.45736	.51430	1.9444	.88928	47
14	.44203	.49278	2.0293	.89700	46	14	.45762	.51467	1.9430	.88915	46	14	.45762	.51467	1.9430	.88915	46	14	.45762	.51467	1.9430	.88915	46
15	.44229	.49315	2.0278	.89687	45	15	.45787	.51503	1.9416	.88902	45	15	.45787	.51503	1.9416	.88902	45	15	.45787	.51503	1.9416	.88902	45
16	.44255	.49351	2.0263	.89674	44	16	.45813	.51540	1.9402	.88888	44	16	.45813	.51540	1.9402	.88888	44	16	.45813	.51540	1.9402	.88888	44
17	.44281	.49387	2.0248	.89662	43	17	.45839	.51577	1.9388	.88875	43	17	.45839	.51577	1.9388	.88875	43	17	.45839	.51577	1.9388	.88875	43
18	.44307	.49423	2.0233	.89649	42	18	.45865	.51614	1.9375	.88862	42	18	.45865	.51614	1.9375	.88862	42	18	.45865	.51614	1.9375	.88862	42
19	.44333	.49459	2.0219	.89636	41	19	.45891	.51651	1.9361	.88848	41	19	.45891	.51651	1.9361	.88848	41	19	.45891	.51651	1.9361	.88848	41
20	.44359	.49495	2.0204	.89623	40	20	.45917	.51688	1.9347	.88835	40	20	.45917	.51688	1.9347	.88835	40	20	.45917	.51688	1.9347	.88835	40
21	.44385	.49532	2.0189	.89610	39	21	.45942	.51724	1.9333	.88822	39	21	.45942	.51724	1.9333	.88822	39	21	.45942	.51724	1.9333	.88822	39
22	.44411	.49568	2.0174	.89597	38	22	.45968	.51761	1.9319	.88808	38	22	.45968	.51761	1.9319	.88808	38	22	.45968	.51761	1.9319	.88808	38
23	.44437	.49604	2.0160	.89584	37	23	.45994	.51798	1.9306	.88795	37	23	.45994	.51798	1.9306	.88795	37	23	.45994	.51798	1.9306	.88795	37
24	.44464	.49640	2.0145	.89571	36	24	.46020	.51835	1.9292	.88782	36	24	.46020	.51835	1.9292	.88782	36	24	.46020	.51835	1.9292	.88782	36
25	.44490	.49677	2.0130	.89558	35	25	.46046	.51872	1.9278	.88768	35	25	.46046	.51872	1.9278	.88768	35	25	.46046	.51872	1.9278	.88768	35
26	.44516	.49713	2.0115	.89545	34	26	.46072	.51909	1.9265	.88755	34	26	.46072	.51909	1.9265	.88755	34	26	.46072	.51909	1.9265	.88755	34
27	.44542	.49749	2.0101	.89532	33	27	.46097	.51946	1.9251	.88741	33	27	.46097	.51946	1.9251	.88741	33	27	.46097	.51946	1.9251	.88741	33
28	.44568	.49786	2.0086	.89519	32	28	.46123	.51983	1.9237	.88728	32	28	.46123	.51983	1.9237	.88728	32	28	.46123	.51983	1.9237	.88728	32
29	.44594	.49822	2.0072	.89506	31	29	.46149	.52020	1.9223	.88715	31	29	.46149	.52020	1.9223	.88715	31	29	.46149	.52020	1.9223	.88715	31
30	.44620	.49858	2.0057	.89493	30	30	.46175	.52057	1.9210	.88701	30	30	.46175	.52057	1.9210	.88701	30	30	.46175	.52057	1.9210	.88701	30
31	.44646	.49894	2.0042	.89480	29	31	.46201	.52094	1.9196	.88688	29	31	.46201	.52094	1.9196	.88688	29	31	.46201	.52094	1.9196	.88688	29
32	.44672	.49931	2.0028	.89467	28	32	.46226	.52131	1.9183	.88674	28	32	.46226	.52131	1.9183	.88674	28	32	.46226	.52131	1.9183	.88674	28
33	.44698	.49967	2.0013	.89454	27	33	.46252	.52168	1.9169	.88661	27	33	.46252	.52168	1.9169	.88661	27	33	.46252	.52168	1.9169	.88661	27
34	.44724	.50004	1.9999	.89441	26	34	.46278	.52205	1.9155	.88647	26	34	.46278	.52205	1.9155	.88647	26	34	.46278	.52205	1.9155	.88647	26
35	.44750	.50040	1.9984	.89428	25	35	.46304	.52242	1.9142	.88634	25	35	.46304	.52242	1.9142	.88634	25	35	.46304	.52242	1.9142	.88634	25
36	.44776	.50076	1.9970	.89415	24	36	.46330	.52279	1.9128	.88620	24	36	.46330	.52279	1.9128	.88620	24	36	.46330	.52279	1.9128	.88620	24
37	.44802	.50113	1.9955	.89402	23	37	.46355	.52316	1.9115	.88607	23	37	.46355	.52316	1.9115	.88607	23	37	.46355	.52316	1.9115	.88607	23
38	.44828	.50149	1.9941	.89389	22	38	.46381	.52353	1.9101	.88593	22	38	.46381	.52353	1.9101	.88593	22	38	.46381	.52353	1.9101	.88593	22
39	.44854	.50185	1.9926	.89376	21	39	.46407	.52390	1.9088	.88580	21	39	.46407	.52390	1.9088	.88580	21	39	.46407	.52390	1.9088	.88580	21
40	.44880	.50222	1.9912	.89363	20	40	.46433	.52427	1.9074	.88566	20	40	.46433	.52427	1.9074	.88566	20	40	.46433	.52427	1.9074	.88566	20
41	.44906	.50258	1.9897	.89350	19	41	.46458	.52464	1.9061	.88553	19	41	.46458	.52464	1.9061	.88553	19	41	.46458	.52464	1.9061	.88553	19
42	.44932	.50295	1.9883	.89337	18	42	.46484	.52501	1.9047	.88539	18	42	.46484	.52501	1.9047	.88539	18	42	.46484	.52501	1.9047	.88539	18
43	.44958	.50331	1.9868	.89324	17	43	.46510	.52538	1.9034	.88526	17	43	.46510	.52538	1.9034	.88526	17	43	.46510	.52538	1.9034	.88526	17
44	.44984	.50368	1.9854	.89311	16	44	.46536	.52575	1.9020	.88512	16	44	.46536	.52575	1.9020	.88512	16	44	.46536	.52575	1.9020	.88512	16
45	.45010	.50404	1.9840	.89298	15	45	.46561	.52613	1.9007	.88499	15	45	.46561	.52613	1.9007	.88499	15	45	.46561	.52613	1.9007	.88499	15
46	.45036	.50441	1.9825	.89285	14	46	.46587	.52650	1.8993	.88485	14	46	.46587	.52650	1.8993	.88485	14	46	.46587	.52650	1.8993	.88485	14
47	.45062	.50477	1.9811	.89272	13	47	.46613	.52687	1.8980	.88472	13	47	.46613	.52687	1.8980	.88472	13	47	.46613	.52687	1.8980	.88472	13
48	.45088	.50514	1.9797	.89259	12	48	.46639	.52724	1.8967	.88458	12	48	.46639	.52724	1.8967	.88458	12	48	.46639	.52724	1.8967	.88458	12
49	.45114	.50550	1.9782	.89245	11	49	.46664	.52761	1.8953	.88445	11	49	.46664	.52761	1.8953	.88445	11	49	.46664	.52761	1.8953	.88445	11
50	.45140	.50587	1.9768	.89232	10	50	.46690	.52798	1.8940	.88431	10	50	.46690	.52798	1.8940	.88431	10	50	.46690	.52798	1.8940	.88431	10
51	.45166	.50623	1.9754	.89219	9	51	.46716	.52836	1.8927	.88417	9	51	.46716	.52836	1.8927	.88417	9	51	.46716	.52836	1.8927	.88417	9
52	.45192	.50660	1.9740	.89206	8	52	.46742	.52873	1.8913	.88404	8	52	.46742	.52873	1.8913	.88404	8	52	.46742	.52873	1.8913	.88404	8
53	.45218	.50696	1.972																				

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

28° (208°)					(331°) 151°					29° (209°)					(330°) 150°				
/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/		
0	.46947	.53171	1.8807	.88295	60	0	.48481	.55431	1.8040	.87462	60	0	.48481	.55431	1.8040	.87462	60		
1	.46973	.53208	1.8794	.88281	59	1	.48506	.55469	1.8028	.87448	59	1	.48506	.55469	1.8028	.87448	59		
2	.46999	.53246	1.8781	.88267	58	2	.48532	.55507	1.8016	.87434	58	2	.48532	.55507	1.8016	.87434	58		
3	.47024	.53283	1.8768	.88254	57	3	.48557	.55545	1.8003	.87420	57	3	.48557	.55545	1.8003	.87420	57		
4	.47050	.53320	1.8755	.88240	56	4	.48583	.55583	1.7991	.87406	56	4	.48583	.55583	1.7991	.87406	56		
5	.47076	.53358	1.8741	.88226	55	5	.48608	.55621	1.7979	.87391	55	5	.48608	.55621	1.7979	.87391	55		
6	.47101	.53395	1.8728	.88213	54	6	.48634	.55659	1.7966	.87377	54	6	.48634	.55659	1.7966	.87377	54		
7	.47127	.53432	1.8715	.88199	53	7	.48659	.55697	1.7954	.87363	53	7	.48659	.55697	1.7954	.87363	53		
8	.47153	.53470	1.8702	.88185	52	8	.48684	.55736	1.7942	.87349	52	8	.48684	.55736	1.7942	.87349	52		
9	.47178	.53507	1.8689	.88172	51	9	.48710	.55774	1.7930	.87335	51	9	.48710	.55774	1.7930	.87335	51		
10	.47204	.53545	1.8676	.88158	50	10	.48735	.55812	1.7917	.87321	50	10	.48735	.55812	1.7917	.87321	50		
11	.47229	.53582	1.8663	.88144	49	11	.48761	.55850	1.7905	.87306	49	11	.48761	.55850	1.7905	.87306	49		
12	.47255	.53620	1.8650	.88130	48	12	.48786	.55888	1.7893	.87292	48	12	.48786	.55888	1.7893	.87292	48		
13	.47281	.53657	1.8637	.88117	47	13	.48811	.55926	1.7881	.87278	47	13	.48811	.55926	1.7881	.87278	47		
14	.47306	.53694	1.8624	.88103	46	14	.48837	.55964	1.7868	.87264	46	14	.48837	.55964	1.7868	.87264	46		
15	.47332	.53732	1.8611	.88089	45	15	.48862	.56003	1.7856	.87250	45	15	.48862	.56003	1.7856	.87250	45		
16	.47358	.53769	1.8598	.88075	44	16	.48888	.56041	1.7844	.87235	44	16	.48888	.56041	1.7844	.87235	44		
17	.47383	.53807	1.8585	.88062	43	17	.48913	.56079	1.7832	.87221	43	17	.48913	.56079	1.7832	.87221	43		
18	.47409	.53844	1.8572	.88048	42	18	.48938	.56117	1.7820	.87207	42	18	.48938	.56117	1.7820	.87207	42		
19	.47434	.53882	1.8559	.88034	41	19	.48964	.56156	1.7808	.87193	41	19	.48964	.56156	1.7808	.87193	41		
20	.47460	.53920	1.8546	.88020	40	20	.48989	.56194	1.7796	.87178	40	20	.48989	.56194	1.7796	.87178	40		
21	.47486	.53957	1.8533	.88006	39	21	.49014	.56232	1.7783	.87164	39	21	.49014	.56232	1.7783	.87164	39		
22	.47511	.53995	1.8520	.87993	38	22	.49040	.56270	1.7771	.87150	38	22	.49040	.56270	1.7771	.87150	38		
23	.47537	.54032	1.8507	.87979	37	23	.49065	.56309	1.7759	.87136	37	23	.49065	.56309	1.7759	.87136	37		
24	.47562	.54070	1.8495	.87965	36	24	.49090	.56347	1.7747	.87121	36	24	.49090	.56347	1.7747	.87121	36		
25	.47588	.54107	1.8482	.87951	35	25	.49116	.56385	1.7735	.87107	35	25	.49116	.56385	1.7735	.87107	35		
26	.47614	.54145	1.8469	.87937	34	26	.49141	.56424	1.7723	.87093	34	26	.49141	.56424	1.7723	.87093	34		
27	.47639	.54183	1.8456	.87923	33	27	.49166	.56462	1.7711	.87079	33	27	.49166	.56462	1.7711	.87079	33		
28	.47665	.54220	1.8443	.87909	32	28	.49192	.56501	1.7699	.87064	32	28	.49192	.56501	1.7699	.87064	32		
29	.47690	.54258	1.8430	.87896	31	29	.49217	.56539	1.7687	.87050	31	29	.49217	.56539	1.7687	.87050	31		
30	.47716	.54296	1.8418	.87882	30	30	.49242	.56577	1.7675	.87036	30	30	.49242	.56577	1.7675	.87036	30		
31	.47741	.54333	1.8405	.87868	29	31	.49268	.56616	1.7663	.87021	29	31	.49268	.56616	1.7663	.87021	29		
32	.47767	.54371	1.8392	.87854	28	32	.49293	.56654	1.7651	.87007	28	32	.49293	.56654	1.7651	.87007	28		
33	.47793	.54409	1.8379	.87840	27	33	.49318	.56693	1.7639	.86993	27	33	.49318	.56693	1.7639	.86993	27		
34	.47818	.54446	1.8367	.87826	26	34	.49344	.56731	1.7627	.86978	26	34	.49344	.56731	1.7627	.86978	26		
35	.47844	.54484	1.8354	.87812	25	35	.49369	.56769	1.7615	.86964	25	35	.49369	.56769	1.7615	.86964	25		
36	.47869	.54522	1.8341	.87798	24	36	.49394	.56808	1.7603	.86949	24	36	.49394	.56808	1.7603	.86949	24		
37	.47895	.54560	1.8329	.87784	23	37	.49419	.56846	1.7591	.86935	23	37	.49419	.56846	1.7591	.86935	23		
38	.47920	.54597	1.8316	.87770	22	38	.49445	.56885	1.7579	.86921	22	38	.49445	.56885	1.7579	.86921	22		
39	.47946	.54635	1.8303	.87756	21	39	.49470	.56923	1.7567	.86906	21	39	.49470	.56923	1.7567	.86906	21		
40	.47971	.54673	1.8291	.87743	20	40	.49495	.56962	1.7555	.86892	20	40	.49495	.56962	1.7555	.86892	20		
41	.47997	.54711	1.8278	.87729	19	41	.49521	.57000	1.7544	.86878	19	41	.49521	.57000	1.7544	.86878	19		
42	.48022	.54748	1.8265	.87715	18	42	.49546	.57039	1.7532	.86863	18	42	.49546	.57039	1.7532	.86863	18		
43	.48048	.54786	1.8253	.87701	17	43	.49571	.57078	1.7520	.86849	17	43	.49571	.57078	1.7520	.86849	17		
44	.48073	.54824	1.8240	.87687	16	44	.49596	.57116	1.7508	.86834	16	44	.49596	.57116	1.7508	.86834	16		
45	.48099	.54862	1.8228	.87673	15	45	.49622	.57155	1.7496	.86820	15	45	.49622	.57155	1.7496	.86820	15		
46	.48124	.54900	1.8215	.87659	14	46	.49647	.57193	1.7485	.86805	14	46	.49647	.57193	1.7485	.86805	14		
47	.48150	.54938	1.8202	.87645	13	47	.49672	.57232	1.7473	.86791	13	47	.49672	.57232	1.7473	.86791	13		
48	.48175	.54975	1.8190	.87631	12	48	.49697	.57271	1.7461	.86777	12	48	.49697	.57271	1.7461	.86777	12		
49	.48201	.55013	1.8177	.87617	11	49	.49723	.57309	1.7449	.86762	11	49	.49723	.57309	1.7449	.86762	11		
50	.48226	.55051	1.8165	.87603	10	50	.49748	.57348	1.7437	.86748	10	50	.49748	.57348	1.7437	.86748	10		
51	.48252	.55089	1.8152	.87589	9	51	.49773	.57386	1.7426	.86733	9	51	.49773	.57386	1.7426	.86733	9		
52	.48277	.55127	1.8140	.87575	8	52	.49798	.57425	1.7414	.86719	8	52	.49798	.57425	1.7414	.86719	8		
53	.48303	.55165	1.8127	.87561	7	53	.49824	.57464	1.7402	.86704	7	53	.49824	.57464	1.7402	.86704	7		
54	.48328	.55203	1.8115	.87546	6	54	.49849	.57503	1.7391	.86690	6	54	.49849	.57503	1.7391	.86690	6		
55	.48354	.55241	1.8103	.87532	5	55	.49874	.57541	1.7379	.86675	5	55	.49874	.57541	1.7379	.86675	5		
56	.48379	.55279	1.8090	.87518	4	56	.49899	.57580	1.7367	.86661	4	56	.49899	.57580	1.7367	.86661	4		
57	.48405	.55317	1.8078	.87504	3	57	.49924	.57619	1.7355	.86646	3	57	.49924	.57619	1.7355	.86646	3		
58	.48430	.55355	1.8065	.87490	2	58	.49950	.57657	1.7344	.86632	2	58	.49950	.57657	1.7344	.86632	2		
59	.48456	.55393	1.8053	.87476	1	59	.49975	.57696	1.7332	.86617	1	59	.49975	.57696	1.7332	.86617	1		
60	.48481	.55431	1.8040	.87462	0	60	.50000	.57735	1.7321	.86603	0	60	.50000	.57735	1.7321	.86603	0		
/	Cos	Ctn	Tan	Sin	/	/	Cos	Ctn	Tan	Sin	/	/	Cos	Ctn	Tan	Sin	/		

118° (298°)

(241°) 61°

119° (299°)

(240°) 60°

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

30° (210°)					(329°) 149°					31° (211°)					(328°) 148°				
°	Sin	Tan	Ctn	Cos	°	Sin	Tan	Ctn	Cos	°	Sin	Tan	Ctn	Cos	°	Sin	Tan	Ctn	Cos
0	.50000	.57735	1.7321	.86603	60	.51504	.60086	1.6643	.85717	60	.51504	.60086	1.6643	.85717	60	.51504	.60086	1.6643	.85717
1	.50025	.57774	1.7309	.86588	59	.51529	.60126	1.6632	.85702	59	.51529	.60126	1.6632	.85702	59	.51529	.60126	1.6632	.85702
2	.50050	.57813	1.7297	.86573	58	.51554	.60165	1.6621	.85687	58	.51554	.60165	1.6621	.85687	58	.51554	.60165	1.6621	.85687
3	.50076	.57851	1.7286	.86559	57	.51579	.60205	1.6610	.85672	57	.51579	.60205	1.6610	.85672	57	.51579	.60205	1.6610	.85672
4	.50101	.57890	1.7274	.86544	56	.51604	.60245	1.6599	.85657	56	.51604	.60245	1.6599	.85657	56	.51604	.60245	1.6599	.85657
5	.50126	.57929	1.7262	.86530	55	.51628	.60284	1.6588	.85642	55	.51628	.60284	1.6588	.85642	55	.51628	.60284	1.6588	.85642
6	.50151	.57968	1.7251	.86515	54	.51653	.60324	1.6577	.85627	54	.51653	.60324	1.6577	.85627	54	.51653	.60324	1.6577	.85627
7	.50176	.58007	1.7239	.86501	53	.51678	.60364	1.6566	.85612	53	.51678	.60364	1.6566	.85612	53	.51678	.60364	1.6566	.85612
8	.50201	.58046	1.7228	.86486	52	.51703	.60403	1.6555	.85597	52	.51703	.60403	1.6555	.85597	52	.51703	.60403	1.6555	.85597
9	.50227	.58085	1.7216	.86471	51	.51728	.60443	1.6545	.85582	51	.51728	.60443	1.6545	.85582	51	.51728	.60443	1.6545	.85582
10	.50252	.58124	1.7205	.86457	50	.51753	.60483	1.6534	.85567	50	.51753	.60483	1.6534	.85567	50	.51753	.60483	1.6534	.85567
11	.50277	.58162	1.7193	.86442	49	.51778	.60522	1.6523	.85551	49	.51778	.60522	1.6523	.85551	49	.51778	.60522	1.6523	.85551
12	.50302	.58201	1.7182	.86427	48	.51803	.60562	1.6512	.85536	48	.51803	.60562	1.6512	.85536	48	.51803	.60562	1.6512	.85536
13	.50327	.58240	1.7170	.86413	47	.51828	.60602	1.6501	.85521	47	.51828	.60602	1.6501	.85521	47	.51828	.60602	1.6501	.85521
14	.50352	.58279	1.7159	.86398	46	.51852	.60642	1.6490	.85506	46	.51852	.60642	1.6490	.85506	46	.51852	.60642	1.6490	.85506
15	.50377	.58318	1.7147	.86384	45	.51877	.60681	1.6479	.85491	45	.51877	.60681	1.6479	.85491	45	.51877	.60681	1.6479	.85491
16	.50403	.58357	1.7136	.86369	44	.51902	.60721	1.6469	.85476	44	.51902	.60721	1.6469	.85476	44	.51902	.60721	1.6469	.85476
17	.50428	.58396	1.7124	.86354	43	.51927	.60761	1.6458	.85461	43	.51927	.60761	1.6458	.85461	43	.51927	.60761	1.6458	.85461
18	.50453	.58435	1.7113	.86340	42	.51952	.60801	1.6447	.85446	42	.51952	.60801	1.6447	.85446	42	.51952	.60801	1.6447	.85446
19	.50478	.58474	1.7102	.86325	41	.51977	.60841	1.6436	.85431	41	.51977	.60841	1.6436	.85431	41	.51977	.60841	1.6436	.85431
20	.50503	.58513	1.7090	.86310	40	.52002	.60881	1.6426	.85416	40	.52002	.60881	1.6426	.85416	40	.52002	.60881	1.6426	.85416
21	.50528	.58552	1.7079	.86295	39	.52026	.60921	1.6415	.85401	39	.52026	.60921	1.6415	.85401	39	.52026	.60921	1.6415	.85401
22	.50553	.58591	1.7067	.86281	38	.52051	.60960	1.6404	.85385	38	.52051	.60960	1.6404	.85385	38	.52051	.60960	1.6404	.85385
23	.50578	.58631	1.7056	.86266	37	.52076	.61000	1.6393	.85370	37	.52076	.61000	1.6393	.85370	37	.52076	.61000	1.6393	.85370
24	.50603	.58670	1.7045	.86251	36	.52101	.61040	1.6383	.85355	36	.52101	.61040	1.6383	.85355	36	.52101	.61040	1.6383	.85355
25	.50628	.58709	1.7033	.86237	35	.52126	.61080	1.6372	.85340	35	.52126	.61080	1.6372	.85340	35	.52126	.61080	1.6372	.85340
26	.50654	.58748	1.7022	.86222	34	.52151	.61120	1.6361	.85325	34	.52151	.61120	1.6361	.85325	34	.52151	.61120	1.6361	.85325
27	.50679	.58787	1.7011	.86207	33	.52176	.61160	1.6351	.85310	33	.52176	.61160	1.6351	.85310	33	.52176	.61160	1.6351	.85310
28	.50704	.58826	1.6999	.86192	32	.52200	.61200	1.6340	.85294	32	.52200	.61200	1.6340	.85294	32	.52200	.61200	1.6340	.85294
29	.50729	.58865	1.6988	.86178	31	.52225	.61240	1.6329	.85279	31	.52225	.61240	1.6329	.85279	31	.52225	.61240	1.6329	.85279
30	.50754	.58905	1.6977	.86163	30	.52250	.61280	1.6319	.85264	30	.52250	.61280	1.6319	.85264	30	.52250	.61280	1.6319	.85264
31	.50779	.58944	1.6965	.86148	29	.52275	.61320	1.6308	.85249	29	.52275	.61320	1.6308	.85249	29	.52275	.61320	1.6308	.85249
32	.50804	.58983	1.6954	.86133	28	.52299	.61360	1.6297	.85234	28	.52299	.61360	1.6297	.85234	28	.52299	.61360	1.6297	.85234
33	.50829	.59022	1.6943	.86119	27	.52324	.61400	1.6287	.85218	27	.52324	.61400	1.6287	.85218	27	.52324	.61400	1.6287	.85218
34	.50854	.59061	1.6932	.86104	26	.52349	.61440	1.6276	.85203	26	.52349	.61440	1.6276	.85203	26	.52349	.61440	1.6276	.85203
35	.50879	.59101	1.6920	.86089	25	.52374	.61480	1.6265	.85188	25	.52374	.61480	1.6265	.85188	25	.52374	.61480	1.6265	.85188
36	.50904	.59140	1.6909	.86074	24	.52399	.61520	1.6255	.85173	24	.52399	.61520	1.6255	.85173	24	.52399	.61520	1.6255	.85173
37	.50929	.59179	1.6898	.86059	23	.52423	.61561	1.6244	.85157	23	.52423	.61561	1.6244	.85157	23	.52423	.61561	1.6244	.85157
38	.50954	.59218	1.6887	.86045	22	.52448	.61601	1.6234	.85142	22	.52448	.61601	1.6234	.85142	22	.52448	.61601	1.6234	.85142
39	.50979	.59258	1.6875	.86030	21	.52473	.61641	1.6223	.85127	21	.52473	.61641	1.6223	.85127	21	.52473	.61641	1.6223	.85127
40	.51004	.59297	1.6864	.86015	20	.52498	.61681	1.6212	.85112	20	.52498	.61681	1.6212	.85112	20	.52498	.61681	1.6212	.85112
41	.51029	.59336	1.6853	.86000	19	.52522	.61721	1.6202	.85096	19	.52522	.61721	1.6202	.85096	19	.52522	.61721	1.6202	.85096
42	.51054	.59376	1.6842	.85985	18	.52547	.61761	1.6191	.85081	18	.52547	.61761	1.6191	.85081	18	.52547	.61761	1.6191	.85081
43	.51079	.59415	1.6831	.85970	17	.52572	.61801	1.6181	.85066	17	.52572	.61801	1.6181	.85066	17	.52572	.61801	1.6181	.85066
44	.51104	.59454	1.6820	.85956	16	.52597	.61842	1.6170	.85051	16	.52597	.61842	1.6170	.85051	16	.52597	.61842	1.6170	.85051
45	.51129	.59494	1.6808	.85941	15	.52621	.61882	1.6160	.85035	15	.52621	.61882	1.6160	.85035	15	.52621	.61882	1.6160	.85035
46	.51154	.59533	1.6797	.85926	14	.52646	.61922	1.6149	.85020	14	.52646	.61922	1.6149	.85020	14	.52646	.61922	1.6149	.85020
47	.51179	.59573	1.6786	.85911	13	.52671	.61962	1.6139	.85005	13	.52671	.61962	1.6139	.85005	13	.52671	.61962	1.6139	.85005
48	.51204	.59612	1.6775	.85896	12	.52696	.62003	1.6128	.84989	12	.52696	.62003	1.6128	.84989	12	.52696	.62003	1.6128	.84989
49	.51229	.59651	1.6764	.85881	11	.52720	.62043	1.6118	.84974	11	.52720	.62043	1.6118	.84974	11	.52720	.62043	1.6118	.84974
50	.51254	.59691	1.6753	.85866	10	.52745	.62083	1.6107	.84959	10	.52745	.62083	1.6107	.84959	10	.52745	.62083	1.6107	.84959
51	.51279	.59730	1.6742	.85851	9	.52770	.62124	1.6097	.84943	9	.52770	.62124	1.6097	.84943	9	.52770	.62124	1.6097	.84943
52	.51304	.59770	1.6731	.85836	8	.52794	.62164	1.6087	.84928	8	.52794	.62164	1.6087	.84928	8	.52794	.62164	1.6087	.84928
53	.51329	.59809	1.6720	.85821	7	.52819	.62204	1.6076	.84913	7	.52819	.62204	1.6076	.84913	7	.52819	.62204	1.6076	.84913
54	.51354	.59849	1.6709	.85806	6	.52844	.62245	1.6066	.84897	6	.52844	.62245	1.6066	.84897	6	.52844	.62245	1.6066	.84897
55	.51379	.59888	1.6698	.85792	5	.52869	.62285	1.6055	.84882	5	.52869	.62285	1.6055	.84882	5	.52869	.62285	1.6055	.84882
56	.51404	.59928	1.6687	.85777	4	.52893	.62325	1.6045	.84866	4	.52893	.62325	1.6045	.84866	4	.52893	.62325	1.6045	.84866
57	.51429	.59967	1.6676	.85762	3	.52918	.62366	1.6034	.84851	3	.52918	.62366	1.6034	.84851	3	.52918	.62366	1.6034	.84851
58	.51454	.60007	1.6665	.85747	2	.52943	.62406	1.6024	.84836	2	.52943	.62406	1.6024	.84836	2	.52943	.62406	1.6024	.84836
59	.51479	.60046	1.6654	.85732	1	.52967	.62446	1.6014	.84820	1	.52967	.62446	1.6014	.84820	1	.52967	.62446	1.6014	.84820
60	.51504	.60086	1.6643	.85717	0	.52992	.62487	1.6003	.84805	0	.52992	.62487	1.6003</						

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

32° (212°)					(327°) 147°					33° (213°)					(326°) 146°				
/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/		
0	.52992	.62487	1.6003	.84805	60	0	.54464	.64941	1.5399	.83867	60	0	.54464	.64941	1.5399	.83867	60		
1	.53017	.62527	1.5993	.84789	59	1	.54488	.64982	1.5389	.83851	59	1	.54488	.64982	1.5389	.83851	59		
2	.53041	.62568	1.5983	.84774	58	2	.54513	.65024	1.5379	.83835	58	2	.54513	.65024	1.5379	.83835	58		
3	.53066	.62608	1.5972	.84759	57	3	.54537	.65065	1.5369	.83819	57	3	.54537	.65065	1.5369	.83819	57		
4	.53091	.62649	1.5962	.84743	56	4	.54561	.65106	1.5359	.83804	56	4	.54561	.65106	1.5359	.83804	56		
5	.53115	.62689	1.5952	.84728	55	5	.54586	.65148	1.5350	.83788	55	5	.54586	.65148	1.5350	.83788	55		
6	.53140	.62730	1.5941	.84712	54	6	.54610	.65189	1.5340	.83772	54	6	.54610	.65189	1.5340	.83772	54		
7	.53164	.62770	1.5931	.84697	53	7	.54635	.65231	1.5330	.83756	53	7	.54635	.65231	1.5330	.83756	53		
8	.53189	.62811	1.5921	.84681	52	8	.54659	.65272	1.5320	.83740	52	8	.54659	.65272	1.5320	.83740	52		
9	.53214	.62852	1.5911	.84666	51	9	.54683	.65314	1.5311	.83724	51	9	.54683	.65314	1.5311	.83724	51		
10	.53238	.62892	1.5900	.84650	50	10	.54708	.65355	1.5301	.83708	50	10	.54708	.65355	1.5301	.83708	50		
11	.53263	.62933	1.5890	.84635	49	11	.54732	.65397	1.5291	.83692	49	11	.54732	.65397	1.5291	.83692	49		
12	.53288	.62973	1.5880	.84619	48	12	.54756	.65438	1.5282	.83676	48	12	.54756	.65438	1.5282	.83676	48		
13	.53312	.63014	1.5869	.84604	47	13	.54781	.65480	1.5272	.83660	47	13	.54781	.65480	1.5272	.83660	47		
14	.53337	.63055	1.5859	.84588	46	14	.54805	.65521	1.5262	.83645	46	14	.54805	.65521	1.5262	.83645	46		
15	.53361	.63095	1.5849	.84573	45	15	.54829	.65563	1.5253	.83629	45	15	.54829	.65563	1.5253	.83629	45		
16	.53386	.63136	1.5839	.84557	44	16	.54854	.65604	1.5243	.83613	44	16	.54854	.65604	1.5243	.83613	44		
17	.53411	.63177	1.5829	.84542	43	17	.54878	.65646	1.5233	.83597	43	17	.54878	.65646	1.5233	.83597	43		
18	.53435	.63217	1.5818	.84526	42	18	.54902	.65688	1.5224	.83581	42	18	.54902	.65688	1.5224	.83581	42		
19	.53460	.63258	1.5808	.84511	41	19	.54927	.65729	1.5214	.83565	41	19	.54927	.65729	1.5214	.83565	41		
20	.53484	.63299	1.5798	.84495	40	20	.54951	.65771	1.5204	.83549	40	20	.54951	.65771	1.5204	.83549	40		
21	.53509	.63340	1.5788	.84480	39	21	.54975	.65813	1.5195	.83533	39	21	.54975	.65813	1.5195	.83533	39		
22	.53534	.63380	1.5778	.84464	38	22	.54999	.65854	1.5185	.83517	38	22	.54999	.65854	1.5185	.83517	38		
23	.53558	.63421	1.5768	.84448	37	23	.55024	.65896	1.5175	.83501	37	23	.55024	.65896	1.5175	.83501	37		
24	.53583	.63462	1.5757	.84433	36	24	.55048	.65938	1.5166	.83485	36	24	.55048	.65938	1.5166	.83485	36		
25	.53607	.63503	1.5747	.84417	35	25	.55072	.65980	1.5156	.83469	35	25	.55072	.65980	1.5156	.83469	35		
26	.53632	.63544	1.5737	.84402	34	26	.55097	.66021	1.5147	.83453	34	26	.55097	.66021	1.5147	.83453	34		
27	.53656	.63584	1.5727	.84386	33	27	.55121	.66063	1.5137	.83437	33	27	.55121	.66063	1.5137	.83437	33		
28	.53681	.63625	1.5717	.84370	32	28	.55145	.66105	1.5127	.83421	32	28	.55145	.66105	1.5127	.83421	32		
29	.53705	.63666	1.5707	.84355	31	29	.55169	.66147	1.5118	.83405	31	29	.55169	.66147	1.5118	.83405	31		
30	.53730	.63707	1.5697	.84339	30	30	.55194	.66189	1.5108	.83389	30	30	.55194	.66189	1.5108	.83389	30		
31	.53754	.63748	1.5687	.84324	29	31	.55218	.66230	1.5099	.83373	29	31	.55218	.66230	1.5099	.83373	29		
32	.53779	.63789	1.5677	.84308	28	32	.55242	.66272	1.5089	.83356	28	32	.55242	.66272	1.5089	.83356	28		
33	.53804	.63830	1.5667	.84292	27	33	.55266	.66314	1.5080	.83340	27	33	.55266	.66314	1.5080	.83340	27		
34	.53828	.63871	1.5657	.84277	26	34	.55291	.66356	1.5070	.83324	26	34	.55291	.66356	1.5070	.83324	26		
35	.53853	.63912	1.5647	.84261	25	35	.55315	.66398	1.5061	.83308	25	35	.55315	.66398	1.5061	.83308	25		
36	.53877	.63953	1.5637	.84245	24	36	.55339	.66440	1.5051	.83292	24	36	.55339	.66440	1.5051	.83292	24		
37	.53902	.63994	1.5627	.84230	23	37	.55363	.66482	1.5042	.83276	23	37	.55363	.66482	1.5042	.83276	23		
38	.53926	.64035	1.5617	.84214	22	38	.55388	.66524	1.5032	.83260	22	38	.55388	.66524	1.5032	.83260	22		
39	.53951	.64076	1.5607	.84198	21	39	.55412	.66566	1.5023	.83244	21	39	.55412	.66566	1.5023	.83244	21		
40	.53975	.64117	1.5597	.84182	20	40	.55436	.66608	1.5013	.83228	20	40	.55436	.66608	1.5013	.83228	20		
41	.54000	.64158	1.5587	.84167	19	41	.55460	.66650	1.5004	.83212	19	41	.55460	.66650	1.5004	.83212	19		
42	.54024	.64199	1.5577	.84151	18	42	.55484	.66692	1.4994	.83195	18	42	.55484	.66692	1.4994	.83195	18		
43	.54049	.64240	1.5567	.84135	17	43	.55509	.66734	1.4985	.83179	17	43	.55509	.66734	1.4985	.83179	17		
44	.54073	.64281	1.5557	.84120	16	44	.55533	.66776	1.4975	.83163	16	44	.55533	.66776	1.4975	.83163	16		
45	.54097	.64322	1.5547	.84104	15	45	.55557	.66818	1.4966	.83147	15	45	.55557	.66818	1.4966	.83147	15		
46	.54122	.64363	1.5537	.84088	14	46	.55581	.66860	1.4957	.83131	14	46	.55581	.66860	1.4957	.83131	14		
47	.54146	.64404	1.5527	.84072	13	47	.55605	.66902	1.4947	.83115	13	47	.55605	.66902	1.4947	.83115	13		
48	.54171	.64446	1.5517	.84057	12	48	.55630	.66944	1.4938	.83098	12	48	.55630	.66944	1.4938	.83098	12		
49	.54195	.64487	1.5507	.84041	11	49	.55654	.66986	1.4928	.83082	11	49	.55654	.66986	1.4928	.83082	11		
50	.54220	.64528	1.5497	.84025	10	50	.55678	.67028	1.4919	.83066	10	50	.55678	.67028	1.4919	.83066	10		
51	.54244	.64569	1.5487	.84009	9	51	.55702	.67071	1.4910	.83050	9	51	.55702	.67071	1.4910	.83050	9		
52	.54269	.64610	1.5477	.83994	8	52	.55726	.67113	1.4900	.83034	8	52	.55726	.67113	1.4900	.83034	8		
53	.54293	.64652	1.5468	.83978	7	53	.55750	.67155	1.4891	.83017	7	53	.55750	.67155	1.4891	.83017	7		
54	.54317	.64693	1.5458	.83962	6	54	.55775	.67197	1.4882	.83001	6	54	.55775	.67197	1.4882	.83001	6		
55	.54342	.64734	1.5448	.83946	5	55	.55799	.67239	1.4872	.82985	5	55	.55799	.67239	1.4872	.82985	5		
56	.54366	.64775	1.5438	.83930	4	56	.55823	.67282	1.4863	.82969	4	56	.55823	.67282	1.4863	.82969	4		
57	.54391	.64817	1.5428	.83915	3	57	.55847	.67324	1.4854	.82953	3	57	.55847	.67324	1.4854	.82953	3		
58	.54415	.64858	1.5418	.83899	2	58	.55871	.67366	1.4844	.82936	2	58	.55871	.67366	1.4844	.82936	2		
59	.54440	.64899	1.5408	.83883	1	59	.55895	.67409	1.4835	.82920	1	59	.55895	.67409	1.4835	.82920	1		
60	.54464	.64941	1.5399	.83867	0	60	.55919	.67451	1.4826	.82904	0	60	.55919	.67451	1.4826	.82904	0		
/	Cos	Ctn	Tan	Sin	/	/	Cos	Ctn	Tan	Sin	/	/	Cos	Ctn	Tan	Sin	/		

122° (302°)

(237°) 57°

123° (303°)

(236°) 56°

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

34° (214°)					(325°) 145°					35° (215°)					(324°) 144°						
/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/				
0	.55919	.67451	1.4826	.82904	60	0	.57358	.70021	1.4281	.81915	60	0	.57358	.70021	1.4281	.81915	60				
1	.55943	.67493	1.4816	.82887	59	1	.57381	.70064	1.4273	.81899	59	1	.57381	.70064	1.4273	.81899	59				
2	.55968	.67536	1.4807	.82871	58	2	.57405	.70107	1.4264	.81882	58	2	.57405	.70107	1.4264	.81882	58				
3	.55992	.67578	1.4798	.82855	57	3	.57429	.70151	1.4255	.81865	57	3	.57429	.70151	1.4255	.81865	57				
4	.56016	.67620	1.4788	.82839	56	4	.57453	.70194	1.4246	.81848	56	4	.57453	.70194	1.4246	.81848	56				
5	.56040	.67663	1.4779	.82822	55	5	.57477	.70238	1.4237	.81832	55	5	.57477	.70238	1.4237	.81832	55				
6	.56064	.67705	1.4770	.82806	54	6	.57501	.70281	1.4229	.81815	54	6	.57501	.70281	1.4229	.81815	54				
7	.56088	.67748	1.4761	.82790	53	7	.57524	.70325	1.4220	.81798	53	7	.57524	.70325	1.4220	.81798	53				
8	.56112	.67790	1.4751	.82773	52	8	.57548	.70368	1.4211	.81782	52	8	.57548	.70368	1.4211	.81782	52				
9	.56136	.67832	1.4742	.82757	51	9	.57572	.70412	1.4202	.81765	51	9	.57572	.70412	1.4202	.81765	51				
10	.56160	.67875	1.4733	.82741	50	10	.57596	.70455	1.4193	.81748	50	10	.57596	.70455	1.4193	.81748	50				
11	.56184	.67917	1.4724	.82724	49	11	.57619	.70499	1.4185	.81731	49	11	.57619	.70499	1.4185	.81731	49				
12	.56208	.67960	1.4715	.82708	48	12	.57643	.70542	1.4176	.81714	48	12	.57643	.70542	1.4176	.81714	48				
13	.56232	.68002	1.4705	.82692	47	13	.57667	.70586	1.4167	.81698	47	13	.57667	.70586	1.4167	.81698	47				
14	.56256	.68045	1.4696	.82675	46	14	.57691	.70629	1.4158	.81681	46	14	.57691	.70629	1.4158	.81681	46				
15	.56280	.68088	1.4687	.82659	45	15	.57715	.70673	1.4150	.81664	45	15	.57715	.70673	1.4150	.81664	45				
16	.56305	.68130	1.4678	.82643	44	16	.57738	.70717	1.4141	.81647	44	16	.57738	.70717	1.4141	.81647	44				
17	.56329	.68173	1.4669	.82626	43	17	.57762	.70760	1.4132	.81631	43	17	.57762	.70760	1.4132	.81631	43				
18	.56353	.68215	1.4659	.82610	42	18	.57786	.70804	1.4124	.81614	42	18	.57786	.70804	1.4124	.81614	42				
19	.56377	.68258	1.4650	.82593	41	19	.57810	.70848	1.4115	.81597	41	19	.57810	.70848	1.4115	.81597	41				
20	.56401	.68301	1.4641	.82577	40	20	.57833	.70891	1.4106	.81580	40	20	.57833	.70891	1.4106	.81580	40				
21	.56425	.68343	1.4632	.82561	39	21	.57857	.70935	1.4097	.81563	39	21	.57857	.70935	1.4097	.81563	39				
22	.56449	.68386	1.4623	.82544	38	22	.57881	.70979	1.4089	.81546	38	22	.57881	.70979	1.4089	.81546	38				
23	.56473	.68429	1.4614	.82528	37	23	.57904	.71023	1.4080	.81530	37	23	.57904	.71023	1.4080	.81530	37				
24	.56497	.68471	1.4605	.82511	36	24	.57928	.71066	1.4071	.81513	36	24	.57928	.71066	1.4071	.81513	36				
25	.56521	.68514	1.4596	.82495	35	25	.57952	.71110	1.4063	.81496	35	25	.57952	.71110	1.4063	.81496	35				
26	.56545	.68557	1.4586	.82478	34	26	.57976	.71154	1.4054	.81479	34	26	.57976	.71154	1.4054	.81479	34				
27	.56569	.68600	1.4577	.82462	33	27	.57999	.71198	1.4045	.81462	33	27	.57999	.71198	1.4045	.81462	33				
28	.56593	.68642	1.4568	.82446	32	28	.58023	.71242	1.4037	.81445	32	28	.58023	.71242	1.4037	.81445	32				
29	.56617	.68685	1.4559	.82429	31	29	.58047	.71285	1.4028	.81428	31	29	.58047	.71285	1.4028	.81428	31				
30	.56641	.68728	1.4550	.82413	30	30	.58070	.71329	1.4019	.81412	30	30	.58070	.71329	1.4019	.81412	30				
31	.56665	.68771	1.4541	.82396	29	31	.58094	.71373	1.4011	.81395	29	31	.58094	.71373	1.4011	.81395	29				
32	.56689	.68814	1.4532	.82380	28	32	.58118	.71417	1.4002	.81378	28	32	.58118	.71417	1.4002	.81378	28				
33	.56713	.68857	1.4523	.82363	27	33	.58141	.71461	1.3994	.81361	27	33	.58141	.71461	1.3994	.81361	27				
34	.56736	.68900	1.4514	.82347	26	34	.58165	.71505	1.3985	.81344	26	34	.58165	.71505	1.3985	.81344	26				
35	.56760	.68942	1.4505	.82330	25	35	.58189	.71549	1.3976	.81327	25	35	.58189	.71549	1.3976	.81327	25				
36	.56784	.68985	1.4496	.82314	24	36	.58212	.71593	1.3968	.81310	24	36	.58212	.71593	1.3968	.81310	24				
37	.56808	.69028	1.4487	.82297	23	37	.58236	.71637	1.3959	.81293	23	37	.58236	.71637	1.3959	.81293	23				
38	.56832	.69071	1.4478	.82281	22	38	.58260	.71681	1.3951	.81276	22	38	.58260	.71681	1.3951	.81276	22				
39	.56856	.69114	1.4469	.82264	21	39	.58283	.71725	1.3942	.81259	21	39	.58283	.71725	1.3942	.81259	21				
40	.56880	.69157	1.4460	.82248	20	40	.58307	.71769	1.3934	.81242	20	40	.58307	.71769	1.3934	.81242	20				
41	.56904	.69200	1.4451	.82231	19	41	.58330	.71813	1.3925	.81225	19	41	.58330	.71813	1.3925	.81225	19				
42	.56928	.69243	1.4442	.82214	18	42	.58354	.71857	1.3916	.81208	18	42	.58354	.71857	1.3916	.81208	18				
43	.56952	.69286	1.4433	.82198	17	43	.58378	.71901	1.3908	.81191	17	43	.58378	.71901	1.3908	.81191	17				
44	.56976	.69329	1.4424	.82181	16	44	.58401	.71946	1.3899	.81174	16	44	.58401	.71946	1.3899	.81174	16				
45	.57000	.69372	1.4415	.82165	15	45	.58425	.71990	1.3891	.81157	15	45	.58425	.71990	1.3891	.81157	15				
46	.57024	.69416	1.4406	.82148	14	46	.58449	.72034	1.3882	.81140	14	46	.58449	.72034	1.3882	.81140	14				
47	.57047	.69459	1.4397	.82132	13	47	.58472	.72078	1.3874	.81123	13	47	.58472	.72078	1.3874	.81123	13				
48	.57071	.69502	1.4388	.82115	12	48	.58496	.72122	1.3865	.81106	12	48	.58496	.72122	1.3865	.81106	12				
49	.57095	.69545	1.4379	.82098	11	49	.58519	.72167	1.3857	.81089	11	49	.58519	.72167	1.3857	.81089	11				
50	.57119	.69588	1.4370	.82082	10	50	.58543	.72211	1.3848	.81072	10	50	.58543	.72211	1.3848	.81072	10				
51	.57143	.69631	1.4361	.82065	9	51	.58567	.72255	1.3840	.81055	9	51	.58567	.72255	1.3840	.81055	9				
52	.57167	.69675	1.4352	.82048	8	52	.58590	.72299	1.3831	.81038	8	52	.58590	.72299	1.3831	.81038	8				
53	.57191	.69718	1.4344	.82032	7	53	.58614	.72344	1.3823	.81021	7	53	.58614	.72344	1.3823	.81021	7				
54	.57215	.69761	1.4335	.82015	6	54	.58637	.72388	1.3814	.81004	6	54	.58637	.72388	1.3814	.81004	6				
55	.57238	.69804	1.4326	.81999	5	55	.58661	.72432	1.3806	.80987	5	55	.58661	.72432	1.3806	.80987	5				
56	.57262	.69847	1.4317	.81982	4	56	.58684	.72477	1.3798	.80970	4	56	.58684	.72477	1.3798	.80970	4				
57	.57286	.69891	1.4308	.81965	3	57	.58708	.72521	1.3789	.80953	3	57	.58708	.72521	1.3789	.80953	3				
58	.57310	.69934	1.4299	.81949	2	58	.58731	.72565	1.3781	.80936	2	58	.58731	.72565	1.3781	.80936	2				
59	.57334	.69977	1.4290	.81932	1	59	.58755	.72610	1.3772	.80919	1	59	.58755	.72610	1.3772	.80919	1				
60	.57358	.70021	1.4281	.81915	0	60	.58779	.72654	1.3764	.80902	0	60	.58779	.72654	1.3764	.80902	0				
/	Cos	Ctn	Tan	Sin	/	/	Cos	Ctn	Tan	Sin	/	/	Cos	Ctn	Tan	Sin	/				
124° (304°)	(235°) 55°					125° (305°)	(234°) 54°					(234°) 54°					(234°) 54°				

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

36° (216°)					(323°) 143°					37° (217°)					(322°) 142°				
/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/		
0	58779	.72654	1.3764	.80902	60	0	60182	.75355	1.3270	.79864	60	1	60205	.75401	1.3262	.79846	59		
1	58802	.72699	1.3755	.80885	59	1	60228	.75447	1.3254	.79829	58	2	60251	.75492	1.3246	.79811	57		
2	58826	.72743	1.3747	.80867	58	2	60274	.75538	1.3238	.79793	56	3	60298	.75584	1.3230	.79776	55		
3	58849	.72788	1.3739	.80850	57	3	60321	.75629	1.3222	.79758	54	4	60344	.75675	1.3214	.79741	53		
4	58873	.72832	1.3730	.80833	56	4	60367	.75721	1.3206	.79723	52	5	60390	.75767	1.3198	.79706	51		
5	58896	.72877	1.3722	.80816	55	5	60414	.75812	1.3190	.79688	50	6	60437	.75858	1.3182	.79671	49		
6	58920	.72921	1.3713	.80799	54	6	60460	.75904	1.3175	.79653	48	7	60483	.75950	1.3167	.79635	47		
7	58943	.72966	1.3705	.80782	53	7	60506	.75996	1.3159	.79618	46	8	60529	.76042	1.3151	.79600	45		
8	58967	.73010	1.3697	.80765	52	8	60553	.76088	1.3143	.79583	44	9	60576	.76134	1.3135	.79565	43		
9	58990	.73055	1.3688	.80748	51	9	60599	.76180	1.3127	.79547	42	10	60622	.76226	1.3119	.79530	41		
10	59014	.73100	1.3680	.80730	50	10	60645	.76272	1.3111	.79512	40	11	60668	.76318	1.3103	.79494	39		
11	59037	.73144	1.3672	.80713	49	11	60691	.76364	1.3095	.79477	38	12	60714	.76410	1.3087	.79459	37		
12	59061	.73189	1.3663	.80696	48	12	60738	.76456	1.3079	.79441	36	13	60761	.76502	1.3072	.79424	35		
13	59084	.73234	1.3655	.80679	47	13	60784	.76548	1.3064	.79406	34	14	60807	.76594	1.3056	.79388	33		
14	59108	.73278	1.3647	.80662	46	14	60830	.76640	1.3048	.79371	32	15	60853	.76686	1.3040	.79353	31		
15	59131	.73323	1.3638	.80644	45	15	60876	.76733	1.3032	.79335	30	16	60899	.76779	1.3024	.79318	29		
16	59154	.73368	1.3630	.80627	44	16	60922	.76825	1.3017	.79300	28	17	60945	.76871	1.3009	.79282	27		
17	59178	.73413	1.3622	.80610	43	17	60968	.76918	1.3001	.79264	26	18	60991	.76964	1.2993	.79247	25		
18	59201	.73457	1.3613	.80593	42	18	61015	.77010	1.2985	.79229	24	19	61038	.77057	1.2977	.79211	23		
19	59225	.73502	1.3605	.80576	41	19	61061	.77103	1.2970	.79193	22	20	61084	.77149	1.2962	.79176	21		
20	59248	.73547	1.3597	.80558	40	20	61107	.77196	1.2954	.79158	20	21	61130	.77242	1.2946	.79140	19		
21	59272	.73592	1.3588	.80541	39	21	61153	.77289	1.2938	.79122	18	22	61176	.77335	1.2931	.79105	17		
22	59295	.73637	1.3580	.80524	38	22	61199	.77382	1.2923	.79087	16	23	61222	.77428	1.2915	.79069	15		
23	59318	.73681	1.3572	.80507	37	23	61245	.77475	1.2907	.79051	14	24	61268	.77521	1.2900	.79033	13		
24	59342	.73726	1.3564	.80489	36	24	61291	.77568	1.2892	.79016	12	25	61291	.77568	1.2892	.79016	12		
25	59365	.73771	1.3555	.80472	35	25	61314	.77615	1.2884	.78998	11	26	61337	.77661	1.2876	.78980	10		
26	59389	.73816	1.3547	.80455	34	26	61360	.77708	1.2869	.78962	9	27	61383	.77754	1.2861	.78944	8		
27	59412	.73861	1.3539	.80438	33	27	61406	.77801	1.2853	.78926	7	28	61429	.77848	1.2846	.78908	6		
28	59436	.73906	1.3531	.80420	32	28	61451	.77895	1.2838	.78891	5	29	61474	.77941	1.2830	.78873	4		
29	59459	.73951	1.3522	.80403	31	29	61497	.77988	1.2822	.78855	3	30	61520	.78035	1.2815	.78837	2		
30	59482	.73996	1.3514	.80386	30	30	61543	.78082	1.2807	.78819	1	31	61566	.78129	1.2799	.78801	0		
31	59506	.74041	1.3506	.80368	29	31						32							
32	59529	.74086	1.3498	.80351	28	32						33							
33	59552	.74131	1.3490	.80334	27	33						34							
34	59576	.74176	1.3481	.80316	26	34						35							
35	59599	.74221	1.3473	.80299	25	35						36							
36	59622	.74267	1.3465	.80282	24	36						37							
37	59646	.74312	1.3457	.80264	23	37						38							
38	59669	.74357	1.3449	.80247	22	38						39							
39	59693	.74402	1.3440	.80230	21	39						40							
40	59716	.74447	1.3432	.80212	20	40						41							
41	59739	.74492	1.3424	.80195	19	41						42							
42	59763	.74538	1.3416	.80178	18	42						43							
43	59786	.74583	1.3408	.80160	17	43						44							
44	59809	.74628	1.3400	.80143	16	44						45							
45	59832	.74674	1.3392	.80125	15	45						46							
46	59856	.74719	1.3384	.80108	14	46						47							
47	59879	.74764	1.3375	.80091	13	47						48							
48	59902	.74810	1.3367	.80073	12	48						49							
49	59926	.74855	1.3359	.80056	11	49						50							
50	59949	.74900	1.3351	.80038	10	50						51							
51	59972	.74946	1.3343	.80021	9	51						52							
52	59995	.74991	1.3335	.80003	8	52						53							
53	60019	.75037	1.3327	.79986	7	53						54							
54	60042	.75082	1.3319	.79968	6	54						55							
55	60065	.75128	1.3311	.79951	5	55						56							
56	60089	.75173	1.3303	.79934	4	56						57							
57	60112	.75219	1.3295	.79916	3	57						58							
58	60135	.75264	1.3287	.79899	2	58						59							
59	60158	.75310	1.3278	.79881	1	59						60							
60	60182	.75355	1.3270	.79864	0	60													
/	Cos	Ctn	Tan	Sin	/	/	Cos	Ctn	Tan	Sin	/								

126° (306°)

(233°) 53°

127° (307°)

(232°) 52°

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

38° (218°)					(321°) 141°					39° (219°)					(320°) 140°						
/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos	/
0	.61566	.78129	1.2799	.78801	60	0	.62932	.80978	1.2349	.77715	60	.62932	.80978	1.2349	.77715	60	.62932	.80978	1.2349	.77715	60
1	.61589	.78175	1.2792	.78783	59	1	.62955	.81027	1.2342	.77696	59	.62955	.81027	1.2342	.77696	59	.62955	.81027	1.2342	.77696	59
2	.61612	.78222	1.2784	.78765	58	2	.62977	.81075	1.2334	.77678	58	.62977	.81075	1.2334	.77678	58	.62977	.81075	1.2334	.77678	58
3	.61635	.78269	1.2776	.78747	57	3	.63000	.81123	1.2327	.77660	57	.63000	.81123	1.2327	.77660	57	.63000	.81123	1.2327	.77660	57
4	.61658	.78316	1.2769	.78729	56	4	.63022	.81171	1.2320	.77641	56	.63022	.81171	1.2320	.77641	56	.63022	.81171	1.2320	.77641	56
5	.61681	.78363	1.2761	.78711	55	5	.63045	.81220	1.2312	.77623	55	.63045	.81220	1.2312	.77623	55	.63045	.81220	1.2312	.77623	55
6	.61704	.78410	1.2753	.78694	54	6	.63068	.81268	1.2305	.77605	54	.63068	.81268	1.2305	.77605	54	.63068	.81268	1.2305	.77605	54
7	.61726	.78457	1.2746	.78676	53	7	.63090	.81316	1.2298	.77586	53	.63090	.81316	1.2298	.77586	53	.63090	.81316	1.2298	.77586	53
8	.61749	.78504	1.2738	.78658	52	8	.63113	.81364	1.2290	.77568	52	.63113	.81364	1.2290	.77568	52	.63113	.81364	1.2290	.77568	52
9	.61772	.78551	1.2731	.78640	51	9	.63135	.81413	1.2283	.77550	51	.63135	.81413	1.2283	.77550	51	.63135	.81413	1.2283	.77550	51
10	.61795	.78598	1.2723	.78622	50	10	.63158	.81461	1.2276	.77531	50	.63158	.81461	1.2276	.77531	50	.63158	.81461	1.2276	.77531	50
11	.61818	.78645	1.2715	.78604	49	11	.63180	.81510	1.2268	.77513	49	.63180	.81510	1.2268	.77513	49	.63180	.81510	1.2268	.77513	49
12	.61841	.78692	1.2708	.78586	48	12	.63203	.81558	1.2261	.77494	48	.63203	.81558	1.2261	.77494	48	.63203	.81558	1.2261	.77494	48
13	.61864	.78739	1.2700	.78568	47	13	.63225	.81606	1.2254	.77476	47	.63225	.81606	1.2254	.77476	47	.63225	.81606	1.2254	.77476	47
14	.61887	.78786	1.2693	.78550	46	14	.63248	.81655	1.2247	.77458	46	.63248	.81655	1.2247	.77458	46	.63248	.81655	1.2247	.77458	46
15	.61909	.78834	1.2685	.78532	45	15	.63271	.81703	1.2239	.77439	45	.63271	.81703	1.2239	.77439	45	.63271	.81703	1.2239	.77439	45
16	.61932	.78881	1.2677	.78514	44	16	.63293	.81752	1.2232	.77421	44	.63293	.81752	1.2232	.77421	44	.63293	.81752	1.2232	.77421	44
17	.61955	.78928	1.2670	.78496	43	17	.63316	.81800	1.2225	.77402	43	.63316	.81800	1.2225	.77402	43	.63316	.81800	1.2225	.77402	43
18	.61978	.78975	1.2662	.78478	42	18	.63338	.81849	1.2218	.77384	42	.63338	.81849	1.2218	.77384	42	.63338	.81849	1.2218	.77384	42
19	.62001	.79022	1.2655	.78460	41	19	.63361	.81898	1.2210	.77366	41	.63361	.81898	1.2210	.77366	41	.63361	.81898	1.2210	.77366	41
20	.62024	.79070	1.2647	.78442	40	20	.63383	.81946	1.2203	.77347	40	.63383	.81946	1.2203	.77347	40	.63383	.81946	1.2203	.77347	40
21	.62046	.79117	1.2640	.78424	39	21	.63406	.81995	1.2196	.77329	39	.63406	.81995	1.2196	.77329	39	.63406	.81995	1.2196	.77329	39
22	.62069	.79164	1.2632	.78405	38	22	.63428	.82044	1.2189	.77310	38	.63428	.82044	1.2189	.77310	38	.63428	.82044	1.2189	.77310	38
23	.62092	.79212	1.2624	.78387	37	23	.63451	.82092	1.2181	.77292	37	.63451	.82092	1.2181	.77292	37	.63451	.82092	1.2181	.77292	37
24	.62115	.79259	1.2617	.78369	36	24	.63473	.82141	1.2174	.77273	36	.63473	.82141	1.2174	.77273	36	.63473	.82141	1.2174	.77273	36
25	.62138	.79306	1.2609	.78351	35	25	.63496	.82190	1.2167	.77255	35	.63496	.82190	1.2167	.77255	35	.63496	.82190	1.2167	.77255	35
26	.62160	.79354	1.2602	.78333	34	26	.63518	.82238	1.2160	.77236	34	.63518	.82238	1.2160	.77236	34	.63518	.82238	1.2160	.77236	34
27	.62183	.79401	1.2594	.78315	33	27	.63540	.82287	1.2153	.77218	33	.63540	.82287	1.2153	.77218	33	.63540	.82287	1.2153	.77218	33
28	.62206	.79449	1.2587	.78297	32	28	.63563	.82336	1.2145	.77199	32	.63563	.82336	1.2145	.77199	32	.63563	.82336	1.2145	.77199	32
29	.62229	.79496	1.2579	.78279	31	29	.63585	.82385	1.2138	.77181	31	.63585	.82385	1.2138	.77181	31	.63585	.82385	1.2138	.77181	31
30	.62251	.79544	1.2572	.78261	30	30	.63608	.82434	1.2131	.77162	30	.63608	.82434	1.2131	.77162	30	.63608	.82434	1.2131	.77162	30
31	.62274	.79591	1.2564	.78243	29	31	.63630	.82483	1.2124	.77144	29	.63630	.82483	1.2124	.77144	29	.63630	.82483	1.2124	.77144	29
32	.62297	.79639	1.2557	.78225	28	32	.63653	.82531	1.2117	.77125	28	.63653	.82531	1.2117	.77125	28	.63653	.82531	1.2117	.77125	28
33	.62320	.79686	1.2549	.78206	27	33	.63675	.82580	1.2109	.77107	27	.63675	.82580	1.2109	.77107	27	.63675	.82580	1.2109	.77107	27
34	.62342	.79734	1.2542	.78188	26	34	.63698	.82629	1.2102	.77088	26	.63698	.82629	1.2102	.77088	26	.63698	.82629	1.2102	.77088	26
35	.62365	.79781	1.2534	.78170	25	35	.63720	.82678	1.2095	.77070	25	.63720	.82678	1.2095	.77070	25	.63720	.82678	1.2095	.77070	25
36	.62388	.79829	1.2527	.78152	24	36	.63742	.82727	1.2088	.77051	24	.63742	.82727	1.2088	.77051	24	.63742	.82727	1.2088	.77051	24
37	.62411	.79877	1.2519	.78134	23	37	.63765	.82776	1.2081	.77033	23	.63765	.82776	1.2081	.77033	23	.63765	.82776	1.2081	.77033	23
38	.62433	.79924	1.2512	.78116	22	38	.63787	.82825	1.2074	.77014	22	.63787	.82825	1.2074	.77014	22	.63787	.82825	1.2074	.77014	22
39	.62456	.79972	1.2504	.78098	21	39	.63810	.82874	1.2066	.76996	21	.63810	.82874	1.2066	.76996	21	.63810	.82874	1.2066	.76996	21
40	.62479	.80020	1.2497	.78079	20	40	.63832	.82923	1.2059	.76977	20	.63832	.82923	1.2059	.76977	20	.63832	.82923	1.2059	.76977	20
41	.62502	.80067	1.2489	.78061	19	41	.63854	.82972	1.2052	.76959	19	.63854	.82972	1.2052	.76959	19	.63854	.82972	1.2052	.76959	19
42	.62524	.80115	1.2482	.78043	18	42	.63877	.83021	1.2045	.76940	18	.63877	.83021	1.2045	.76940	18	.63877	.83021	1.2045	.76940	18
43	.62547	.80163	1.2475	.78025	17	43	.63899	.83070	1.2038	.76921	17	.63899	.83070	1.2038	.76921	17	.63899	.83070	1.2038	.76921	17
44	.62570	.80211	1.2467	.78007	16	44	.63922	.83120	1.2031	.76903	16	.63922	.83120	1.2031	.76903	16	.63922	.83120	1.2031	.76903	16
45	.62592	.80258	1.2460	.77988	15	45	.63944	.83169	1.2024	.76884	15	.63944	.83169	1.2024	.76884	15	.63944	.83169	1.2024	.76884	15
46	.62615	.80306	1.2452	.77970	14	46	.63966	.83218	1.2017	.76866	14	.63966	.83218	1.2017	.76866	14	.63966	.83218	1.2017	.76866	14
47	.62638	.80354	1.2445	.77952	13	47	.63989	.83268	1.2010	.76847	13	.63989	.83268	1.2010	.76847	13	.63989	.83268	1.2010	.76847	13
48	.62660	.80402	1.2437	.77934	12	48	.64011	.83317	1.2002	.76828	12	.64011	.83317	1.2002	.76828	12	.64011	.83317	1.2002	.76828	12
49	.62683	.80450	1.2430	.77916	11	49	.64033	.83366	1.1995	.76810	11	.64033	.83366	1.1995	.76810	11	.64033	.83366	1.1995	.76810	11
50	.62706	.80498	1.2423	.77897	10	50	.64056	.83415	1.1988	.76791	10	.64056	.83415	1.1988	.76791	10	.64056	.83415	1.1988	.76791	10
51	.62728	.80546	1.2415	.77879	9	51	.64078	.83465	1.1981	.76772	9	.64078	.83465	1.1981	.76772	9	.64078	.83465	1.1981	.76772	9
52	.62751	.80594	1.2408	.77861	8	52	.64100	.83514	1.1974	.76754	8	.64100	.83514	1.1974	.76754	8	.64100	.83514	1.1974	.76754	8
53	.62774	.80642	1.2401	.77843	7	53	.64123	.83564	1.1967	.76735	7	.64123	.83564	1.1967	.76735	7	.64123	.83564	1.1967	.76735	7
54	.62796	.80690	1.2393	.77824	6	54	.64145	.83613	1.1960	.76717	6	.64145	.83613	1.1960	.76717	6	.64145	.83613	1.1960	.76717	6
55	.62819	.80738	1.2386	.77806	5	55	.64167	.83662	1.1953	.76698	5	.64167	.83662	1.1953	.76698	5	.64167	.83662	1.1953	.76698	5
56	.62842	.80786	1.2378	.77788	4	56	.64190	.83712	1.1946	.76679	4	.64190	.83712	1							

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

40° (220°)					(319°) 139°					41° (221°)					(318°) 138°				
°	Sin	Tan	Ctn	Cos	°	Sin	Tan	Ctn	Cos	°	Sin	Tan	Ctn	Cos	°	Sin	Tan	Ctn	Cos
0	.64279	.83910	1.1918	.76604	60	.65606	.86929	1.1504	.75471	60	.65606	.86929	1.1504	.75471	60	.65606	.86929	1.1504	.75471
1	.64301	.83960	1.1910	.76586	59	.65628	.86980	1.1497	.75452	59	.65628	.86980	1.1497	.75452	59	.65628	.86980	1.1497	.75452
2	.64323	.84009	1.1903	.76567	58	.65650	.87031	1.1490	.75433	58	.65650	.87031	1.1490	.75433	58	.65650	.87031	1.1490	.75433
3	.64346	.84059	1.1896	.76548	57	.65672	.87082	1.1483	.75414	57	.65672	.87082	1.1483	.75414	57	.65672	.87082	1.1483	.75414
4	.64368	.84108	1.1889	.76530	56	.65694	.87133	1.1477	.75395	56	.65694	.87133	1.1477	.75395	56	.65694	.87133	1.1477	.75395
5	.64390	.84158	1.1882	.76511	55	.65716	.87184	1.1470	.75375	55	.65716	.87184	1.1470	.75375	55	.65716	.87184	1.1470	.75375
6	.64412	.84208	1.1875	.76492	54	.65738	.87236	1.1463	.75356	54	.65738	.87236	1.1463	.75356	54	.65738	.87236	1.1463	.75356
7	.64435	.84258	1.1868	.76473	53	.65759	.87287	1.1456	.75337	53	.65759	.87287	1.1456	.75337	53	.65759	.87287	1.1456	.75337
8	.64457	.84307	1.1861	.76455	52	.65781	.87338	1.1450	.75318	52	.65781	.87338	1.1450	.75318	52	.65781	.87338	1.1450	.75318
9	.64479	.84357	1.1854	.76436	51	.65803	.87389	1.1443	.75299	51	.65803	.87389	1.1443	.75299	51	.65803	.87389	1.1443	.75299
10	.64501	.84407	1.1847	.76417	50	.65825	.87441	1.1436	.75280	50	.65825	.87441	1.1436	.75280	50	.65825	.87441	1.1436	.75280
11	.64524	.84457	1.1840	.76398	49	.65847	.87492	1.1430	.75261	49	.65847	.87492	1.1430	.75261	49	.65847	.87492	1.1430	.75261
12	.64546	.84507	1.1833	.76380	48	.65869	.87543	1.1423	.75241	48	.65869	.87543	1.1423	.75241	48	.65869	.87543	1.1423	.75241
13	.64568	.84556	1.1826	.76361	47	.65891	.87595	1.1416	.75222	47	.65891	.87595	1.1416	.75222	47	.65891	.87595	1.1416	.75222
14	.64590	.84606	1.1819	.76342	46	.65913	.87646	1.1410	.75203	46	.65913	.87646	1.1410	.75203	46	.65913	.87646	1.1410	.75203
15	.64612	.84656	1.1812	.76323	45	.65935	.87698	1.1403	.75184	45	.65935	.87698	1.1403	.75184	45	.65935	.87698	1.1403	.75184
16	.64635	.84706	1.1806	.76304	44	.65956	.87749	1.1396	.75165	44	.65956	.87749	1.1396	.75165	44	.65956	.87749	1.1396	.75165
17	.64657	.84756	1.1799	.76286	43	.65978	.87801	1.1389	.75146	43	.65978	.87801	1.1389	.75146	43	.65978	.87801	1.1389	.75146
18	.64679	.84806	1.1792	.76267	42	.66000	.87852	1.1383	.75126	42	.66000	.87852	1.1383	.75126	42	.66000	.87852	1.1383	.75126
19	.64701	.84856	1.1785	.76248	41	.66022	.87904	1.1376	.75107	41	.66022	.87904	1.1376	.75107	41	.66022	.87904	1.1376	.75107
20	.64723	.84906	1.1778	.76229	40	.66044	.87955	1.1369	.75088	40	.66044	.87955	1.1369	.75088	40	.66044	.87955	1.1369	.75088
21	.64746	.84956	1.1771	.76210	39	.66066	.88007	1.1363	.75069	39	.66066	.88007	1.1363	.75069	39	.66066	.88007	1.1363	.75069
22	.64768	.85006	1.1764	.76192	38	.66088	.88059	1.1356	.75050	38	.66088	.88059	1.1356	.75050	38	.66088	.88059	1.1356	.75050
23	.64790	.85057	1.1757	.76173	37	.66109	.88110	1.1349	.75030	37	.66109	.88110	1.1349	.75030	37	.66109	.88110	1.1349	.75030
24	.64812	.85107	1.1750	.76154	36	.66131	.88162	1.1343	.75011	36	.66131	.88162	1.1343	.75011	36	.66131	.88162	1.1343	.75011
25	.64834	.85157	1.1743	.76135	35	.66153	.88214	1.1336	.74992	35	.66153	.88214	1.1336	.74992	35	.66153	.88214	1.1336	.74992
26	.64856	.85207	1.1736	.76116	34	.66175	.88265	1.1329	.74973	34	.66175	.88265	1.1329	.74973	34	.66175	.88265	1.1329	.74973
27	.64878	.85257	1.1729	.76097	33	.66197	.88317	1.1323	.74953	33	.66197	.88317	1.1323	.74953	33	.66197	.88317	1.1323	.74953
28	.64901	.85308	1.1722	.76078	32	.66218	.88369	1.1316	.74934	32	.66218	.88369	1.1316	.74934	32	.66218	.88369	1.1316	.74934
29	.64923	.85358	1.1715	.76059	31	.66240	.88421	1.1310	.74915	31	.66240	.88421	1.1310	.74915	31	.66240	.88421	1.1310	.74915
30	.64945	.85408	1.1708	.76041	30	.66262	.88473	1.1303	.74896	30	.66262	.88473	1.1303	.74896	30	.66262	.88473	1.1303	.74896
31	.64967	.85458	1.1702	.76022	29	.66284	.88524	1.1296	.74877	29	.66284	.88524	1.1296	.74877	29	.66284	.88524	1.1296	.74877
32	.64989	.85509	1.1695	.76003	28	.66306	.88576	1.1290	.74857	28	.66306	.88576	1.1290	.74857	28	.66306	.88576	1.1290	.74857
33	.65011	.85559	1.1688	.75984	27	.66327	.88628	1.1283	.74838	27	.66327	.88628	1.1283	.74838	27	.66327	.88628	1.1283	.74838
34	.65033	.85609	1.1681	.75965	26	.66349	.88680	1.1276	.74818	26	.66349	.88680	1.1276	.74818	26	.66349	.88680	1.1276	.74818
35	.65055	.85660	1.1674	.75946	25	.66371	.88732	1.1270	.74799	25	.66371	.88732	1.1270	.74799	25	.66371	.88732	1.1270	.74799
36	.65077	.85710	1.1667	.75927	24	.66393	.88784	1.1263	.74780	24	.66393	.88784	1.1263	.74780	24	.66393	.88784	1.1263	.74780
37	.65100	.85761	1.1660	.75908	23	.66414	.88836	1.1257	.74760	23	.66414	.88836	1.1257	.74760	23	.66414	.88836	1.1257	.74760
38	.65122	.85811	1.1653	.75889	22	.66436	.88888	1.1250	.74741	22	.66436	.88888	1.1250	.74741	22	.66436	.88888	1.1250	.74741
39	.65144	.85862	1.1647	.75870	21	.66458	.88940	1.1243	.74722	21	.66458	.88940	1.1243	.74722	21	.66458	.88940	1.1243	.74722
40	.65166	.85912	1.1640	.75851	20	.66480	.88992	1.1237	.74703	20	.66480	.88992	1.1237	.74703	20	.66480	.88992	1.1237	.74703
41	.65188	.85963	1.1633	.75832	19	.66501	.89045	1.1230	.74683	19	.66501	.89045	1.1230	.74683	19	.66501	.89045	1.1230	.74683
42	.65210	.86014	1.1626	.75813	18	.66523	.89097	1.1224	.74664	18	.66523	.89097	1.1224	.74664	18	.66523	.89097	1.1224	.74664
43	.65232	.86064	1.1619	.75794	17	.66545	.89149	1.1217	.74644	17	.66545	.89149	1.1217	.74644	17	.66545	.89149	1.1217	.74644
44	.65254	.86115	1.1612	.75775	16	.66566	.89201	1.1211	.74625	16	.66566	.89201	1.1211	.74625	16	.66566	.89201	1.1211	.74625
45	.65276	.86166	1.1606	.75756	15	.66588	.89253	1.1204	.74606	15	.66588	.89253	1.1204	.74606	15	.66588	.89253	1.1204	.74606
46	.65298	.86216	1.1599	.75738	14	.66610	.89305	1.1197	.74586	14	.66610	.89305	1.1197	.74586	14	.66610	.89305	1.1197	.74586
47	.65320	.86267	1.1592	.75719	13	.66632	.89358	1.1191	.74567	13	.66632	.89358	1.1191	.74567	13	.66632	.89358	1.1191	.74567
48	.65342	.86318	1.1585	.75700	12	.66653	.89410	1.1184	.74548	12	.66653	.89410	1.1184	.74548	12	.66653	.89410	1.1184	.74548
49	.65364	.86368	1.1578	.75680	11	.66675	.89463	1.1178	.74528	11	.66675	.89463	1.1178	.74528	11	.66675	.89463	1.1178	.74528
50	.65386	.86419	1.1571	.75661	10	.66697	.89515	1.1171	.74509	10	.66697	.89515	1.1171	.74509	10	.66697	.89515	1.1171	.74509
51	.65408	.86470	1.1565	.75642	9	.66718	.89567	1.1165	.74489	9	.66718	.89567	1.1165	.74489	9	.66718	.89567	1.1165	.74489
52	.65430	.86521	1.1558	.75623	8	.66740	.89620	1.1158	.74470	8	.66740	.89620	1.1158	.74470	8	.66740	.89620	1.1158	.74470
53	.65452	.86572	1.1551	.75604	7	.66762	.89672	1.1152	.74451	7	.66762	.89672	1.1152	.74451	7	.66762	.89672	1.1152	.74451
54	.65474	.86623	1.1544	.75585	6	.66783	.89725	1.1145	.74431	6	.66783	.89725	1.1145	.74431	6	.66783	.89725	1.1145	.74431
55	.65496	.86674	1.1538	.75566	5	.66805	.89777	1.1139	.74412	5	.66805	.89777	1.1139	.74412	5	.66805	.89777	1.1139	.74412
56	.65518	.86725	1.1531	.75547	4	.66827	.89830	1.1132	.74392	4	.66827	.89830	1.1132	.74392	4	.66827	.89830	1.1132	.74392
57	.65540	.86776	1.1524	.75528	3	.66848	.89883	1.1126	.74373	3	.66848	.89883	1.1126	.74373	3	.66848	.89883	1.1126	.74373
58	.65562	.86827	1.1517	.75509	2	.66870	.89935	1.1119	.74353	2	.66870	.89935	1.1119	.74353	2	.66870	.89935	1.1119	.74353
59	.65584	.86878	1.1510	.75490	1	.66891	.89988	1.1113	.74334	1	.66891	.89988	1.1113	.74334	1	.66891	.89988	1.1113	.74334
60	.65606	.86929	1.1504	.75471	0	.66913	.90040	1.1106	.74314	0	.66913	.90040	1.1106						

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

42° (222°)					(317°) 137°					43° (223°)					(316°) 136°						
/	Sin	Tan	Ctn	Cos	/	/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos	/	Sin	Tan	Ctn	Cos	/
0	.66913	.90040	1.1106	.74314	60	0	.68200	.93252	1.0724	.73135	60	.68200	.93252	1.0724	.73135	60	.68200	.93252	1.0724	.73135	60
1	.66935	.90093	1.1100	.74295	59	1	.68221	.93306	1.0717	.73116	59	.68221	.93306	1.0717	.73116	59	.68221	.93306	1.0717	.73116	59
2	.66956	.90146	1.1093	.74276	58	2	.68242	.93360	1.0711	.73096	58	.68242	.93360	1.0711	.73096	58	.68242	.93360	1.0711	.73096	58
3	.66978	.90199	1.1087	.74256	57	3	.68264	.93415	1.0705	.73076	57	.68264	.93415	1.0705	.73076	57	.68264	.93415	1.0705	.73076	57
4	.66999	.90251	1.1080	.74237	56	4	.68285	.93469	1.0699	.73056	56	.68285	.93469	1.0699	.73056	56	.68285	.93469	1.0699	.73056	56
5	.67021	.90304	1.1074	.74217	55	5	.68306	.93524	1.0692	.73036	55	.68306	.93524	1.0692	.73036	55	.68306	.93524	1.0692	.73036	55
6	.67043	.90357	1.1067	.74198	54	6	.68327	.93578	1.0686	.73016	54	.68327	.93578	1.0686	.73016	54	.68327	.93578	1.0686	.73016	54
7	.67064	.90410	1.1061	.74178	53	7	.68349	.93633	1.0680	.72996	53	.68349	.93633	1.0680	.72996	53	.68349	.93633	1.0680	.72996	53
8	.67086	.90463	1.1054	.74159	52	8	.68370	.93688	1.0674	.72976	52	.68370	.93688	1.0674	.72976	52	.68370	.93688	1.0674	.72976	52
9	.67107	.90516	1.1048	.74139	51	9	.68391	.93742	1.0668	.72957	51	.68391	.93742	1.0668	.72957	51	.68391	.93742	1.0668	.72957	51
10	.67129	.90569	1.1041	.74120	50	10	.68412	.93797	1.0661	.72937	50	.68412	.93797	1.0661	.72937	50	.68412	.93797	1.0661	.72937	50
11	.67151	.90621	1.1035	.74100	49	11	.68434	.93852	1.0655	.72917	49	.68434	.93852	1.0655	.72917	49	.68434	.93852	1.0655	.72917	49
12	.67172	.90674	1.1028	.74080	48	12	.68455	.93906	1.0649	.72897	48	.68455	.93906	1.0649	.72897	48	.68455	.93906	1.0649	.72897	48
13	.67194	.90727	1.1022	.74061	47	13	.68476	.93961	1.0643	.72877	47	.68476	.93961	1.0643	.72877	47	.68476	.93961	1.0643	.72877	47
14	.67215	.90781	1.1016	.74041	46	14	.68497	.94016	1.0637	.72857	46	.68497	.94016	1.0637	.72857	46	.68497	.94016	1.0637	.72857	46
15	.67237	.90834	1.1009	.74022	45	15	.68518	.94071	1.0630	.72837	45	.68518	.94071	1.0630	.72837	45	.68518	.94071	1.0630	.72837	45
16	.67258	.90887	1.1003	.74002	44	16	.68539	.94125	1.0624	.72817	44	.68539	.94125	1.0624	.72817	44	.68539	.94125	1.0624	.72817	44
17	.67280	.90940	1.0996	.73983	43	17	.68561	.94180	1.0618	.72797	43	.68561	.94180	1.0618	.72797	43	.68561	.94180	1.0618	.72797	43
18	.67301	.90993	1.0990	.73963	42	18	.68582	.94235	1.0612	.72777	42	.68582	.94235	1.0612	.72777	42	.68582	.94235	1.0612	.72777	42
19	.67323	.91046	1.0983	.73944	41	19	.68603	.94290	1.0606	.72757	41	.68603	.94290	1.0606	.72757	41	.68603	.94290	1.0606	.72757	41
20	.67344	.91099	1.0977	.73924	40	20	.68624	.94345	1.0599	.72737	40	.68624	.94345	1.0599	.72737	40	.68624	.94345	1.0599	.72737	40
21	.67366	.91153	1.0971	.73904	39	21	.68645	.94400	1.0593	.72717	39	.68645	.94400	1.0593	.72717	39	.68645	.94400	1.0593	.72717	39
22	.67387	.91206	1.0964	.73885	38	22	.68666	.94455	1.0587	.72697	38	.68666	.94455	1.0587	.72697	38	.68666	.94455	1.0587	.72697	38
23	.67409	.91259	1.0958	.73865	37	23	.68688	.94510	1.0581	.72677	37	.68688	.94510	1.0581	.72677	37	.68688	.94510	1.0581	.72677	37
24	.67430	.91313	1.0951	.73846	36	24	.68709	.94565	1.0575	.72657	36	.68709	.94565	1.0575	.72657	36	.68709	.94565	1.0575	.72657	36
25	.67452	.91366	1.0945	.73826	35	25	.68730	.94620	1.0569	.72637	35	.68730	.94620	1.0569	.72637	35	.68730	.94620	1.0569	.72637	35
26	.67473	.91419	1.0939	.73806	34	26	.68751	.94676	1.0562	.72617	34	.68751	.94676	1.0562	.72617	34	.68751	.94676	1.0562	.72617	34
27	.67495	.91473	1.0932	.73787	33	27	.68772	.94731	1.0556	.72597	33	.68772	.94731	1.0556	.72597	33	.68772	.94731	1.0556	.72597	33
28	.67516	.91526	1.0926	.73767	32	28	.68793	.94786	1.0550	.72577	32	.68793	.94786	1.0550	.72577	32	.68793	.94786	1.0550	.72577	32
29	.67538	.91580	1.0919	.73747	31	29	.68814	.94841	1.0544	.72557	31	.68814	.94841	1.0544	.72557	31	.68814	.94841	1.0544	.72557	31
30	.67559	.91633	1.0913	.73728	30	30	.68835	.94896	1.0538	.72537	30	.68835	.94896	1.0538	.72537	30	.68835	.94896	1.0538	.72537	30
31	.67580	.91687	1.0907	.73708	29	31	.68857	.94952	1.0532	.72517	29	.68857	.94952	1.0532	.72517	29	.68857	.94952	1.0532	.72517	29
32	.67602	.91740	1.0900	.73688	28	32	.68878	.95007	1.0526	.72497	28	.68878	.95007	1.0526	.72497	28	.68878	.95007	1.0526	.72497	28
33	.67623	.91794	1.0894	.73669	27	33	.68899	.95062	1.0519	.72477	27	.68899	.95062	1.0519	.72477	27	.68899	.95062	1.0519	.72477	27
34	.67645	.91847	1.0888	.73649	26	34	.68920	.95118	1.0513	.72457	26	.68920	.95118	1.0513	.72457	26	.68920	.95118	1.0513	.72457	26
35	.67666	.91901	1.0881	.73629	25	35	.68941	.95173	1.0507	.72437	25	.68941	.95173	1.0507	.72437	25	.68941	.95173	1.0507	.72437	25
36	.67688	.91955	1.0875	.73610	24	36	.68962	.95229	1.0501	.72417	24	.68962	.95229	1.0501	.72417	24	.68962	.95229	1.0501	.72417	24
37	.67709	.92008	1.0869	.73590	23	37	.68983	.95284	1.0495	.72397	23	.68983	.95284	1.0495	.72397	23	.68983	.95284	1.0495	.72397	23
38	.67730	.92062	1.0862	.73570	22	38	.69004	.95340	1.0489	.72377	22	.69004	.95340	1.0489	.72377	22	.69004	.95340	1.0489	.72377	22
39	.67752	.92116	1.0856	.73551	21	39	.69025	.95395	1.0483	.72357	21	.69025	.95395	1.0483	.72357	21	.69025	.95395	1.0483	.72357	21
40	.67773	.92170	1.0850	.73531	20	40	.69046	.95451	1.0477	.72337	20	.69046	.95451	1.0477	.72337	20	.69046	.95451	1.0477	.72337	20
41	.67795	.92224	1.0843	.73511	19	41	.69067	.95506	1.0470	.72317	19	.69067	.95506	1.0470	.72317	19	.69067	.95506	1.0470	.72317	19
42	.67816	.92277	1.0837	.73491	18	42	.69088	.95562	1.0464	.72297	18	.69088	.95562	1.0464	.72297	18	.69088	.95562	1.0464	.72297	18
43	.67837	.92331	1.0831	.73472	17	43	.69109	.95618	1.0458	.72277	17	.69109	.95618	1.0458	.72277	17	.69109	.95618	1.0458	.72277	17
44	.67859	.92385	1.0824	.73452	16	44	.69130	.95673	1.0452	.72257	16	.69130	.95673	1.0452	.72257	16	.69130	.95673	1.0452	.72257	16
45	.67880	.92439	1.0818	.73432	15	45	.69151	.95729	1.0446	.72236	15	.69151	.95729	1.0446	.72236	15	.69151	.95729	1.0446	.72236	15
46	.67901	.92493	1.0812	.73413	14	46	.69172	.95785	1.0440	.72216	14	.69172	.95785	1.0440	.72216	14	.69172	.95785	1.0440	.72216	14
47	.67923	.92547	1.0805	.73393	13	47	.69193	.95841	1.0434	.72196	13	.69193	.95841	1.0434	.72196	13	.69193	.95841	1.0434	.72196	13
48	.67944	.92601	1.0799	.73373	12	48	.69214	.95897	1.0428	.72176	12	.69214	.95897	1.0428	.72176	12	.69214	.95897	1.0428	.72176	12
49	.67965	.92655	1.0793	.73353	11	49	.69235	.95952	1.0422	.72156	11	.69235	.95952	1.0422	.72156	11	.69235	.95952	1.0422	.72156	11
50	.67987	.92709	1.0786	.73333	10	50	.69256	.96008	1.0416	.72136	10	.69256	.96008	1.0416	.72136	10	.69256	.96008	1.0416	.72136	10
51	.68008	.92763	1.0780	.73314	9	51	.69277	.96064	1.0410	.72116	9	.69277	.96064	1.0410	.72116	9	.69277	.96064	1.0410	.72116	9
52	.68029	.92817	1.0774	.73294	8	52	.69298	.96120	1.0404	.72096	8	.69298	.96120	1.0404	.72096	8	.69298	.96120	1.0404	.72096	8
53	.68051	.92872	1.0768	.73274	7	53	.69319	.96176	1.0398	.72076	7	.69319	.96176	1.0398	.72076	7	.69319	.96176	1.0398	.72076	7
54	.68072	.92926	1.0761	.73254	6	54	.69340	.96232	1.0392	.72055	6	.69340	.96232	1.0392	.72055	6	.69340	.96232	1.0392	.72055	6
55	.68093	.92980	1.0755	.73234	5	55	.69361	.96288	1.0385	.72035	5	.69361	.96288	1.0385	.72035	5	.69361	.96288	1.0385	.72035	5
56	.68115	.93034	1.0749	.73215	4	56	.69382	.96344	1.0379	.72015	4	.69382	.96344	1							

Table 6.5(c) (Cont'd)

NATURAL TRIGONOMETRIC FUNCTIONS OF ANGLES IN DEGREES

44° (224°)				(315°) 135°			
°	Sin	Tan	Ctn	Cos	°		
0	.69466	.96569	1.0355	.71934	60		
1	.69487	.96625	1.0349	.71914	59		
2	.69508	.96681	1.0343	.71894	58		
3	.69529	.96738	1.0337	.71873	57		
4	.69549	.96794	1.0331	.71853	56		
5	.69570	.96850	1.0325	.71833	55		
6	.69591	.96907	1.0319	.71813	54		
7	.69612	.96963	1.0313	.71792	53		
8	.69633	.97020	1.0307	.71772	52		
9	.69654	.97076	1.0301	.71752	51		
10	.69675	.97133	1.0295	.71732	50		
11	.69696	.97189	1.0289	.71711	49		
12	.69717	.97246	1.0283	.71691	48		
13	.69737	.97302	1.0277	.71671	47		
14	.69758	.97359	1.0271	.71650	46		
15	.69779	.97416	1.0265	.71630	45		
16	.69800	.97472	1.0259	.71610	44		
17	.69821	.97529	1.0253	.71590	43		
18	.69842	.97586	1.0247	.71569	42		
19	.69862	.97643	1.0241	.71549	41		
20	.69883	.97700	1.0235	.71529	40		
21	.69904	.97756	1.0230	.71508	39		
22	.69925	.97813	1.0224	.71488	38		
23	.69946	.97870	1.0218	.71468	37		
24	.69966	.97927	1.0212	.71447	36		
25	.69987	.97984	1.0206	.71427	35		
26	.70008	.98041	1.0200	.71407	34		
27	.70029	.98098	1.0194	.71386	33		
28	.70049	.98155	1.0188	.71366	32		
29	.70070	.98213	1.0182	.71345	31		
30	.70091	.98270	1.0176	.71325	30		
31	.70112	.98327	1.0170	.71305	29		
32	.70132	.98384	1.0164	.71284	28		
33	.70153	.98441	1.0158	.71264	27		
34	.70174	.98499	1.0152	.71243	26		
35	.70195	.98556	1.0147	.71223	25		
36	.70215	.98613	1.0141	.71203	24		
37	.70236	.98671	1.0135	.71182	23		
38	.70257	.98728	1.0129	.71162	22		
39	.70277	.98786	1.0123	.71141	21		
40	.70298	.98843	1.0117	.71121	20		
41	.70319	.98901	1.0111	.71100	19		
42	.70339	.98958	1.0105	.71080	18		
43	.70360	.99016	1.0099	.71059	17		
44	.70381	.99073	1.0094	.71039	16		
45	.70401	.99131	1.0088	.71019	15		
46	.70422	.99189	1.0082	.70998	14		
47	.70443	.99247	1.0076	.70978	13		
48	.70463	.99304	1.0070	.70957	12		
49	.70484	.99362	1.0064	.70937	11		
50	.70505	.99420	1.0058	.70916	10		
51	.70525	.99478	1.0052	.70896	9		
52	.70546	.99536	1.0047	.70875	8		
53	.70567	.99594	1.0041	.70855	7		
54	.70587	.99652	1.0035	.70834	6		
55	.70608	.99710	1.0029	.70813	5		
56	.70628	.99768	1.0023	.70793	4		
57	.70649	.99826	1.0017	.70772	3		
58	.70670	.99884	1.0012	.70752	2		
59	.70690	.99942	1.0006	.70731	1		
60	.70711	1.0000	1.0000	.70711	0		
°	Cos	Ctn	Tan	Sin	°		

134 (314°)

(225°) 45°

Table 6.5(d)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

0° (180°)			(359°) 179°			1° (181°)			(358°) 178°			2° (182°)			(357°) 177°		
/	Sec	Csc	/	/	Sec	Csc	/	/	Sec	Csc	/	/	Sec	Csc	/	/	
0	1.0000		60	0	1.0002	57.299	60	0	1.0006	28.654	60	0	1.0006	28.654	60		
1	1.0000	3437.7	59	1	1.0002	56.359	59	1	1.0006	28.417	59	1	1.0006	28.417	59		
2	1.0000	1718.9	58	2	1.0002	55.451	58	2	1.0006	28.184	58	2	1.0006	28.184	58		
3	1.0000	1145.9	57	3	1.0002	54.570	57	3	1.0006	27.955	57	3	1.0006	27.955	57		
4	1.0000	859.44	56	4	1.0002	53.718	56	4	1.0007	27.730	56	4	1.0007	27.730	56		
5	1.0000	687.55	55	5	1.0002	52.892	55	5	1.0007	27.508	55	5	1.0007	27.508	55		
6	1.0000	572.96	54	6	1.0002	52.090	54	6	1.0007	27.290	54	6	1.0007	27.290	54		
7	1.0000	491.11	53	7	1.0002	51.313	53	7	1.0007	27.075	53	7	1.0007	27.075	53		
8	1.0000	429.72	52	8	1.0002	50.558	52	8	1.0007	26.864	52	8	1.0007	26.864	52		
9	1.0000	381.97	51	9	1.0002	49.826	51	9	1.0007	26.655	51	9	1.0007	26.655	51		
10	1.0000	343.78	50	10	1.0002	49.114	50	10	1.0007	26.451	50	10	1.0007	26.451	50		
11	1.0000	312.52	49	11	1.0002	48.422	49	11	1.0007	26.249	49	11	1.0007	26.249	49		
12	1.0000	286.48	48	12	1.0002	47.750	48	12	1.0007	26.050	48	12	1.0007	26.050	48		
13	1.0000	264.44	47	13	1.0002	47.096	47	13	1.0007	25.854	47	13	1.0007	25.854	47		
14	1.0000	245.55	46	14	1.0002	46.460	46	14	1.0008	25.661	46	14	1.0008	25.661	46		
15	1.0000	229.18	45	15	1.0002	45.840	45	15	1.0008	25.471	45	15	1.0008	25.471	45		
16	1.0000	214.86	44	16	1.0002	45.237	44	16	1.0008	25.284	44	16	1.0008	25.284	44		
17	1.0000	202.22	43	17	1.0003	44.650	43	17	1.0008	25.100	43	17	1.0008	25.100	43		
18	1.0000	190.99	42	18	1.0003	44.077	42	18	1.0008	24.918	42	18	1.0008	24.918	42		
19	1.0000	180.93	41	19	1.0003	43.520	41	19	1.0008	24.739	41	19	1.0008	24.739	41		
20	1.0000	171.89	40	20	1.0003	42.976	40	20	1.0008	24.562	40	20	1.0008	24.562	40		
21	1.0000	163.70	39	21	1.0003	42.445	39	21	1.0008	24.388	39	21	1.0008	24.388	39		
22	1.0000	156.26	38	22	1.0003	41.928	38	22	1.0009	24.216	38	22	1.0009	24.216	38		
23	1.0000	149.47	37	23	1.0003	41.423	37	23	1.0009	24.047	37	23	1.0009	24.047	37		
24	1.0000	143.24	36	24	1.0003	40.930	36	24	1.0009	23.880	36	24	1.0009	23.880	36		
25	1.0000	137.51	35	25	1.0003	40.448	35	25	1.0009	23.716	35	25	1.0009	23.716	35		
26	1.0000	132.22	34	26	1.0003	39.978	34	26	1.0009	23.553	34	26	1.0009	23.553	34		
27	1.0000	127.33	33	27	1.0003	39.519	33	27	1.0009	23.393	33	27	1.0009	23.393	33		
28	1.0000	122.78	32	28	1.0003	39.070	32	28	1.0009	23.235	32	28	1.0009	23.235	32		
29	1.0000	118.54	31	29	1.0003	38.631	31	29	1.0009	23.079	31	29	1.0009	23.079	31		
30	1.0000	114.59	30	30	1.0003	38.202	30	30	1.0010	22.926	30	30	1.0010	22.926	30		
31	1.0000	110.90	29	31	1.0004	37.782	29	31	1.0010	22.774	29	31	1.0010	22.774	29		
32	1.0000	107.43	28	32	1.0004	37.371	28	32	1.0010	22.624	28	32	1.0010	22.624	28		
33	1.0000	104.18	27	33	1.0004	36.970	27	33	1.0010	22.476	27	33	1.0010	22.476	27		
34	1.0000	101.11	26	34	1.0004	36.576	26	34	1.0010	22.330	26	34	1.0010	22.330	26		
35	1.0001	98.223	25	35	1.0004	36.191	25	35	1.0010	22.187	25	35	1.0010	22.187	25		
36	1.0001	95.495	24	36	1.0004	35.815	24	36	1.0010	22.044	24	36	1.0010	22.044	24		
37	1.0001	92.914	23	37	1.0004	35.445	23	37	1.0010	21.904	23	37	1.0010	21.904	23		
38	1.0001	90.469	22	38	1.0004	35.084	22	38	1.0011	21.766	22	38	1.0011	21.766	22		
39	1.0001	88.149	21	39	1.0004	34.730	21	39	1.0011	21.629	21	39	1.0011	21.629	21		
40	1.0001	85.946	20	40	1.0004	34.382	20	40	1.0011	21.494	20	40	1.0011	21.494	20		
41	1.0001	83.849	19	41	1.0004	34.042	19	41	1.0011	21.360	19	41	1.0011	21.360	19		
42	1.0001	81.853	18	42	1.0004	33.708	18	42	1.0011	21.229	18	42	1.0011	21.229	18		
43	1.0001	79.950	17	43	1.0004	33.381	17	43	1.0011	21.098	17	43	1.0011	21.098	17		
44	1.0001	78.133	16	44	1.0005	33.060	16	44	1.0011	20.970	16	44	1.0011	20.970	16		
45	1.0001	76.397	15	45	1.0005	32.746	15	45	1.0012	20.843	15	45	1.0012	20.843	15		
46	1.0001	74.736	14	46	1.0005	32.437	14	46	1.0012	20.717	14	46	1.0012	20.717	14		
47	1.0001	73.146	13	47	1.0005	32.134	13	47	1.0012	20.593	13	47	1.0012	20.593	13		
48	1.0001	71.622	12	48	1.0005	31.836	12	48	1.0012	20.471	12	48	1.0012	20.471	12		
49	1.0001	70.160	11	49	1.0005	31.544	11	49	1.0012	20.350	11	49	1.0012	20.350	11		
50	1.0001	68.757	10	50	1.0005	31.258	10	50	1.0012	20.230	10	50	1.0012	20.230	10		
51	1.0001	67.409	9	51	1.0005	30.976	9	51	1.0012	20.112	9	51	1.0012	20.112	9		
52	1.0001	66.113	8	52	1.0005	30.700	8	52	1.0013	19.995	8	52	1.0013	19.995	8		
53	1.0001	64.866	7	53	1.0005	30.428	7	53	1.0013	19.880	7	53	1.0013	19.880	7		
54	1.0001	63.665	6	54	1.0006	30.161	6	54	1.0013	19.766	6	54	1.0013	19.766	6		
55	1.0001	62.507	5	55	1.0006	29.899	5	55	1.0013	19.653	5	55	1.0013	19.653	5		
56	1.0001	61.391	4	56	1.0006	29.641	4	56	1.0013	19.541	4	56	1.0013	19.541	4		
57	1.0001	60.314	3	57	1.0006	29.388	3	57	1.0013	19.431	3	57	1.0013	19.431	3		
58	1.0001	59.274	2	58	1.0006	29.139	2	58	1.0013	19.322	2	58	1.0013	19.322	2		
59	1.0001	58.270	1	59	1.0006	28.894	1	59	1.0014	19.214	1	59	1.0014	19.214	1		
60	1.0002	57.299	0	60	1.0006	28.654	0	60	1.0014	19.107	0	60	1.0014	19.107	0		
/	Csc	Sec	/	/	Csc	Sec	/	/	Csc	Sec	/	/	Csc	Sec	/	/	
90° (270°)			(269°) 89°			91° (271°)			(268°) 88°			92° (272°)			(267°) 87°		

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

3° (183°)			(356°) 176°			4° (184°)			(355°) 175°			5° (185°)			(354°) 174°		
/	Sec	Csc	/			/	Sec	Csc	/			/	Sec	Csc	/		
0	1.0014	19.107	60			0	1.0024	14.336	60			0	1.0038	11.474	60		
1	1.0014	19.002	59			1	1.0025	14.276	59			1	1.0038	11.436	59		
2	1.0014	18.898	58			2	1.0025	14.217	58			2	1.0039	11.398	58		
3	1.0014	18.794	57			3	1.0025	14.159	57			3	1.0039	11.360	57		
4	1.0014	18.692	56			4	1.0025	14.101	56			4	1.0039	11.323	56		
5	1.0014	18.591	55			5	1.0025	14.044	55			5	1.0039	11.286	55		
6	1.0015	18.492	54			6	1.0026	13.987	54			6	1.0040	11.249	54		
7	1.0015	18.393	53			7	1.0026	13.930	53			7	1.0040	11.213	53		
8	1.0015	18.295	52			8	1.0026	13.874	52			8	1.0040	11.176	52		
9	1.0015	18.198	51			9	1.0026	13.818	51			9	1.0041	11.140	51		
10	1.0015	18.103	50			10	1.0027	13.763	50			10	1.0041	11.105	50		
11	1.0015	18.008	49			11	1.0027	13.708	49			11	1.0041	11.069	49		
12	1.0016	17.914	48			12	1.0027	13.654	48			12	1.0041	11.034	48		
13	1.0016	17.822	47			13	1.0027	13.600	47			13	1.0042	10.998	47		
14	1.0016	17.730	46			14	1.0027	13.547	46			14	1.0042	10.963	46		
15	1.0016	17.639	45			15	1.0028	13.494	45			15	1.0042	10.929	45		
16	1.0016	17.549	44			16	1.0028	13.441	44			16	1.0042	10.894	44		
17	1.0016	17.460	43			17	1.0028	13.389	43			17	1.0043	10.860	43		
18	1.0017	17.372	42			18	1.0028	13.337	42			18	1.0043	10.826	42		
19	1.0017	17.285	41			19	1.0028	13.286	41			19	1.0043	10.792	41		
20	1.0017	17.198	40			20	1.0029	13.235	40			20	1.0043	10.758	40		
21	1.0017	17.113	39			21	1.0029	13.184	39			21	1.0044	10.725	39		
22	1.0017	17.028	38			22	1.0029	13.134	38			22	1.0044	10.692	38		
23	1.0017	16.945	37			23	1.0029	13.084	37			23	1.0044	10.659	37		
24	1.0018	16.862	36			24	1.0030	13.035	36			24	1.0045	10.626	36		
25	1.0018	16.779	35			25	1.0030	12.985	35			25	1.0045	10.593	35		
26	1.0018	16.698	34			26	1.0030	12.937	34			26	1.0045	10.561	34		
27	1.0018	16.618	33			27	1.0030	12.888	33			27	1.0045	10.529	33		
28	1.0018	16.538	32			28	1.0030	12.840	32			28	1.0046	10.497	32		
29	1.0019	16.459	31			29	1.0031	12.793	31			29	1.0046	10.465	31		
30	1.0019	16.380	30			30	1.0031	12.745	30			30	1.0046	10.433	30		
31	1.0019	16.303	29			31	1.0031	12.699	29			31	1.0047	10.402	29		
32	1.0019	16.226	28			32	1.0031	12.652	28			32	1.0047	10.371	28		
33	1.0019	16.150	27			33	1.0032	12.606	27			33	1.0047	10.340	27		
34	1.0019	16.075	26			34	1.0032	12.560	26			34	1.0047	10.309	26		
35	1.0020	16.000	25			35	1.0032	12.514	25			35	1.0048	10.278	25		
36	1.0020	15.926	24			36	1.0032	12.469	24			36	1.0048	10.248	24		
37	1.0020	15.853	23			37	1.0033	12.424	23			37	1.0048	10.217	23		
38	1.0020	15.780	22			38	1.0033	12.379	22			38	1.0049	10.187	22		
39	1.0020	15.708	21			39	1.0033	12.335	21			39	1.0049	10.157	21		
40	1.0021	15.637	20			40	1.0033	12.291	20			40	1.0049	10.128	20		
41	1.0021	15.566	19			41	1.0034	12.248	19			41	1.0049	10.098	19		
42	1.0021	15.496	18			42	1.0034	12.204	18			42	1.0050	10.068	18		
43	1.0021	15.427	17			43	1.0034	12.161	17			43	1.0050	10.039	17		
44	1.0021	15.358	16			44	1.0034	12.119	16			44	1.0050	10.010	16		
45	1.0021	15.290	15			45	1.0034	12.076	15			45	1.0051	9.9812	15		
46	1.0022	15.222	14			46	1.0035	12.034	14			46	1.0051	9.9525	14		
47	1.0022	15.155	13			47	1.0035	11.992	13			47	1.0051	9.9239	13		
48	1.0022	15.089	12			48	1.0035	11.951	12			48	1.0051	9.8955	12		
49	1.0022	15.023	11			49	1.0035	11.909	11			49	1.0052	9.8672	11		
50	1.0022	14.958	10			50	1.0036	11.868	10			50	1.0052	9.8391	10		
51	1.0023	14.893	9			51	1.0036	11.828	9			51	1.0052	9.8112	9		
52	1.0023	14.829	8			52	1.0036	11.787	8			52	1.0053	9.7834	8		
53	1.0023	14.766	7			53	1.0036	11.747	7			53	1.0053	9.7558	7		
54	1.0023	14.703	6			54	1.0037	11.707	6			54	1.0053	9.7283	6		
55	1.0023	14.640	5			55	1.0037	11.668	5			55	1.0054	9.7010	5		
56	1.0024	14.578	4			56	1.0037	11.628	4			56	1.0054	9.6739	4		
57	1.0024	14.517	3			57	1.0037	11.589	3			57	1.0054	9.6469	3		
58	1.0024	14.456	2			58	1.0038	11.551	2			58	1.0054	9.6200	2		
59	1.0024	14.395	1			59	1.0038	11.512	1			59	1.0055	9.5933	1		
60	1.0024	14.336	0			60	1.0038	11.474	0			60	1.0055	9.5668	0		
/	Csc	Sec	/			/	Csc	Sec	/			/	Csc	Sec	/		

93° 273°

(266°) 86°

94° 274°

(265°) 85°

95° (275°)

(264°) 84°

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

6° (186°)				7° (187°)				8° (188°)			
(353°) 173°				(352°) 172°				(351°) 171°			
/	Sec	Csc	/	/	Sec	Csc	/	/	Sec	Csc	/
0	1.0055	9.5668	60	0	1.0075	8.2055	60	0	1.0098	7.1853	60
1	1.0055	9.5404	59	1	1.0075	8.1861	59	1	1.0099	7.1705	59
2	1.0056	9.5141	58	2	1.0076	8.1668	58	2	1.0099	7.1557	58
3	1.0056	9.4880	57	3	1.0076	8.1476	57	3	1.0100	7.1410	57
4	1.0056	9.4620	56	4	1.0077	8.1285	56	4	1.0100	7.1263	56
5	1.0057	9.4362	55	5	1.0077	8.1095	55	5	1.0100	7.1117	55
6	1.0057	9.4105	54	6	1.0077	8.0905	54	6	1.0101	7.0972	54
7	1.0057	9.3850	53	7	1.0078	8.0717	53	7	1.0101	7.0827	53
8	1.0058	9.3596	52	8	1.0078	8.0529	52	8	1.0102	7.0683	52
9	1.0058	9.3343	51	9	1.0078	8.0342	51	9	1.0102	7.0539	51
10	1.0058	9.3092	50	10	1.0079	8.0156	50	10	1.0102	7.0396	50
11	1.0059	9.2842	49	11	1.0079	7.9971	49	11	1.0103	7.0254	49
12	1.0059	9.2593	48	12	1.0079	7.9787	48	12	1.0103	7.0112	48
13	1.0059	9.2346	47	13	1.0080	7.9604	47	13	1.0104	6.9971	47
14	1.0059	9.2100	46	14	1.0080	7.9422	46	14	1.0104	6.9830	46
15	1.0060	9.1855	45	15	1.0081	7.9240	45	15	1.0105	6.9690	45
16	1.0060	9.1612	44	16	1.0081	7.9059	44	16	1.0105	6.9550	44
17	1.0060	9.1370	43	17	1.0081	7.8879	43	17	1.0105	6.9411	43
18	1.0061	9.1129	42	18	1.0082	7.8700	42	18	1.0106	6.9273	42
19	1.0061	9.0890	41	19	1.0082	7.8522	41	19	1.0106	6.9135	41
20	1.0061	9.0652	40	20	1.0082	7.8344	40	20	1.0107	6.8998	40
21	1.0062	9.0415	39	21	1.0083	7.8168	39	21	1.0107	6.8861	39
22	1.0062	9.0179	38	22	1.0083	7.7992	38	22	1.0108	6.8725	38
23	1.0062	8.9944	37	23	1.0084	7.7817	37	23	1.0108	6.8589	37
24	1.0063	8.9711	36	24	1.0084	7.7642	36	24	1.0108	6.8454	36
25	1.0063	8.9479	35	25	1.0084	7.7469	35	25	1.0109	6.8320	35
26	1.0063	8.9248	34	26	1.0085	7.7296	34	26	1.0109	6.8186	34
27	1.0064	8.9019	33	27	1.0085	7.7124	33	27	1.0110	6.8052	33
28	1.0064	8.8790	32	28	1.0086	7.6953	32	28	1.0110	6.7919	32
29	1.0064	8.8563	31	29	1.0086	7.6783	31	29	1.0111	6.7787	31
30	1.0065	8.8337	30	30	1.0086	7.6613	30	30	1.0111	6.7655	30
31	1.0065	8.8112	29	31	1.0087	7.6444	29	31	1.0112	6.7523	29
32	1.0065	8.7888	28	32	1.0087	7.6276	28	32	1.0112	6.7392	28
33	1.0066	8.7665	27	33	1.0087	7.6109	27	33	1.0112	6.7262	27
34	1.0066	8.7444	26	34	1.0088	7.5942	26	34	1.0113	6.7132	26
35	1.0066	8.7223	25	35	1.0088	7.5776	25	35	1.0113	6.7003	25
36	1.0067	8.7004	24	36	1.0089	7.5611	24	36	1.0114	6.6874	24
37	1.0067	8.6786	23	37	1.0089	7.5446	23	37	1.0114	6.6745	23
38	1.0067	8.6569	22	38	1.0089	7.5282	22	38	1.0115	6.6618	22
39	1.0068	8.6353	21	39	1.0090	7.5119	21	39	1.0115	6.6490	21
40	1.0068	8.6138	20	40	1.0090	7.4957	20	40	1.0116	6.6363	20
41	1.0068	8.5924	19	41	1.0091	7.4795	19	41	1.0116	6.6237	19
42	1.0069	8.5711	18	42	1.0091	7.4635	18	42	1.0116	6.6111	18
43	1.0069	8.5500	17	43	1.0091	7.4474	17	43	1.0117	6.5986	17
44	1.0069	8.5289	16	44	1.0092	7.4315	16	44	1.0117	6.5861	16
45	1.0070	8.5079	15	45	1.0092	7.4156	15	45	1.0118	6.5736	15
46	1.0070	8.4871	14	46	1.0093	7.3998	14	46	1.0118	6.5612	14
47	1.0070	8.4663	13	47	1.0093	7.3840	13	47	1.0119	6.5489	13
48	1.0071	8.4457	12	48	1.0093	7.3684	12	48	1.0119	6.5366	12
49	1.0071	8.4251	11	49	1.0094	7.3527	11	49	1.0120	6.5243	11
50	1.0072	8.4047	10	50	1.0094	7.3372	10	50	1.0120	6.5121	10
51	1.0072	8.3843	9	51	1.0095	7.3217	9	51	1.0120	6.4999	9
52	1.0072	8.3641	8	52	1.0095	7.3063	8	52	1.0121	6.4878	8
53	1.0073	8.3439	7	53	1.0095	7.2909	7	53	1.0121	6.4757	7
54	1.0073	8.3238	6	54	1.0096	7.2757	6	54	1.0122	6.4637	6
55	1.0073	8.3039	5	55	1.0096	7.2604	5	55	1.0122	6.4517	5
56	1.0074	8.2840	4	56	1.0097	7.2453	4	56	1.0123	6.4398	4
57	1.0074	8.2642	3	57	1.0097	7.2302	3	57	1.0123	6.4279	3
58	1.0074	8.2446	2	58	1.0097	7.2152	2	58	1.0124	6.4160	2
59	1.0075	8.2250	1	59	1.0098	7.2002	1	59	1.0124	6.4042	1
60	1.0075	8.2055	0	60	1.0098	7.1853	0	60	1.0125	6.3925	0
/	Csc	Sec	/	/	Csc	Sec	/	/	Csc	Sec	/
96° (276°)				97° (277°)				98° (278°)			
(263°) 83°				(262°) 82°				(261°) 81°			

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

9° 189°			10° 190°			11° 191°		
(350° 170°)			(349° 169°)			(348° 168°)		
/	Sec	Csc	/	Sec	Csc	/	Sec	Csc
0	1.0125	6.3925	0	1.0154	5.7588	0	1.0187	5.2408
1	1.0125	6.3807	1	1.0155	5.7493	1	1.0188	5.2330
2	1.0126	6.3691	2	1.0155	5.7398	2	1.0188	5.2252
3	1.0126	6.3574	3	1.0156	5.7304	3	1.0189	5.2174
4	1.0127	6.3458	4	1.0156	5.7210	4	1.0189	5.2097
5	1.0127	6.3343	5	1.0157	5.7117	5	1.0190	5.2019
6	1.0127	6.3228	6	1.0157	5.7023	6	1.0191	5.1942
7	1.0128	6.3113	7	1.0158	5.6930	7	1.0191	5.1865
8	1.0128	6.2999	8	1.0158	5.6838	8	1.0192	5.1789
9	1.0129	6.2885	9	1.0159	5.6745	9	1.0192	5.1712
10	1.0129	6.2772	10	1.0160	5.6653	10	1.0193	5.1636
11	1.0130	6.2659	11	1.0160	5.6562	11	1.0194	5.1560
12	1.0130	6.2546	12	1.0161	5.6470	12	1.0194	5.1484
13	1.0131	6.2434	13	1.0161	5.6379	13	1.0195	5.1409
14	1.0131	6.2323	14	1.0162	5.6288	14	1.0195	5.1333
15	1.0132	6.2211	15	1.0162	5.6198	15	1.0196	5.1258
16	1.0132	6.2100	16	1.0163	5.6107	16	1.0197	5.1183
17	1.0133	6.1990	17	1.0163	5.6017	17	1.0197	5.1109
18	1.0133	6.1880	18	1.0164	5.5928	18	1.0198	5.1034
19	1.0134	6.1770	19	1.0164	5.5838	19	1.0198	5.0960
20	1.0134	6.1661	20	1.0165	5.5749	20	1.0199	5.0886
21	1.0135	6.1552	21	1.0165	5.5660	21	1.0199	5.0813
22	1.0135	6.1443	22	1.0166	5.5572	22	1.0200	5.0739
23	1.0136	6.1335	23	1.0166	5.5484	23	1.0201	5.0666
24	1.0136	6.1227	24	1.0167	5.5396	24	1.0201	5.0593
25	1.0137	6.1120	25	1.0168	5.5308	25	1.0202	5.0520
26	1.0137	6.1013	26	1.0168	5.5221	26	1.0202	5.0447
27	1.0138	6.0906	27	1.0169	5.5134	27	1.0203	5.0375
28	1.0138	6.0800	28	1.0169	5.5047	28	1.0204	5.0302
29	1.0139	6.0694	29	1.0170	5.4960	29	1.0204	5.0230
30	1.0139	6.0589	30	1.0170	5.4874	30	1.0205	5.0159
31	1.0140	6.0483	31	1.0171	5.4788	31	1.0205	5.0087
32	1.0140	6.0379	32	1.0171	5.4702	32	1.0206	5.0016
33	1.0141	6.0274	33	1.0172	5.4617	33	1.0207	4.9944
34	1.0141	6.0170	34	1.0173	5.4532	34	1.0207	4.9873
35	1.0142	6.0067	35	1.0173	5.4447	35	1.0208	4.9803
36	1.0142	5.9963	36	1.0174	5.4362	36	1.0209	4.9732
37	1.0143	5.9860	37	1.0174	5.4278	37	1.0209	4.9662
38	1.0143	5.9758	38	1.0175	5.4194	38	1.0210	4.9591
39	1.0144	5.9656	39	1.0175	5.4110	39	1.0210	4.9521
40	1.0144	5.9554	40	1.0176	5.4026	40	1.0211	4.9452
41	1.0145	5.9452	41	1.0176	5.3943	41	1.0212	4.9382
42	1.0145	5.9351	42	1.0177	5.3860	42	1.0212	4.9313
43	1.0146	5.9250	43	1.0178	5.3777	43	1.0213	4.9244
44	1.0146	5.9150	44	1.0178	5.3695	44	1.0213	4.9175
45	1.0147	5.9049	45	1.0179	5.3612	45	1.0214	4.9106
46	1.0147	5.8950	46	1.0179	5.3530	46	1.0215	4.9037
47	1.0148	5.8850	47	1.0180	5.3449	47	1.0215	4.8969
48	1.0148	5.8751	48	1.0180	5.3367	48	1.0216	4.8901
49	1.0149	5.8652	49	1.0181	5.3286	49	1.0217	4.8833
50	1.0149	5.8554	50	1.0181	5.3205	50	1.0217	4.8765
51	1.0150	5.8456	51	1.0182	5.3124	51	1.0218	4.8697
52	1.0150	5.8358	52	1.0183	5.3044	52	1.0218	4.8630
53	1.0151	5.8261	53	1.0183	5.2963	53	1.0219	4.8563
54	1.0151	5.8164	54	1.0184	5.2883	54	1.0220	4.8496
55	1.0152	5.8067	55	1.0184	5.2804	55	1.0220	4.8429
56	1.0152	5.7970	56	1.0185	5.2724	56	1.0221	4.8362
57	1.0153	5.7874	57	1.0185	5.2645	57	1.0222	4.8296
58	1.0153	5.7778	58	1.0186	5.2566	58	1.0222	4.8229
59	1.0154	5.7683	59	1.0187	5.2487	59	1.0223	4.8163
60	1.0154	5.7588	60	1.0187	5.2408	60	1.0223	4.8097
/	Csc	Sec	/	Csc	Sec	/	Csc	Sec
99° (279°)	(260°) 80°		100° (260°)	(259°) 79°		101° (281°)	(258°) 78°	

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

12° (192°)				(347°) 167°				13° (193°)				(346°) 166°				14° (194°)				(345°) 165°			
/	Sec	Csc	/	/	Sec	Csc	/	/	Sec	Csc	/	/	Sec	Csc	/	/	Sec	Csc	/	/			
0	1.0223	4.8097	60	0	1.0263	4.4454	60	0	1.0306	4.1336	60	0	1.0306	4.1336	60	0	1.0306	4.1336	60	0			
1	1.0224	4.8032	59	1	1.0264	4.4398	59	1	1.0307	4.1287	59	1	1.0307	4.1287	59	1	1.0307	4.1287	59	1			
2	1.0225	4.7966	58	2	1.0264	4.4342	58	2	1.0308	4.1239	58	2	1.0308	4.1239	58	2	1.0308	4.1239	58	2			
3	1.0225	4.7901	57	3	1.0265	4.4287	57	3	1.0308	4.1191	57	3	1.0308	4.1191	57	3	1.0308	4.1191	57	3			
4	1.0226	4.7836	56	4	1.0266	4.4231	56	4	1.0309	4.1144	56	4	1.0309	4.1144	56	4	1.0309	4.1144	56	4			
5	1.0227	4.7771	55	5	1.0266	4.4176	55	5	1.0310	4.1096	55	5	1.0310	4.1096	55	5	1.0310	4.1096	55	5			
6	1.0227	4.7706	54	6	1.0267	4.4121	54	6	1.0311	4.1048	54	6	1.0311	4.1048	54	6	1.0311	4.1048	54	6			
7	1.0228	4.7641	53	7	1.0268	4.4066	53	7	1.0311	4.1001	53	7	1.0311	4.1001	53	7	1.0311	4.1001	53	7			
8	1.0228	4.7577	52	8	1.0269	4.4011	52	8	1.0312	4.0954	52	8	1.0312	4.0954	52	8	1.0312	4.0954	52	8			
9	1.0229	4.7512	51	9	1.0269	4.3956	51	9	1.0313	4.0906	51	9	1.0313	4.0906	51	9	1.0313	4.0906	51	9			
10	1.0230	4.7448	50	10	1.0270	4.3901	50	10	1.0314	4.0859	50	10	1.0314	4.0859	50	10	1.0314	4.0859	50	10			
11	1.0230	4.7384	49	11	1.0271	4.3847	49	11	1.0314	4.0812	49	11	1.0314	4.0812	49	11	1.0314	4.0812	49	11			
12	1.0231	4.7321	48	12	1.0271	4.3792	48	12	1.0315	4.0765	48	12	1.0315	4.0765	48	12	1.0315	4.0765	48	12			
13	1.0232	4.7257	47	13	1.0272	4.3738	47	13	1.0316	4.0718	47	13	1.0316	4.0718	47	13	1.0316	4.0718	47	13			
14	1.0232	4.7194	46	14	1.0273	4.3684	46	14	1.0317	4.0672	46	14	1.0317	4.0672	46	14	1.0317	4.0672	46	14			
15	1.0233	4.7130	45	15	1.0273	4.3630	45	15	1.0317	4.0625	45	15	1.0317	4.0625	45	15	1.0317	4.0625	45	15			
16	1.0234	4.7067	44	16	1.0274	4.3576	44	16	1.0318	4.0579	44	16	1.0318	4.0579	44	16	1.0318	4.0579	44	16			
17	1.0234	4.7004	43	17	1.0275	4.3522	43	17	1.0319	4.0532	43	17	1.0319	4.0532	43	17	1.0319	4.0532	43	17			
18	1.0235	4.6942	42	18	1.0276	4.3469	42	18	1.0320	4.0486	42	18	1.0320	4.0486	42	18	1.0320	4.0486	42	18			
19	1.0236	4.6879	41	19	1.0276	4.3415	41	19	1.0321	4.0440	41	19	1.0321	4.0440	41	19	1.0321	4.0440	41	19			
20	1.0236	4.6817	40	20	1.0277	4.3362	40	20	1.0321	4.0394	40	20	1.0321	4.0394	40	20	1.0321	4.0394	40	20			
21	1.0237	4.6755	39	21	1.0278	4.3309	39	21	1.0322	4.0348	39	21	1.0322	4.0348	39	21	1.0322	4.0348	39	21			
22	1.0238	4.6693	38	22	1.0278	4.3256	38	22	1.0323	4.0302	38	22	1.0323	4.0302	38	22	1.0323	4.0302	38	22			
23	1.0238	4.6631	37	23	1.0279	4.3203	37	23	1.0324	4.0256	37	23	1.0324	4.0256	37	23	1.0324	4.0256	37	23			
24	1.0239	4.6569	36	24	1.0280	4.3150	36	24	1.0324	4.0211	36	24	1.0324	4.0211	36	24	1.0324	4.0211	36	24			
25	1.0240	4.6507	35	25	1.0281	4.3098	35	25	1.0325	4.0165	35	25	1.0325	4.0165	35	25	1.0325	4.0165	35	25			
26	1.0240	4.6446	34	26	1.0281	4.3045	34	26	1.0326	4.0120	34	26	1.0326	4.0120	34	26	1.0326	4.0120	34	26			
27	1.0241	4.6385	33	27	1.0282	4.2993	33	27	1.0327	4.0075	33	27	1.0327	4.0075	33	27	1.0327	4.0075	33	27			
28	1.0241	4.6324	32	28	1.0283	4.2941	32	28	1.0327	4.0029	32	28	1.0327	4.0029	32	28	1.0327	4.0029	32	28			
29	1.0242	4.6263	31	29	1.0283	4.2889	31	29	1.0328	3.9984	31	29	1.0328	3.9984	31	29	1.0328	3.9984	31	29			
30	1.0243	4.6202	30	30	1.0284	4.2837	30	30	1.0329	3.9939	30	30	1.0329	3.9939	30	30	1.0329	3.9939	30	30			
31	1.0243	4.6142	29	31	1.0285	4.2785	29	31	1.0330	3.9894	29	31	1.0330	3.9894	29	31	1.0330	3.9894	29	31			
32	1.0244	4.6081	28	32	1.0286	4.2733	28	32	1.0331	3.9850	28	32	1.0331	3.9850	28	32	1.0331	3.9850	28	32			
33	1.0245	4.6021	27	33	1.0286	4.2681	27	33	1.0331	3.9805	27	33	1.0331	3.9805	27	33	1.0331	3.9805	27	33			
34	1.0245	4.5961	26	34	1.0287	4.2630	26	34	1.0332	3.9760	26	34	1.0332	3.9760	26	34	1.0332	3.9760	26	34			
35	1.0246	4.5901	25	35	1.0288	4.2579	25	35	1.0333	3.9716	25	35	1.0333	3.9716	25	35	1.0333	3.9716	25	35			
36	1.0247	4.5841	24	36	1.0288	4.2527	24	36	1.0334	3.9672	24	36	1.0334	3.9672	24	36	1.0334	3.9672	24	36			
37	1.0247	4.5782	23	37	1.0289	4.2476	23	37	1.0334	3.9627	23	37	1.0334	3.9627	23	37	1.0334	3.9627	23	37			
38	1.0248	4.5722	22	38	1.0290	4.2425	22	38	1.0335	3.9583	22	38	1.0335	3.9583	22	38	1.0335	3.9583	22	38			
39	1.0249	4.5663	21	39	1.0291	4.2375	21	39	1.0336	3.9539	21	39	1.0336	3.9539	21	39	1.0336	3.9539	21	39			
40	1.0249	4.5604	20	40	1.0291	4.2324	20	40	1.0337	3.9495	20	40	1.0337	3.9495	20	40	1.0337	3.9495	20	40			
41	1.0250	4.5545	19	41	1.0292	4.2273	19	41	1.0338	3.9451	19	41	1.0338	3.9451	19	41	1.0338	3.9451	19	41			
42	1.0251	4.5486	18	42	1.0293	4.2223	18	42	1.0338	3.9408	18	42	1.0338	3.9408	18	42	1.0338	3.9408	18	42			
43	1.0251	4.5428	17	43	1.0294	4.2173	17	43	1.0339	3.9364	17	43	1.0339	3.9364	17	43	1.0339	3.9364	17	43			
44	1.0252	4.5369	16	44	1.0294	4.2122	16	44	1.0340	3.9320	16	44	1.0340	3.9320	16	44	1.0340	3.9320	16	44			
45	1.0253	4.5311	15	45	1.0295	4.2072	15	45	1.0341	3.9277	15	45	1.0341	3.9277	15	45	1.0341	3.9277	15	45			
46	1.0253	4.5253	14	46	1.0296	4.2022	14	46	1.0342	3.9234	14	46	1.0342	3.9234	14	46	1.0342	3.9234	14	46			
47	1.0254	4.5195	13	47	1.0297	4.1973	13	47	1.0342	3.9190	13	47	1.0342	3.9190	13	47	1.0342	3.9190	13	47			
48	1.0255	4.5137	12	48	1.0297	4.1923	12	48	1.0343	3.9147	12	48	1.0343	3.9147	12	48	1.0343	3.9147	12	48			
49	1.0256	4.5079	11	49	1.0298	4.1873	11	49	1.0344	3.9104	11	49	1.0344	3.9104	11	49	1.0344	3.9104	11	49			
50	1.0256	4.5022	10	50	1.0299	4.1824	10	50	1.0345	3.9061	10	50	1.0345	3.9061	10	50	1.0345	3.9061	10	50			
51	1.0257	4.4964	9	51	1.0299	4.1774	9	51	1.0346	3.9018	9	51	1.0346	3.9018	9	51	1.0346	3.9018	9	51			
52	1.0258	4.4907	8	52	1.0300	4.1725	8	52	1.0346	3.8976	8	52	1.0346	3.8976	8	52	1.0346	3.8976	8	52			
53	1.0258	4.4850	7	53	1.0301	4.1676	7	53	1.0347	3.8933	7	53	1.0347	3.8933	7	53	1.0347	3.8933	7	53			
54	1.0259	4.4793	6	54	1.0302	4.1627	6	54	1.0348	3.8890	6	54	1.0348	3.8890	6	54	1.0348	3.8890	6	54			
55	1.0260	4.4736	5	55	1.0302	4.1578	5	55	1.0349	3.8848	5	55	1.0349	3.8848	5	55	1.0349	3.8848	5	55			
56	1.0260	4.4679	4	56	1.0303	4.1529	4	56	1.0350	3.8806	4	56	1.0350	3.8806	4	56	1.0350	3.8806	4	56			
57	1.0261	4.4623	3	57	1.0304	4.1481	3	57	1.0350	3.8763	3	57	1.0350	3.8763	3	57	1.0350	3.8763	3	57			
58	1.0262	4.4566	2	58	1.0305	4.1432	2	58	1.0351	3.8721	2	58	1.0351	3.8721	2	58	1.0351	3.8721	2	58			
59	1.0262	4.4510	1	59	1.0305	4.1384	1	59	1.0352	3.8679	1	59	1.0352	3.8679	1	59	1.0352	3.8679	1	59			
60	1.0263	4.4454	0	60	1.0306	4.1336	0	60	1.0353	3.8637	0	60	1.0353	3.8637	0	60	1.0353	3.8637	0	60			
/	Csc	Sec	/	/	Csc	Sec	/	/	Csc	Sec	/	/	Csc	Sec	/	/	Csc	Sec	/	/			
102° (282°)				(257°) 77°				103° (283°)				(256°) 76°				104° (284°)				(255°) 75°			

102° (282°)

(257°) 77°

103° (283°)

(256°) 76°

104° (284°)

(255°) 75°

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

15° (195°)				(344°) 164°				16° (196°)				(343°) 163°				17° (197°)				(342°) 162°			
/	Sec	Csc	/	/	Sec	Csc	/	/	Sec	Csc	/	/	Sec	Csc	/	/	Sec	Csc	/	/	Sec	Csc	/
0	1.0353	3.8637	60	0	1.0403	3.6280	60	0	1.0457	3.4203	60	0	1.0457	3.4203	60	0	1.0457	3.4203	60	0	1.0457	3.4203	60
1	1.0354	3.8595	59	1	1.0404	3.6243	59	1	1.0458	3.4171	59	1	1.0458	3.4171	59	1	1.0458	3.4171	59	1	1.0458	3.4171	59
2	1.0354	3.8553	58	2	1.0405	3.6206	58	2	1.0459	3.4138	58	2	1.0459	3.4138	58	2	1.0459	3.4138	58	2	1.0459	3.4138	58
3	1.0355	3.8512	57	3	1.0406	3.6169	57	3	1.0460	3.4106	57	3	1.0460	3.4106	57	3	1.0460	3.4106	57	3	1.0460	3.4106	57
4	1.0356	3.8470	56	4	1.0406	3.6133	56	4	1.0461	3.4073	56	4	1.0461	3.4073	56	4	1.0461	3.4073	56	4	1.0461	3.4073	56
5	1.0357	3.8428	55	5	1.0407	3.6097	55	5	1.0462	3.4041	55	5	1.0462	3.4041	55	5	1.0462	3.4041	55	5	1.0462	3.4041	55
6	1.0358	3.8387	54	6	1.0408	3.6060	54	6	1.0463	3.4009	54	6	1.0463	3.4009	54	6	1.0463	3.4009	54	6	1.0463	3.4009	54
7	1.0358	3.8346	53	7	1.0409	3.6024	53	7	1.0463	3.3977	53	7	1.0463	3.3977	53	7	1.0463	3.3977	53	7	1.0463	3.3977	53
8	1.0359	3.8304	52	8	1.0410	3.5988	52	8	1.0464	3.3945	52	8	1.0464	3.3945	52	8	1.0464	3.3945	52	8	1.0464	3.3945	52
9	1.0360	3.8263	51	9	1.0411	3.5951	51	9	1.0465	3.3913	51	9	1.0465	3.3913	51	9	1.0465	3.3913	51	9	1.0465	3.3913	51
10	1.0361	3.8222	50	10	1.0412	3.5915	50	10	1.0466	3.3881	50	10	1.0466	3.3881	50	10	1.0466	3.3881	50	10	1.0466	3.3881	50
11	1.0362	3.8181	49	11	1.0413	3.5879	49	11	1.0467	3.3849	49	11	1.0467	3.3849	49	11	1.0467	3.3849	49	11	1.0467	3.3849	49
12	1.0363	3.8140	48	12	1.0413	3.5843	48	12	1.0468	3.3817	48	12	1.0468	3.3817	48	12	1.0468	3.3817	48	12	1.0468	3.3817	48
13	1.0363	3.8100	47	13	1.0414	3.5808	47	13	1.0469	3.3785	47	13	1.0469	3.3785	47	13	1.0469	3.3785	47	13	1.0469	3.3785	47
14	1.0364	3.8059	46	14	1.0415	3.5772	46	14	1.0470	3.3754	46	14	1.0470	3.3754	46	14	1.0470	3.3754	46	14	1.0470	3.3754	46
15	1.0365	3.8018	45	15	1.0416	3.5736	45	15	1.0471	3.3722	45	15	1.0471	3.3722	45	15	1.0471	3.3722	45	15	1.0471	3.3722	45
16	1.0366	3.7978	44	16	1.0417	3.5700	44	16	1.0472	3.3691	44	16	1.0472	3.3691	44	16	1.0472	3.3691	44	16	1.0472	3.3691	44
17	1.0367	3.7937	43	17	1.0418	3.5665	43	17	1.0473	3.3659	43	17	1.0473	3.3659	43	17	1.0473	3.3659	43	17	1.0473	3.3659	43
18	1.0367	3.7897	42	18	1.0419	3.5629	42	18	1.0474	3.3628	42	18	1.0474	3.3628	42	18	1.0474	3.3628	42	18	1.0474	3.3628	42
19	1.0368	3.7857	41	19	1.0420	3.5594	41	19	1.0475	3.3596	41	19	1.0475	3.3596	41	19	1.0475	3.3596	41	19	1.0475	3.3596	41
20	1.0369	3.7817	40	20	1.0421	3.5559	40	20	1.0476	3.3565	40	20	1.0476	3.3565	40	20	1.0476	3.3565	40	20	1.0476	3.3565	40
21	1.0370	3.7777	39	21	1.0421	3.5523	39	21	1.0477	3.3534	39	21	1.0477	3.3534	39	21	1.0477	3.3534	39	21	1.0477	3.3534	39
22	1.0371	3.7737	38	22	1.0422	3.5488	38	22	1.0478	3.3502	38	22	1.0478	3.3502	38	22	1.0478	3.3502	38	22	1.0478	3.3502	38
23	1.0372	3.7697	37	23	1.0423	3.5453	37	23	1.0479	3.3471	37	23	1.0479	3.3471	37	23	1.0479	3.3471	37	23	1.0479	3.3471	37
24	1.0372	3.7657	36	24	1.0424	3.5418	36	24	1.0480	3.3440	36	24	1.0480	3.3440	36	24	1.0480	3.3440	36	24	1.0480	3.3440	36
25	1.0373	3.7617	35	25	1.0425	3.5383	35	25	1.0480	3.3409	35	25	1.0480	3.3409	35	25	1.0480	3.3409	35	25	1.0480	3.3409	35
26	1.0374	3.7577	34	26	1.0426	3.5348	34	26	1.0481	3.3378	34	26	1.0481	3.3378	34	26	1.0481	3.3378	34	26	1.0481	3.3378	34
27	1.0375	3.7538	33	27	1.0427	3.5313	33	27	1.0482	3.3347	33	27	1.0482	3.3347	33	27	1.0482	3.3347	33	27	1.0482	3.3347	33
28	1.0376	3.7498	32	28	1.0428	3.5279	32	28	1.0483	3.3317	32	28	1.0483	3.3317	32	28	1.0483	3.3317	32	28	1.0483	3.3317	32
29	1.0377	3.7459	31	29	1.0429	3.5244	31	29	1.0484	3.3286	31	29	1.0484	3.3286	31	29	1.0484	3.3286	31	29	1.0484	3.3286	31
30	1.0377	3.7420	30	30	1.0429	3.5209	30	30	1.0485	3.3255	30	30	1.0485	3.3255	30	30	1.0485	3.3255	30	30	1.0485	3.3255	30
31	1.0378	3.7381	29	31	1.0430	3.5175	29	31	1.0486	3.3224	29	31	1.0486	3.3224	29	31	1.0486	3.3224	29	31	1.0486	3.3224	29
32	1.0379	3.7341	28	32	1.0431	3.5140	28	32	1.0487	3.3194	28	32	1.0487	3.3194	28	32	1.0487	3.3194	28	32	1.0487	3.3194	28
33	1.0380	3.7302	27	33	1.0432	3.5106	27	33	1.0488	3.3163	27	33	1.0488	3.3163	27	33	1.0488	3.3163	27	33	1.0488	3.3163	27
34	1.0381	3.7263	26	34	1.0433	3.5072	26	34	1.0489	3.3133	26	34	1.0489	3.3133	26	34	1.0489	3.3133	26	34	1.0489	3.3133	26
35	1.0382	3.7225	25	35	1.0434	3.5037	25	35	1.0490	3.3102	25	35	1.0490	3.3102	25	35	1.0490	3.3102	25	35	1.0490	3.3102	25
36	1.0382	3.7186	24	36	1.0435	3.5003	24	36	1.0491	3.3072	24	36	1.0491	3.3072	24	36	1.0491	3.3072	24	36	1.0491	3.3072	24
37	1.0383	3.7147	23	37	1.0436	3.4969	23	37	1.0492	3.3042	23	37	1.0492	3.3042	23	37	1.0492	3.3042	23	37	1.0492	3.3042	23
38	1.0384	3.7108	22	38	1.0437	3.4935	22	38	1.0493	3.3012	22	38	1.0493	3.3012	22	38	1.0493	3.3012	22	38	1.0493	3.3012	22
39	1.0385	3.7070	21	39	1.0438	3.4901	21	39	1.0494	3.2981	21	39	1.0494	3.2981	21	39	1.0494	3.2981	21	39	1.0494	3.2981	21
40	1.0386	3.7032	20	40	1.0439	3.4867	20	40	1.0495	3.2951	20	40	1.0495	3.2951	20	40	1.0495	3.2951	20	40	1.0495	3.2951	20
41	1.0387	3.6993	19	41	1.0439	3.4833	19	41	1.0496	3.2921	19	41	1.0496	3.2921	19	41	1.0496	3.2921	19	41	1.0496	3.2921	19
42	1.0388	3.6955	18	42	1.0440	3.4799	18	42	1.0497	3.2891	18	42	1.0497	3.2891	18	42	1.0497	3.2891	18	42	1.0497	3.2891	18
43	1.0388	3.6917	17	43	1.0441	3.4766	17	43	1.0498	3.2861	17	43	1.0498	3.2861	17	43	1.0498	3.2861	17	43	1.0498	3.2861	17
44	1.0389	3.6879	16	44	1.0442	3.4732	16	44	1.0499	3.2831	16	44	1.0499	3.2831	16	44	1.0499	3.2831	16	44	1.0499	3.2831	16
45	1.0390	3.6840	15	45	1.0443	3.4699	15	45	1.0500	3.2801	15	45	1.0500	3.2801	15	45	1.0500	3.2801	15	45	1.0500	3.2801	15
46	1.0391	3.6803	14	46	1.0444	3.4665	14	46	1.0501	3.2772	14	46	1.0501	3.2772	14	46	1.0501	3.2772	14	46	1.0501	3.2772	14
47	1.0392	3.6765	13	47	1.0445	3.4632	13	47	1.0502	3.2742	13	47	1.0502	3.2742	13	47	1.0502	3.2742	13	47	1.0502	3.2742	13
48	1.0393	3.6727	12	48	1.0446	3.4598	12	48	1.0503	3.2712	12	48	1.0503	3.2712	12	48	1.0503	3.2712	12	48	1.0503	3.2712	12
49	1.0394	3.6689	11	49	1.0447	3.4565	11	49	1.0504	3.2683	11	49	1.0504	3.2683	11	49	1.0504	3.2683	11	49	1.0504	3.2683	11
50	1.0394	3.6652	10	50	1.0448	3.4532	10	50	1.0505	3.2653	10	50	1.0505	3.2653	10	50	1.0505	3.2653	10	50	1.0505	3.2653	10

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

18° (198°)			(341°) 161°			19° (199°)			(340°) 160°			20° (200°)			(339°) 159°		
/	Sec	Csc	/			/	Sec	Csc	/			/	Sec	Csc	/		
0	1.0515	3.2361	60			0	1.0576	3.0716	60			0	1.0642	2.9238	60		
1	1.0516	3.2332	59			1	1.0577	3.0690	59			1	1.0643	2.9215	59		
2	1.0517	3.2303	58			2	1.0578	3.0664	58			2	1.0644	2.9191	58		
3	1.0518	3.2274	57			3	1.0579	3.0638	57			3	1.0645	2.9168	57		
4	1.0519	3.2245	56			4	1.0580	3.0612	56			4	1.0646	2.9145	56		
5	1.0520	3.2217	55			5	1.0582	3.0586	55			5	1.0647	2.9122	55		
6	1.0521	3.2188	54			6	1.0583	3.0561	54			6	1.0649	2.9099	54		
7	1.0522	3.2159	53			7	1.0584	3.0535	53			7	1.0650	2.9075	53		
8	1.0523	3.2131	52			8	1.0585	3.0509	52			8	1.0651	2.9052	52		
9	1.0524	3.2102	51			9	1.0586	3.0484	51			9	1.0652	2.9029	51		
10	1.0525	3.2074	50			10	1.0587	3.0458	50			10	1.0653	2.9006	50		
11	1.0526	3.2045	49			11	1.0588	3.0433	49			11	1.0654	2.8983	49		
12	1.0527	3.2017	48			12	1.0589	3.0407	48			12	1.0655	2.8960	48		
13	1.0528	3.1989	47			13	1.0590	3.0382	47			13	1.0657	2.8938	47		
14	1.0529	3.1960	46			14	1.0591	3.0357	46			14	1.0658	2.8915	46		
15	1.0530	3.1932	45			15	1.0592	3.0331	45			15	1.0659	2.8892	45		
16	1.0531	3.1904	44			16	1.0593	3.0306	44			16	1.0660	2.8869	44		
17	1.0532	3.1876	43			17	1.0594	3.0281	43			17	1.0661	2.8846	43		
18	1.0533	3.1848	42			18	1.0595	3.0256	42			18	1.0662	2.8824	42		
19	1.0534	3.1820	41			19	1.0597	3.0231	41			19	1.0663	2.8801	41		
20	1.0535	3.1792	40			20	1.0598	3.0206	40			20	1.0665	2.8779	40		
21	1.0536	3.1764	39			21	1.0599	3.0181	39			21	1.0666	2.8756	39		
22	1.0537	3.1736	38			22	1.0600	3.0156	38			22	1.0667	2.8733	38		
23	1.0538	3.1708	37			23	1.0601	3.0131	37			23	1.0668	2.8711	37		
24	1.0539	3.1681	36			24	1.0602	3.0106	36			24	1.0669	2.8688	36		
25	1.0540	3.1653	35			25	1.0603	3.0081	35			25	1.0670	2.8666	35		
26	1.0541	3.1625	34			26	1.0604	3.0056	34			26	1.0671	2.8644	34		
27	1.0542	3.1598	33			27	1.0605	3.0031	33			27	1.0673	2.8621	33		
28	1.0543	3.1570	32			28	1.0606	3.0007	32			28	1.0674	2.8599	32		
29	1.0544	3.1543	31			29	1.0607	2.9982	31			29	1.0675	2.8577	31		
30	1.0545	3.1515	30			30	1.0608	2.9957	30			30	1.0676	2.8555	30		
31	1.0546	3.1488	29			31	1.0610	2.9933	29			31	1.0677	2.8532	29		
32	1.0547	3.1461	28			32	1.0611	2.9908	28			32	1.0678	2.8510	28		
33	1.0548	3.1433	27			33	1.0612	2.9884	27			33	1.0680	2.8488	27		
34	1.0549	3.1406	26			34	1.0613	2.9859	26			34	1.0681	2.8466	26		
35	1.0550	3.1379	25			35	1.0614	2.9835	25			35	1.0682	2.8444	25		
36	1.0551	3.1352	24			36	1.0615	2.9811	24			36	1.0683	2.8422	24		
37	1.0552	3.1325	23			37	1.0616	2.9786	23			37	1.0684	2.8400	23		
38	1.0553	3.1298	22			38	1.0617	2.9762	22			38	1.0685	2.8378	22		
39	1.0554	3.1271	21			39	1.0618	2.9738	21			39	1.0687	2.8356	21		
40	1.0555	3.1244	20			40	1.0619	2.9713	20			40	1.0688	2.8334	20		
41	1.0556	3.1217	19			41	1.0621	2.9689	19			41	1.0689	2.8312	19		
42	1.0557	3.1190	18			42	1.0622	2.9665	18			42	1.0690	2.8291	18		
43	1.0558	3.1163	17			43	1.0623	2.9641	17			43	1.0691	2.8269	17		
44	1.0559	3.1137	16			44	1.0624	2.9617	16			44	1.0692	2.8247	16		
45	1.0560	3.1110	15			45	1.0625	2.9593	15			45	1.0694	2.8225	15		
46	1.0561	3.1083	14			46	1.0626	2.9569	14			46	1.0695	2.8204	14		
47	1.0563	3.1057	13			47	1.0627	2.9545	13			47	1.0696	2.8182	13		
48	1.0564	3.1030	12			48	1.0628	2.9521	12			48	1.0697	2.8161	12		
49	1.0565	3.1004	11			49	1.0629	2.9498	11			49	1.0698	2.8139	11		
50	1.0566	3.0977	10			50	1.0631	2.9474	10			50	1.0700	2.8117	10		
51	1.0567	3.0951	9			51	1.0632	2.9450	9			51	1.0701	2.8096	9		
52	1.0568	3.0925	8			52	1.0633	2.9426	8			52	1.0702	2.8075	8		
53	1.0569	3.0898	7			53	1.0634	2.9403	7			53	1.0703	2.8053	7		
54	1.0570	3.0872	6			54	1.0635	2.9379	6			54	1.0704	2.8032	6		
55	1.0571	3.0846	5			55	1.0636	2.9355	5			55	1.0705	2.8010	5		
56	1.0572	3.0820	4			56	1.0637	2.9332	4			56	1.0707	2.7989	4		
57	1.0573	3.0794	3			57	1.0638	2.9308	3			57	1.0708	2.7968	3		
58	1.0574	3.0768	2			58	1.0640	2.9285	2			58	1.0709	2.7947	2		
59	1.0575	3.0742	1			59	1.0641	2.9261	1			59	1.0710	2.7925	1		
60	1.0576	3.0716	0			60	1.0642	2.9238	0			60	1.0711	2.7904	0		
/	Csc	Sec	/			/	Csc	Sec	/			/	Csc	Sec	/		
108° (288°)			(251°) 71°			109° (289°)			(250°) 70°			110° (290°)			(249°) 69°		

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

21° (201°)			338° 158°			22° (202°)			337° 157°			23° (203°)			(336°) 156°		
/	Sec	Csc	/			/	Sec	Csc	/			/	Sec	Csc	/		
0	1.0711	2.7904	60			0	1.0785	2.6695	60			0	1.0864	2.5593	60		
1	1.0713	2.7883	59			1	1.0787	2.6675	59			1	1.0865	2.5576	59		
2	1.0714	2.7862	58			2	1.0788	2.6656	58			2	1.0866	2.5558	58		
3	1.0715	2.7841	57			3	1.0789	2.6637	57			3	1.0868	2.5541	57		
4	1.0716	2.7820	56			4	1.0790	2.6618	56			4	1.0869	2.5523	56		
5	1.0717	2.7799	55			5	1.0792	2.6599	55			5	1.0870	2.5506	55		
6	1.0719	2.7778	54			6	1.0793	2.6580	54			6	1.0872	2.5488	54		
7	1.0720	2.7757	53			7	1.0794	2.6561	53			7	1.0873	2.5471	53		
8	1.0721	2.7736	52			8	1.0796	2.6542	52			8	1.0874	2.5454	52		
9	1.0722	2.7715	51			9	1.0797	2.6523	51			9	1.0876	2.5436	51		
10	1.0723	2.7695	50			10	1.0798	2.6504	50			10	1.0877	2.5419	50		
11	1.0725	2.7674	49			11	1.0799	2.6485	49			11	1.0878	2.5402	49		
12	1.0726	2.7653	48			12	1.0801	2.6466	48			12	1.0880	2.5384	48		
13	1.0727	2.7632	47			13	1.0802	2.6447	47			13	1.0881	2.5367	47		
14	1.0728	2.7612	46			14	1.0803	2.6429	46			14	1.0883	2.5350	46		
15	1.0730	2.7591	45			15	1.0804	2.6410	45			15	1.0884	2.5333	45		
16	1.0731	2.7570	44			16	1.0806	2.6391	44			16	1.0885	2.5316	44		
17	1.0732	2.7550	43			17	1.0807	2.6372	43			17	1.0887	2.5299	43		
18	1.0733	2.7529	42			18	1.0808	2.6354	42			18	1.0888	2.5282	42		
19	1.0734	2.7509	41			19	1.0810	2.6335	41			19	1.0889	2.5264	41		
20	1.0736	2.7488	40			20	1.0811	2.6316	40			20	1.0891	2.5247	40		
21	1.0737	2.7468	39			21	1.0812	2.6298	39			21	1.0892	2.5230	39		
22	1.0738	2.7447	38			22	1.0814	2.6279	38			22	1.0893	2.5213	38		
23	1.0739	2.7427	37			23	1.0815	2.6260	37			23	1.0895	2.5196	37		
24	1.0740	2.7407	36			24	1.0816	2.6242	36			24	1.0896	2.5180	36		
25	1.0742	2.7386	35			25	1.0817	2.6223	35			25	1.0898	2.5163	35		
26	1.0743	2.7366	34			26	1.0819	2.6205	34			26	1.0899	2.5146	34		
27	1.0744	2.7346	33			27	1.0820	2.6186	33			27	1.0900	2.5129	33		
28	1.0745	2.7325	32			28	1.0821	2.6168	32			28	1.0902	2.5112	32		
29	1.0747	2.7305	31			29	1.0823	2.6150	31			29	1.0903	2.5095	31		
30	1.0748	2.7285	30			30	1.0824	2.6131	30			30	1.0904	2.5078	30		
31	1.0749	2.7265	29			31	1.0825	2.6113	29			31	1.0906	2.5062	29		
32	1.0750	2.7245	28			32	1.0827	2.6095	28			32	1.0907	2.5045	28		
33	1.0752	2.7225	27			33	1.0828	2.6076	27			33	1.0909	2.5028	27		
34	1.0753	2.7205	26			34	1.0829	2.6058	26			34	1.0910	2.5012	26		
35	1.0754	2.7185	25			35	1.0830	2.6040	25			35	1.0911	2.4995	25		
36	1.0755	2.7165	24			36	1.0832	2.6022	24			36	1.0913	2.4978	24		
37	1.0757	2.7145	23			37	1.0833	2.6003	23			37	1.0914	2.4962	23		
38	1.0758	2.7125	22			38	1.0834	2.5985	22			38	1.0915	2.4945	22		
39	1.0759	2.7105	21			39	1.0836	2.5967	21			39	1.0917	2.4928	21		
40	1.0760	2.7085	20			40	1.0837	2.5949	20			40	1.0918	2.4912	20		
41	1.0761	2.7065	19			41	1.0838	2.5931	19			41	1.0920	2.4895	19		
42	1.0763	2.7046	18			42	1.0840	2.5913	18			42	1.0921	2.4879	18		
43	1.0764	2.7026	17			43	1.0841	2.5895	17			43	1.0922	2.4862	17		
44	1.0765	2.7006	16			44	1.0842	2.5877	16			44	1.0924	2.4846	16		
45	1.0766	2.6986	15			45	1.0844	2.5859	15			45	1.0925	2.4830	15		
46	1.0768	2.6967	14			46	1.0845	2.5841	14			46	1.0927	2.4813	14		
47	1.0769	2.6947	13			47	1.0846	2.5823	13			47	1.0928	2.4797	13		
48	1.0770	2.6927	12			48	1.0843	2.5805	12			48	1.0929	2.4780	12		
49	1.0771	2.6908	11			49	1.0849	2.5788	11			49	1.0931	2.4764	11		
50	1.0773	2.6888	10			50	1.0850	2.5770	10			50	1.0932	2.4748	10		
51	1.0774	2.6869	9			51	1.0852	2.5752	9			51	1.0934	2.4731	9		
52	1.0775	2.6849	8			52	1.0853	2.5734	8			52	1.0935	2.4715	8		
53	1.0777	2.6830	7			53	1.0854	2.5716	7			53	1.0936	2.4699	7		
54	1.0778	2.6811	6			54	1.0856	2.5699	6			54	1.0938	2.4683	6		
55	1.0779	2.6791	5			55	1.0857	2.5681	5			55	1.0939	2.4667	5		
56	1.0780	2.6772	4			56	1.0858	2.5663	4			56	1.0941	2.4650	4		
57	1.0782	2.6752	3			57	1.0860	2.5646	3			57	1.0942	2.4634	3		
58	1.0783	2.6733	2			58	1.0861	2.5628	2			58	1.0944	2.4618	2		
59	1.0784	2.6714	1			59	1.0862	2.5611	1			59	1.0945	2.4602	1		
60	1.0785	2.6695	0			60	1.0864	2.5593	0			60	1.0946	2.4586	0		
/	Csc	Sec	/			/	Csc	Sec	/			/	Csc	Sec	/		
111° 231°			248° 68°			112° 232°			247° 67°			113° 233°			246° 66°		

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

24° (204°)			(335°) 155°			25° (205°)			(334°) 154°			26° (206°)			(333°) 153°		
/	Sec	Csc	/	Sec	Csc	/	Sec	Csc	/	Sec	Csc	/	Sec	Csc	/	Sec	Csc
0	1.0946	2.4586	60			0	1.1034	2.3662	60			0	1.1126	2.2812	60		
1	1.0948	2.4570	59			1	1.1035	2.3647	59			1	1.1128	2.2798	59		
2	1.0949	2.4554	58			2	1.1037	2.3633	58			2	1.1129	2.2785	58		
3	1.0951	2.4538	57			3	1.1038	2.3618	57			3	1.1131	2.2771	57		
4	1.0952	2.4522	56			4	1.1040	2.3603	56			4	1.1132	2.2757	56		
5	1.0953	2.4506	55			5	1.1041	2.3588	55			5	1.1134	2.2744	55		
6	1.0955	2.4490	54			6	1.1043	2.3574	54			6	1.1136	2.2730	54		
7	1.0956	2.4474	53			7	1.1044	2.3559	53			7	1.1137	2.2717	53		
8	1.0958	2.4458	52			8	1.1046	2.3545	52			8	1.1139	2.2703	52		
9	1.0959	2.4442	51			9	1.1047	2.3530	51			9	1.1140	2.2690	51		
10	1.0961	2.4426	50			10	1.1049	2.3515	50			10	1.1142	2.2677	50		
11	1.0962	2.4411	49			11	1.1050	2.3501	49			11	1.1143	2.2663	49		
12	1.0963	2.4395	48			12	1.1052	2.3486	48			12	1.1145	2.2650	48		
13	1.0965	2.4379	47			13	1.1053	2.3472	47			13	1.1147	2.2636	47		
14	1.0966	2.4363	46			14	1.1055	2.3457	46			14	1.1148	2.2623	46		
15	1.0968	2.4348	45			15	1.1056	2.3443	45			15	1.1150	2.2610	45		
16	1.0969	2.4332	44			16	1.1058	2.3428	44			16	1.1151	2.2596	44		
17	1.0971	2.4316	43			17	1.1059	2.3414	43			17	1.1153	2.2583	43		
18	1.0972	2.4300	42			18	1.1061	2.3400	42			18	1.1155	2.2570	42		
19	1.0974	2.4285	41			19	1.1062	2.3385	41			19	1.1156	2.2556	41		
20	1.0975	2.4269	40			20	1.1064	2.3371	40			20	1.1158	2.2543	40		
21	1.0976	2.4254	39			21	1.1066	2.3356	39			21	1.1159	2.2530	39		
22	1.0978	2.4238	38			22	1.1067	2.3342	38			22	1.1161	2.2517	38		
23	1.0979	2.4222	37			23	1.1069	2.3328	37			23	1.1163	2.2504	37		
24	1.0981	2.4207	36			24	1.1070	2.3314	36			24	1.1164	2.2490	36		
25	1.0982	2.4191	35			25	1.1072	2.3299	35			25	1.1166	2.2477	35		
26	1.0984	2.4176	34			26	1.1073	2.3285	34			26	1.1168	2.2464	34		
27	1.0985	2.4160	33			27	1.1075	2.3271	33			27	1.1169	2.2451	33		
28	1.0987	2.4145	32			28	1.1076	2.3257	32			28	1.1171	2.2438	32		
29	1.0988	2.4130	31			29	1.1078	2.3242	31			29	1.1172	2.2425	31		
30	1.0989	2.4114	30			30	1.1079	2.3228	30			30	1.1174	2.2412	30		
31	1.0991	2.4099	29			31	1.1081	2.3214	29			31	1.1176	2.2399	29		
32	1.0992	2.4083	28			32	1.1082	2.3200	28			32	1.1177	2.2385	28		
33	1.0994	2.4068	27			33	1.1084	2.3186	27			33	1.1179	2.2372	27		
34	1.0995	2.4053	26			34	1.1085	2.3172	26			34	1.1180	2.2359	26		
35	1.0997	2.4038	25			35	1.1087	2.3158	25			35	1.1182	2.2346	25		
36	1.0998	2.4022	24			36	1.1089	2.3144	24			36	1.1184	2.2333	24		
37	1.1000	2.4007	23			37	1.1090	2.3130	23			37	1.1185	2.2320	23		
38	1.1001	2.3992	22			38	1.1092	2.3115	22			38	1.1187	2.2308	22		
39	1.1003	2.3977	21			39	1.1093	2.3101	21			39	1.1189	2.2295	21		
40	1.1004	2.3961	20			40	1.1095	2.3088	20			40	1.1190	2.2282	20		
41	1.1006	2.3946	19			41	1.1096	2.3074	19			41	1.1192	2.2269	19		
42	1.1007	2.3931	18			42	1.1098	2.3060	18			42	1.1194	2.2256	18		
43	1.1009	2.3916	17			43	1.1099	2.3046	17			43	1.1195	2.2243	17		
44	1.1010	2.3901	16			44	1.1101	2.3032	16			44	1.1197	2.2230	16		
45	1.1011	2.3886	15			45	1.1102	2.3018	15			45	1.1198	2.2217	15		
46	1.1013	2.3871	14			46	1.1104	2.3004	14			46	1.1200	2.2205	14		
47	1.1014	2.3856	13			47	1.1106	2.2990	13			47	1.1202	2.2192	13		
48	1.1016	2.3841	12			48	1.1107	2.2976	12			48	1.1203	2.2179	12		
49	1.1017	2.3826	11			49	1.1109	2.2962	11			49	1.1205	2.2166	11		
50	1.1019	2.3811	10			50	1.1110	2.2949	10			50	1.1207	2.2153	10		
51	1.1020	2.3796	9			51	1.1112	2.2935	9			51	1.1208	2.2141	9		
52	1.1022	2.3781	8			52	1.1113	2.2921	8			52	1.1210	2.2128	8		
53	1.1023	2.3766	7			53	1.1115	2.2907	7			53	1.1212	2.2115	7		
54	1.1025	2.3751	6			54	1.1117	2.2894	6			54	1.1213	2.2103	6		
55	1.1026	2.3736	5			55	1.1118	2.2880	5			55	1.1215	2.2090	5		
56	1.1028	2.3721	4			56	1.1120	2.2866	4			56	1.1217	2.2077	4		
57	1.1029	2.3706	3			57	1.1121	2.2853	3			57	1.1218	2.2065	3		
58	1.1031	2.3692	2			58	1.1123	2.2839	2			58	1.1220	2.2052	2		
59	1.1032	2.3677	1			59	1.1124	2.2825	1			59	1.1222	2.2039	1		
60	1.1034	2.3662	0			60	1.1126	2.2812	0			60	1.1223	2.2027	0		
/	Csc	Sec	/	Csc	Sec	/	Csc	Sec	/	Csc	Sec	/	Csc	Sec	/	Csc	Sec
114° (294°)			(245°) 65°			115° (295°)			(244°) 64°			116° (296°)			(243°) 63°		

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

27° (207°)				(332°) 152°				28° (208°)				(331°) 151°				29° (209°)				(330°) 150°			
/	Sec	Csc	/	/	Sec	Csc	/	/	Sec	Csc	/	/	Sec	Csc	/	/	Sec	Csc	/	/	Sec	Csc	/
0	1.1223	2.2027	60	0	1.1326	2.1301	60	0	1.1434	2.0627	60	0	1.1434	2.0627	60	0	1.1434	2.0627	60	0	1.1434	2.0627	60
1	1.1225	2.2014	59	1	1.1327	2.1289	59	1	1.1435	2.0616	59	1	1.1435	2.0616	59	1	1.1435	2.0616	59	1	1.1435	2.0616	59
2	1.1227	2.2002	58	2	1.1329	2.1277	58	2	1.1437	2.0605	58	2	1.1437	2.0605	58	2	1.1437	2.0605	58	2	1.1437	2.0605	58
3	1.1228	2.1989	57	3	1.1331	2.1266	57	3	1.1439	2.0594	57	3	1.1439	2.0594	57	3	1.1439	2.0594	57	3	1.1439	2.0594	57
4	1.1230	2.1977	56	4	1.1333	2.1254	56	4	1.1441	2.0583	56	4	1.1441	2.0583	56	4	1.1441	2.0583	56	4	1.1441	2.0583	56
5	1.1232	2.1964	55	5	1.1334	2.1242	55	5	1.1443	2.0573	55	5	1.1443	2.0573	55	5	1.1443	2.0573	55	5	1.1443	2.0573	55
6	1.1233	2.1952	54	6	1.1336	2.1231	54	6	1.1445	2.0562	54	6	1.1445	2.0562	54	6	1.1445	2.0562	54	6	1.1445	2.0562	54
7	1.1235	2.1939	53	7	1.1338	2.1219	53	7	1.1446	2.0551	53	7	1.1446	2.0551	53	7	1.1446	2.0551	53	7	1.1446	2.0551	53
8	1.1237	2.1927	52	8	1.1340	2.1208	52	8	1.1448	2.0540	52	8	1.1448	2.0540	52	8	1.1448	2.0540	52	8	1.1448	2.0540	52
9	1.1238	2.1914	51	9	1.1342	2.1196	51	9	1.1450	2.0530	51	9	1.1450	2.0530	51	9	1.1450	2.0530	51	9	1.1450	2.0530	51
10	1.1240	2.1902	50	10	1.1343	2.1185	50	10	1.1452	2.0519	50	10	1.1452	2.0519	50	10	1.1452	2.0519	50	10	1.1452	2.0519	50
11	1.1242	2.1890	49	11	1.1345	2.1173	49	11	1.1454	2.0508	49	11	1.1454	2.0508	49	11	1.1454	2.0508	49	11	1.1454	2.0508	49
12	1.1243	2.1877	48	12	1.1347	2.1162	48	12	1.1456	2.0498	48	12	1.1456	2.0498	48	12	1.1456	2.0498	48	12	1.1456	2.0498	48
13	1.1245	2.1865	47	13	1.1349	2.1150	47	13	1.1458	2.0487	47	13	1.1458	2.0487	47	13	1.1458	2.0487	47	13	1.1458	2.0487	47
14	1.1247	2.1852	46	14	1.1350	2.1139	46	14	1.1460	2.0476	46	14	1.1460	2.0476	46	14	1.1460	2.0476	46	14	1.1460	2.0476	46
15	1.1248	2.1840	45	15	1.1352	2.1127	45	15	1.1461	2.0466	45	15	1.1461	2.0466	45	15	1.1461	2.0466	45	15	1.1461	2.0466	45
16	1.1250	2.1828	44	16	1.1354	2.1116	44	16	1.1463	2.0455	44	16	1.1463	2.0455	44	16	1.1463	2.0455	44	16	1.1463	2.0455	44
17	1.1252	2.1815	43	17	1.1356	2.1105	43	17	1.1465	2.0445	43	17	1.1465	2.0445	43	17	1.1465	2.0445	43	17	1.1465	2.0445	43
18	1.1253	2.1803	42	18	1.1357	2.1093	42	18	1.1467	2.0434	42	18	1.1467	2.0434	42	18	1.1467	2.0434	42	18	1.1467	2.0434	42
19	1.1255	2.1791	41	19	1.1359	2.1082	41	19	1.1469	2.0423	41	19	1.1469	2.0423	41	19	1.1469	2.0423	41	19	1.1469	2.0423	41
20	1.1257	2.1779	40	20	1.1361	2.1070	40	20	1.1471	2.0413	40	20	1.1471	2.0413	40	20	1.1471	2.0413	40	20	1.1471	2.0413	40
21	1.1259	2.1766	39	21	1.1363	2.1059	39	21	1.1473	2.0402	39	21	1.1473	2.0402	39	21	1.1473	2.0402	39	21	1.1473	2.0402	39
22	1.1260	2.1754	38	22	1.1365	2.1048	38	22	1.1474	2.0392	38	22	1.1474	2.0392	38	22	1.1474	2.0392	38	22	1.1474	2.0392	38
23	1.1262	2.1742	37	23	1.1366	2.1036	37	23	1.1476	2.0381	37	23	1.1476	2.0381	37	23	1.1476	2.0381	37	23	1.1476	2.0381	37
24	1.1264	2.1730	36	24	1.1368	2.1025	36	24	1.1478	2.0371	36	24	1.1478	2.0371	36	24	1.1478	2.0371	36	24	1.1478	2.0371	36
25	1.1265	2.1718	35	25	1.1370	2.1014	35	25	1.1480	2.0360	35	25	1.1480	2.0360	35	25	1.1480	2.0360	35	25	1.1480	2.0360	35
26	1.1267	2.1705	34	26	1.1372	2.1002	34	26	1.1482	2.0350	34	26	1.1482	2.0350	34	26	1.1482	2.0350	34	26	1.1482	2.0350	34
27	1.1269	2.1693	33	27	1.1374	2.0991	33	27	1.1484	2.0339	33	27	1.1484	2.0339	33	27	1.1484	2.0339	33	27	1.1484	2.0339	33
28	1.1270	2.1681	32	28	1.1375	2.0980	32	28	1.1486	2.0329	32	28	1.1486	2.0329	32	28	1.1486	2.0329	32	28	1.1486	2.0329	32
29	1.1272	2.1669	31	29	1.1377	2.0969	31	29	1.1488	2.0318	31	29	1.1488	2.0318	31	29	1.1488	2.0318	31	29	1.1488	2.0318	31
30	1.1274	2.1657	30	30	1.1379	2.0957	30	30	1.1490	2.0308	30	30	1.1490	2.0308	30	30	1.1490	2.0308	30	30	1.1490	2.0308	30
31	1.1276	2.1645	29	31	1.1381	2.0946	29	31	1.1491	2.0297	29	31	1.1491	2.0297	29	31	1.1491	2.0297	29	31	1.1491	2.0297	29
32	1.1277	2.1633	28	32	1.1383	2.0935	28	32	1.1493	2.0287	28	32	1.1493	2.0287	28	32	1.1493	2.0287	28	32	1.1493	2.0287	28
33	1.1279	2.1621	27	33	1.1384	2.0924	27	33	1.1495	2.0276	27	33	1.1495	2.0276	27	33	1.1495	2.0276	27	33	1.1495	2.0276	27
34	1.1281	2.1609	26	34	1.1386	2.0913	26	34	1.1497	2.0266	26	34	1.1497	2.0266	26	34	1.1497	2.0266	26	34	1.1497	2.0266	26
35	1.1282	2.1596	25	35	1.1388	2.0901	25	35	1.1499	2.0256	25	35	1.1499	2.0256	25	35	1.1499	2.0256	25	35	1.1499	2.0256	25
36	1.1284	2.1584	24	36	1.1390	2.0890	24	36	1.1501	2.0245	24	36	1.1501	2.0245	24	36	1.1501	2.0245	24	36	1.1501	2.0245	24
37	1.1286	2.1572	23	37	1.1392	2.0879	23	37	1.1503	2.0235	23	37	1.1503	2.0235	23	37	1.1503	2.0235	23	37	1.1503	2.0235	23
38	1.1288	2.1560	22	38	1.1393	2.0868	22	38	1.1505	2.0225	22	38	1.1505	2.0225	22	38	1.1505	2.0225	22	38	1.1505	2.0225	22
39	1.1289	2.1549	21	39	1.1395	2.0857	21	39	1.1507	2.0214	21	39	1.1507	2.0214	21	39	1.1507	2.0214	21	39	1.1507	2.0214	21
40	1.1291	2.1537	20	40	1.1397	2.0846	20	40	1.1509	2.0204	20	40	1.1509	2.0204	20	40	1.1509	2.0204	20	40	1.1509	2.0204	20
41	1.1293	2.1525	19	41	1.1399	2.0835	19	41	1.1510	2.0194	19	41	1.1510	2.0194	19	41	1.1510	2.0194	19	41	1.1510	2.0194	19
42	1.1294	2.1513	18	42	1.1401	2.0824	18	42	1.1512	2.0183	18	42	1.1512	2.0183	18	42	1.1512	2.0183	18	42	1.1512	2.0183	18
43	1.1296	2.1501	17	43	1.1402	2.0813	17	43	1.1514	2.0173	17	43	1.1514	2.0173	17	43	1.1514	2.0173	17	43	1.1514	2.0173	17
44	1.1298	2.1489	16	44	1.1404	2.0802	16	44	1.1516	2.0163	16	44	1.1516	2.0163	16	44	1.1516	2.0163	16	44	1.1516	2.0163	16
45	1.1300	2.1477	15	45	1.1406	2.0791	15	45	1.1518	2.0152	15	45	1.1518	2.0152	15	45	1.1518	2.0152	15	45	1.1518	2.0152	15
46	1.1301	2.1465	14	46	1.1408	2.0779	14	46	1.1520	2.0142	14	46	1.1520	2.0142	14	46	1.1520	2.0142	14	46	1.1520	2.0142	14
47	1.1303	2.1453	13	47	1.1410	2.0768	13	47	1.1522	2.0132	13	47	1.1522	2.0132	13	47	1.1522	2.0132	13	47	1.1522	2.0132	13
48	1.1305	2.1441	12	48	1.1412	2.0757	12	48	1.1524	2.0122	12	48	1.1524	2.0122	12	48	1.1524	2.0122	12	48	1.1524	2.0122	12
49	1.1307	2.1430	11	49	1.1413	2.0747	11	49	1.1526	2.0112	11	49	1.1526	2.0112	11	49	1.1526	2.0112	11	49	1.1526	2.0112	11
50	1.1308	2.1418	10	50	1.1415	2.0736	10	50	1.1528	2.0101	10	50	1.1528	2.0101	10	50	1.1528	2.0101	10	50	1.1528	2.0101	10
51	1.1310	2.1406	9	51	1.1417	2.0725	9	51	1.1530	2.0091	9	51	1.1530	2.0091	9	51	1.1530	2.0091	9	51	1.1530	2.0091	9
52	1.1312	2.1394	8	52	1.1419	2.0714	8	52	1.1532	2.0081	8	52	1.1532	2.0081	8	52	1.1532	2.0081	8	52	1.1532	2.0081	8
53	1.1313	2.1382	7	53	1.1421	2.0703	7	53	1.1533	2.0071	7	53	1.1533	2.0071	7	53	1.1533	2.0071	7	53	1.1533	2	

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

30° (210°)			(329°) 149°			31° (211°)			(328°) 148°			32° (212°)			(327°) 147°		
/	Sec	Csc	/			/	Sec	Csc	/			/	Sec	Csc	/		
0	1.1547	2.0000	60			0	1.1666	1.9416	60			0	1.1792	1.8871	60		
1	1.1549	1.9990	59			1	1.1668	1.9407	59			1	1.1794	1.8862	59		
2	1.1551	1.9980	58			2	1.1670	1.9397	58			2	1.1796	1.8853	58		
3	1.1553	1.9970	57			3	1.1672	1.9388	57			3	1.1798	1.8844	57		
4	1.1555	1.9960	56			4	1.1675	1.9379	56			4	1.1800	1.8836	56		
5	1.1557	1.9950	55			5	1.1677	1.9369	55			5	1.1803	1.8827	55		
6	1.1559	1.9940	54			6	1.1679	1.9360	54			6	1.1805	1.8818	54		
7	1.1561	1.9930	53			7	1.1681	1.9351	53			7	1.1807	1.8810	53		
8	1.1563	1.9920	52			8	1.1683	1.9341	52			8	1.1809	1.8801	52		
9	1.1565	1.9910	51			9	1.1685	1.9332	51			9	1.1811	1.8792	51		
10	1.1566	1.9900	50			10	1.1687	1.9323	50			10	1.1813	1.8783	50		
11	1.1568	1.9890	49			11	1.1689	1.9313	49			11	1.1815	1.8775	49		
12	1.1570	1.9880	48			12	1.1691	1.9304	48			12	1.1818	1.8766	48		
13	1.1572	1.9870	47			13	1.1693	1.9295	47			13	1.1820	1.8757	47		
14	1.1574	1.9860	46			14	1.1695	1.9285	46			14	1.1822	1.8749	46		
15	1.1576	1.9850	45			15	1.1697	1.9276	45			15	1.1824	1.8740	45		
16	1.1578	1.9840	44			16	1.1699	1.9267	44			16	1.1826	1.8731	44		
17	1.1580	1.9830	43			17	1.1701	1.9258	43			17	1.1828	1.8723	43		
18	1.1582	1.9821	42			18	1.1703	1.9249	42			18	1.1831	1.8714	42		
19	1.1584	1.9811	41			19	1.1705	1.9239	41			19	1.1833	1.8706	41		
20	1.1586	1.9801	40			20	1.1707	1.9230	40			20	1.1835	1.8697	40		
21	1.1588	1.9791	39			21	1.1710	1.9221	39			21	1.1837	1.8688	39		
22	1.1590	1.9781	38			22	1.1712	1.9212	38			22	1.1839	1.8680	38		
23	1.1592	1.9771	37			23	1.1714	1.9203	37			23	1.1842	1.8671	37		
24	1.1594	1.9762	36			24	1.1716	1.9194	36			24	1.1844	1.8663	36		
25	1.1596	1.9752	35			25	1.1718	1.9184	35			25	1.1846	1.8654	35		
26	1.1598	1.9742	34			26	1.1720	1.9175	34			26	1.1848	1.8646	34		
27	1.1600	1.9732	33			27	1.1722	1.9166	33			27	1.1850	1.8637	33		
28	1.1602	1.9722	32			28	1.1724	1.9157	32			28	1.1852	1.8629	32		
29	1.1604	1.9713	31			29	1.1726	1.9148	31			29	1.1855	1.8620	31		
30	1.1606	1.9703	30			30	1.1728	1.9139	30			30	1.1857	1.8612	30		
31	1.1608	1.9693	29			31	1.1730	1.9130	29			31	1.1859	1.8603	29		
32	1.1610	1.9684	28			32	1.1732	1.9121	28			32	1.1861	1.8595	28		
33	1.1612	1.9674	27			33	1.1735	1.9112	27			33	1.1863	1.8586	27		
34	1.1614	1.9664	26			34	1.1737	1.9103	26			34	1.1866	1.8578	26		
35	1.1616	1.9654	25			35	1.1739	1.9094	25			35	1.1868	1.8569	25		
36	1.1618	1.9645	24			36	1.1741	1.9084	24			36	1.1870	1.8561	24		
37	1.1620	1.9635	23			37	1.1743	1.9075	23			37	1.1872	1.8552	23		
38	1.1622	1.9625	22			38	1.1745	1.9066	22			38	1.1875	1.8544	22		
39	1.1624	1.9616	21			39	1.1747	1.9057	21			39	1.1877	1.8535	21		
40	1.1626	1.9606	20			40	1.1749	1.9048	20			40	1.1879	1.8527	20		
41	1.1628	1.9597	19			41	1.1751	1.9039	19			41	1.1881	1.8519	19		
42	1.1630	1.9587	18			42	1.1753	1.9031	18			42	1.1883	1.8510	18		
43	1.1632	1.9577	17			43	1.1756	1.9022	17			43	1.1886	1.8502	17		
44	1.1634	1.9568	16			44	1.1758	1.9013	16			44	1.1888	1.8494	16		
45	1.1636	1.9558	15			45	1.1760	1.9004	15			45	1.1890	1.8485	15		
46	1.1638	1.9549	14			46	1.1762	1.8995	14			46	1.1892	1.8477	14		
47	1.1640	1.9539	13			47	1.1764	1.8986	13			47	1.1895	1.8468	13		
48	1.1642	1.9530	12			48	1.1766	1.8977	12			48	1.1897	1.8460	12		
49	1.1644	1.9520	11			49	1.1768	1.8968	11			49	1.1899	1.8452	11		
50	1.1646	1.9511	10			50	1.1770	1.8959	10			50	1.1901	1.8443	10		
51	1.1648	1.9501	9			51	1.1773	1.8950	9			51	1.1903	1.8435	9		
52	1.1650	1.9492	8			52	1.1775	1.8941	8			52	1.1906	1.8427	8		
53	1.1652	1.9482	7			53	1.1777	1.8933	7			53	1.1908	1.8419	7		
54	1.1654	1.9473	6			54	1.1779	1.8924	6			54	1.1910	1.8410	6		
55	1.1656	1.9463	5			55	1.1781	1.8915	5			55	1.1912	1.8402	5		
56	1.1658	1.9454	4			56	1.1783	1.8906	4			56	1.1915	1.8394	4		
57	1.1660	1.9444	3			57	1.1785	1.8897	3			57	1.1917	1.8385	3		
58	1.1662	1.9435	2			58	1.1788	1.8888	2			58	1.1919	1.8377	2		
59	1.1664	1.9425	1			59	1.1790	1.8880	1			59	1.1921	1.8369	1		
60	1.1666	1.9416	0			60	1.1792	1.8871	0			60	1.1924	1.8361	0		
/	Csc	Sec	/			/	Csc	Sec	/			/	Csc	Sec	/		

120° (300°)

(239°) 59°

121° (301°)

(238°) 58°

122° (302°)

(237°) 57°

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

33° (213°)			(326°) 146°			34° (214°)			(325°) 145°			35° (215°)			(324°) 144°		
/	Sec	Csc	/			/	Sec	Csc	/			/	Sec	Csc	/		
0	1.1924	1.8361	60			0	1.2062	1.7883	60			0	1.2208	1.7434	60		
1	1.1926	1.8353	59			1	1.2065	1.7875	59			1	1.2210	1.7427	59		
2	1.1928	1.8344	58			2	1.2067	1.7868	58			2	1.2213	1.7420	58		
3	1.1930	1.8336	57			3	1.2069	1.7860	57			3	1.2215	1.7413	57		
4	1.1933	1.8328	56			4	1.2072	1.7852	56			4	1.2218	1.7406	56		
5	1.1935	1.8320	55			5	1.2074	1.7844	55			5	1.2220	1.7398	55		
6	1.1937	1.8312	54			6	1.2076	1.7837	54			6	1.2223	1.7391	54		
7	1.1939	1.8303	53			7	1.2079	1.7829	53			7	1.2225	1.7384	53		
8	1.1942	1.8295	52			8	1.2081	1.7821	52			8	1.2228	1.7377	52		
9	1.1944	1.8287	51			9	1.2084	1.7814	51			9	1.2230	1.7370	51		
10	1.1946	1.8279	50			10	1.2086	1.7806	50			10	1.2233	1.7362	50		
11	1.1949	1.8271	49			11	1.2088	1.7799	49			11	1.2235	1.7355	49		
12	1.1951	1.8263	48			12	1.2091	1.7791	48			12	1.2238	1.7348	48		
13	1.1953	1.8255	47			13	1.2093	1.7783	47			13	1.2240	1.7341	47		
14	1.1955	1.8247	46			14	1.2096	1.7776	46			14	1.2243	1.7334	46		
15	1.1958	1.8238	45			15	1.2098	1.7768	45			15	1.2245	1.7327	45		
16	1.1960	1.8230	44			16	1.2100	1.7761	44			16	1.2248	1.7320	44		
17	1.1962	1.8222	43			17	1.2103	1.7753	43			17	1.2250	1.7312	43		
18	1.1964	1.8214	42			18	1.2105	1.7745	42			18	1.2253	1.7305	42		
19	1.1967	1.8206	41			19	1.2108	1.7738	41			19	1.2255	1.7298	41		
20	1.1969	1.8198	40			20	1.2110	1.7730	40			20	1.2258	1.7291	40		
21	1.1971	1.8190	39			21	1.2112	1.7723	39			21	1.2260	1.7284	39		
22	1.1974	1.8182	38			22	1.2115	1.7715	38			22	1.2263	1.7277	38		
23	1.1976	1.8174	37			23	1.2117	1.7708	37			23	1.2265	1.7270	37		
24	1.1978	1.8166	36			24	1.2120	1.7700	36			24	1.2268	1.7263	36		
25	1.1981	1.8158	35			25	1.2122	1.7693	35			25	1.2271	1.7256	35		
26	1.1983	1.8150	34			26	1.2124	1.7685	34			26	1.2273	1.7249	34		
27	1.1985	1.8142	33			27	1.2127	1.7678	33			27	1.2276	1.7242	33		
28	1.1987	1.8134	32			28	1.2129	1.7670	32			28	1.2278	1.7235	32		
29	1.1990	1.8126	31			29	1.2132	1.7663	31			29	1.2281	1.7228	31		
30	1.1992	1.8118	30			30	1.2134	1.7655	30			30	1.2283	1.7221	30		
31	1.1994	1.8110	29			31	1.2136	1.7648	29			31	1.2286	1.7213	29		
32	1.1997	1.8102	28			32	1.2139	1.7640	28			32	1.2288	1.7206	28		
33	1.1999	1.8094	27			33	1.2141	1.7633	27			33	1.2291	1.7199	27		
34	1.2001	1.8086	26			34	1.2144	1.7625	26			34	1.2293	1.7192	26		
35	1.2004	1.8078	25			35	1.2146	1.7618	25			35	1.2296	1.7185	25		
36	1.2006	1.8070	24			36	1.2149	1.7610	24			36	1.2299	1.7177	24		
37	1.2008	1.8062	23			37	1.2151	1.7603	23			37	1.2301	1.7172	23		
38	1.2011	1.8055	22			38	1.2154	1.7596	22			38	1.2304	1.7165	22		
39	1.2013	1.8047	21			39	1.2156	1.7588	21			39	1.2306	1.7158	21		
40	1.2015	1.8039	20			40	1.2158	1.7581	20			40	1.2309	1.7151	20		
41	1.2018	1.8031	19			41	1.2161	1.7573	19			41	1.2311	1.7144	19		
42	1.2020	1.8023	18			42	1.2163	1.7566	18			42	1.2314	1.7137	18		
43	1.2022	1.8015	17			43	1.2166	1.7559	17			43	1.2317	1.7130	17		
44	1.2025	1.8007	16			44	1.2168	1.7551	16			44	1.2319	1.7123	16		
45	1.2027	1.8000	15			45	1.2171	1.7544	15			45	1.2322	1.7116	15		
46	1.2029	1.7992	14			46	1.2173	1.7537	14			46	1.2324	1.7109	14		
47	1.2032	1.7984	13			47	1.2176	1.7529	13			47	1.2327	1.7102	13		
48	1.2034	1.7976	12			48	1.2178	1.7522	12			48	1.2329	1.7095	12		
49	1.2036	1.7968	11			49	1.2181	1.7515	11			49	1.2332	1.7088	11		
50	1.2039	1.7960	10			50	1.2183	1.7507	10			50	1.2335	1.7081	10		
51	1.2041	1.7953	9			51	1.2185	1.7500	9			51	1.2337	1.7075	9		
52	1.2043	1.7945	8			52	1.2188	1.7493	8			52	1.2340	1.7068	8		
53	1.2046	1.7937	7			53	1.2190	1.7485	7			53	1.2342	1.7061	7		
54	1.2048	1.7929	6			54	1.2193	1.7478	6			54	1.2345	1.7054	6		
55	1.2050	1.7922	5			55	1.2195	1.7471	5			55	1.2348	1.7047	5		
56	1.2053	1.7914	4			56	1.2198	1.7463	4			56	1.2350	1.7040	4		
57	1.2055	1.7906	3			57	1.2200	1.7456	3			57	1.2353	1.7033	3		
58	1.2057	1.7898	2			58	1.2203	1.7449	2			58	1.2355	1.7027	2		
59	1.2059	1.7891	1			59	1.2205	1.7442	1			59	1.2358	1.7020	1		
60	1.2062	1.7883	0			60	1.2208	1.7434	0			60	1.2361	1.7013	0		
/	Csc	Sec	/			/	Csc	Sec	/			/	Csc	Sec	/		

123° (303°)

(236°) 56°

124° (304°)

(235°) 55°

125° (305°)

(234°) 54°

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

36° (216°)			(323°) 143°			37° (217°)			(322°) 142°			38° (218°)			(321°) 141°		
/	Sec	Csc	/			/	Sec	Csc	/			/	Sec	Csc	/		
0	1.2361	1.7013	60			0	1.2521	1.6616	60			0	1.2690	1.6243	60		
1	1.2363	1.7006	59			1	1.2524	1.6610	59			1	1.2693	1.6237	59		
2	1.2366	1.6999	58			2	1.2527	1.6604	58			2	1.2696	1.6231	58		
3	1.2369	1.6993	57			3	1.2530	1.6597	57			3	1.2699	1.6225	57		
4	1.2371	1.6986	56			4	1.2532	1.6591	56			4	1.2702	1.6219	56		
5	1.2374	1.6979	55			5	1.2535	1.6584	55			5	1.2705	1.6213	55		
6	1.2376	1.6972	54			6	1.2538	1.6578	54			6	1.2708	1.6207	54		
7	1.2379	1.6966	53			7	1.2541	1.6572	53			7	1.2710	1.6201	53		
8	1.2382	1.6959	52			8	1.2543	1.6565	52			8	1.2713	1.6195	52		
9	1.2384	1.6952	51			9	1.2546	1.6559	51			9	1.2716	1.6189	51		
10	1.2387	1.6945	50			10	1.2549	1.6553	50			10	1.2719	1.6183	50		
11	1.2390	1.6939	49			11	1.2552	1.6546	49			11	1.2722	1.6177	49		
12	1.2392	1.6932	48			12	1.2554	1.6540	48			12	1.2725	1.6171	48		
13	1.2395	1.6925	47			13	1.2557	1.6534	47			13	1.2728	1.6165	47		
14	1.2397	1.6918	46			14	1.2560	1.6527	46			14	1.2731	1.6159	46		
15	1.2400	1.6912	45			15	1.2563	1.6521	45			15	1.2734	1.6153	45		
16	1.2403	1.6905	44			16	1.2566	1.6515	44			16	1.2737	1.6147	44		
17	1.2405	1.6898	43			17	1.2568	1.6508	43			17	1.2740	1.6141	43		
18	1.2408	1.6892	42			18	1.2571	1.6502	42			18	1.2742	1.6135	42		
19	1.2411	1.6885	41			19	1.2574	1.6496	41			19	1.2745	1.6129	41		
20	1.2413	1.6878	40			20	1.2577	1.6489	40			20	1.2748	1.6123	40		
21	1.2416	1.6871	39			21	1.2579	1.6483	39			21	1.2751	1.6117	39		
22	1.2419	1.6865	38			22	1.2582	1.6477	38			22	1.2754	1.6111	38		
23	1.2421	1.6858	37			23	1.2585	1.6471	37			23	1.2757	1.6105	37		
24	1.2424	1.6852	36			24	1.2588	1.6464	36			24	1.2760	1.6099	36		
25	1.2427	1.6845	35			25	1.2591	1.6458	35			25	1.2763	1.6093	35		
26	1.2429	1.6838	34			26	1.2593	1.6452	34			26	1.2766	1.6087	34		
27	1.2432	1.6832	33			27	1.2596	1.6446	33			27	1.2769	1.6082	33		
28	1.2435	1.6825	32			28	1.2599	1.6439	32			28	1.2772	1.6076	32		
29	1.2437	1.6818	31			29	1.2602	1.6433	31			29	1.2775	1.6070	31		
30	1.2440	1.6812	30			30	1.2605	1.6427	30			30	1.2778	1.6064	30		
31	1.2443	1.6805	29			31	1.2608	1.6421	29			31	1.2781	1.6058	29		
32	1.2445	1.6799	28			32	1.2610	1.6414	28			32	1.2784	1.6052	28		
33	1.2448	1.6792	27			33	1.2613	1.6408	27			33	1.2787	1.6046	27		
34	1.2451	1.6785	26			34	1.2616	1.6402	26			34	1.2790	1.6040	26		
35	1.2453	1.6779	25			35	1.2619	1.6396	25			35	1.2793	1.6035	25		
36	1.2456	1.6772	24			36	1.2622	1.6390	24			36	1.2796	1.6029	24		
37	1.2459	1.6766	23			37	1.2624	1.6383	23			37	1.2799	1.6023	23		
38	1.2462	1.6759	22			38	1.2627	1.6377	22			38	1.2802	1.6017	22		
39	1.2464	1.6753	21			39	1.2630	1.6371	21			39	1.2804	1.6011	21		
40	1.2467	1.6746	20			40	1.2633	1.6365	20			40	1.2807	1.6005	20		
41	1.2470	1.6739	19			41	1.2636	1.6359	19			41	1.2810	1.6000	19		
42	1.2472	1.6733	18			42	1.2639	1.6353	18			42	1.2813	1.5994	18		
43	1.2475	1.6726	17			43	1.2641	1.6346	17			43	1.2816	1.5988	17		
44	1.2478	1.6720	16			44	1.2644	1.6340	16			44	1.2819	1.5982	16		
45	1.2480	1.6713	15			45	1.2647	1.6334	15			45	1.2822	1.5976	15		
46	1.2483	1.6707	14			46	1.2650	1.6328	14			46	1.2825	1.5971	14		
47	1.2486	1.6700	13			47	1.2653	1.6322	13			47	1.2828	1.5965	13		
48	1.2489	1.6694	12			48	1.2656	1.6316	12			48	1.2831	1.5959	12		
49	1.2491	1.6687	11			49	1.2659	1.6310	11			49	1.2834	1.5953	11		
50	1.2494	1.6681	10			50	1.2661	1.6303	10			50	1.2837	1.5948	10		
51	1.2497	1.6674	9			51	1.2664	1.6297	9			51	1.2840	1.5942	9		
52	1.2499	1.6668	8			52	1.2667	1.6291	8			52	1.2843	1.5936	8		
53	1.2502	1.6661	7			53	1.2670	1.6285	7			53	1.2846	1.5930	7		
54	1.2505	1.6655	6			54	1.2673	1.6279	6			54	1.2849	1.5925	6		
55	1.2508	1.6649	5			55	1.2676	1.6273	5			55	1.2852	1.5919	5		
56	1.2510	1.6642	4			56	1.2679	1.6267	4			56	1.2855	1.5913	4		
57	1.2513	1.6636	3			57	1.2682	1.6261	3			57	1.2859	1.5907	3		
58	1.2516	1.6629	2			58	1.2684	1.6255	2			58	1.2862	1.5902	2		
59	1.2519	1.6623	1			59	1.2687	1.6249	1			59	1.2865	1.5896	1		
60	1.2521	1.6616	0			60	1.2690	1.6243	0			60	1.2868	1.5890	0		
/	Csc	Sec	/			/	Csc	Sec	/			/	Csc	Sec	/		

126° (306°)

(233°) 53°

127° (307°)

(232°) 52°

128° (308°)

(231°) 51°

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

39° (219°)				(320°) 140°				40° (220°)				(319°) 139°				41° (221°)				(318°) 138°			
/	Sec	Csc	/	/	Sec	Csc	/	/	Sec	Csc	/	/	Sec	Csc	/	/	Sec	Csc	/	/			
0	1.2868	1.5890	60	0	1.3054	1.5557	60	0	1.3250	1.5243	60	0	1.3250	1.5243	60	0	1.3250	1.5243	60	0			
1	1.2871	1.5884	59	1	1.3057	1.5552	59	1	1.3253	1.5237	59	1	1.3253	1.5237	59	1	1.3253	1.5237	59	1			
2	1.2874	1.5879	58	2	1.3060	1.5546	58	2	1.3257	1.5232	58	2	1.3257	1.5232	58	2	1.3257	1.5232	58	2			
3	1.2877	1.5873	57	3	1.3064	1.5541	57	3	1.3260	1.5227	57	3	1.3260	1.5227	57	3	1.3260	1.5227	57	3			
4	1.2880	1.5867	56	4	1.3067	1.5536	56	4	1.3264	1.5222	56	4	1.3264	1.5222	56	4	1.3264	1.5222	56	4			
5	1.2883	1.5862	55	5	1.3070	1.5530	55	5	1.3267	1.5217	55	5	1.3267	1.5217	55	5	1.3267	1.5217	55	5			
6	1.2886	1.5856	54	6	1.3073	1.5525	54	6	1.3270	1.5212	54	6	1.3270	1.5212	54	6	1.3270	1.5212	54	6			
7	1.2889	1.5850	53	7	1.3076	1.5520	53	7	1.3274	1.5207	53	7	1.3274	1.5207	53	7	1.3274	1.5207	53	7			
8	1.2892	1.5845	52	8	1.3080	1.5514	52	8	1.3277	1.5202	52	8	1.3277	1.5202	52	8	1.3277	1.5202	52	8			
9	1.2895	1.5839	51	9	1.3083	1.5509	51	9	1.3280	1.5197	51	9	1.3280	1.5197	51	9	1.3280	1.5197	51	9			
10	1.2898	1.5833	50	10	1.3086	1.5504	50	10	1.3284	1.5192	50	10	1.3284	1.5192	50	10	1.3284	1.5192	50	10			
11	1.2901	1.5828	49	11	1.3089	1.5498	49	11	1.3287	1.5187	49	11	1.3287	1.5187	49	11	1.3287	1.5187	49	11			
12	1.2904	1.5822	48	12	1.3093	1.5493	48	12	1.3291	1.5182	48	12	1.3291	1.5182	48	12	1.3291	1.5182	48	12			
13	1.2907	1.5816	47	13	1.3096	1.5488	47	13	1.3294	1.5177	47	13	1.3294	1.5177	47	13	1.3294	1.5177	47	13			
14	1.2910	1.5811	46	14	1.3099	1.5482	46	14	1.3297	1.5172	46	14	1.3297	1.5172	46	14	1.3297	1.5172	46	14			
15	1.2913	1.5805	45	15	1.3102	1.5477	45	15	1.3301	1.5167	45	15	1.3301	1.5167	45	15	1.3301	1.5167	45	15			
16	1.2916	1.5800	44	16	1.3105	1.5472	44	16	1.3304	1.5162	44	16	1.3304	1.5162	44	16	1.3304	1.5162	44	16			
17	1.2919	1.5794	43	17	1.3109	1.5466	43	17	1.3307	1.5156	43	17	1.3307	1.5156	43	17	1.3307	1.5156	43	17			
18	1.2923	1.5788	42	18	1.3112	1.5461	42	18	1.3311	1.5151	42	18	1.3311	1.5151	42	18	1.3311	1.5151	42	18			
19	1.2926	1.5783	41	19	1.3115	1.5456	41	19	1.3314	1.5146	41	19	1.3314	1.5146	41	19	1.3314	1.5146	41	19			
20	1.2929	1.5777	40	20	1.3118	1.5450	40	20	1.3318	1.5141	40	20	1.3318	1.5141	40	20	1.3318	1.5141	40	20			
21	1.2932	1.5771	39	21	1.3122	1.5445	39	21	1.3321	1.5136	39	21	1.3321	1.5136	39	21	1.3321	1.5136	39	21			
22	1.2935	1.5766	38	22	1.3125	1.5440	38	22	1.3325	1.5131	38	22	1.3325	1.5131	38	22	1.3325	1.5131	38	22			
23	1.2938	1.5760	37	23	1.3128	1.5435	37	23	1.3328	1.5126	37	23	1.3328	1.5126	37	23	1.3328	1.5126	37	23			
24	1.2941	1.5755	36	24	1.3131	1.5429	36	24	1.3331	1.5121	36	24	1.3331	1.5121	36	24	1.3331	1.5121	36	24			
25	1.2944	1.5749	35	25	1.3135	1.5424	35	25	1.3335	1.5116	35	25	1.3335	1.5116	35	25	1.3335	1.5116	35	25			
26	1.2947	1.5744	34	26	1.3138	1.5419	34	26	1.3338	1.5111	34	26	1.3338	1.5111	34	26	1.3338	1.5111	34	26			
27	1.2950	1.5738	33	27	1.3141	1.5413	33	27	1.3342	1.5107	33	27	1.3342	1.5107	33	27	1.3342	1.5107	33	27			
28	1.2953	1.5732	32	28	1.3144	1.5408	32	28	1.3345	1.5102	32	28	1.3345	1.5102	32	28	1.3345	1.5102	32	28			
29	1.2957	1.5727	31	29	1.3148	1.5403	31	29	1.3348	1.5097	31	29	1.3348	1.5097	31	29	1.3348	1.5097	31	29			
30	1.2960	1.5721	30	30	1.3151	1.5398	30	30	1.3352	1.5092	30	30	1.3352	1.5092	30	30	1.3352	1.5092	30	30			
31	1.2963	1.5716	29	31	1.3154	1.5392	29	31	1.3355	1.5087	29	31	1.3355	1.5087	29	31	1.3355	1.5087	29	31			
32	1.2966	1.5710	28	32	1.3157	1.5387	28	32	1.3359	1.5082	28	32	1.3359	1.5082	28	32	1.3359	1.5082	28	32			
33	1.2969	1.5705	27	33	1.3161	1.5382	27	33	1.3362	1.5077	27	33	1.3362	1.5077	27	33	1.3362	1.5077	27	33			
34	1.2972	1.5699	26	34	1.3164	1.5377	26	34	1.3366	1.5072	26	34	1.3366	1.5072	26	34	1.3366	1.5072	26	34			
35	1.2975	1.5694	25	35	1.3167	1.5372	25	35	1.3369	1.5067	25	35	1.3369	1.5067	25	35	1.3369	1.5067	25	35			
36	1.2978	1.5688	24	36	1.3171	1.5366	24	36	1.3373	1.5062	24	36	1.3373	1.5062	24	36	1.3373	1.5062	24	36			
37	1.2981	1.5683	23	37	1.3174	1.5361	23	37	1.3376	1.5057	23	37	1.3376	1.5057	23	37	1.3376	1.5057	23	37			
38	1.2985	1.5677	22	38	1.3177	1.5356	22	38	1.3380	1.5052	22	38	1.3380	1.5052	22	38	1.3380	1.5052	22	38			
39	1.2988	1.5672	21	39	1.3180	1.5351	21	39	1.3383	1.5047	21	39	1.3383	1.5047	21	39	1.3383	1.5047	21	39			
40	1.2991	1.5666	20	40	1.3184	1.5345	20	40	1.3386	1.5042	20	40	1.3386	1.5042	20	40	1.3386	1.5042	20	40			
41	1.2994	1.5661	19	41	1.3187	1.5340	19	41	1.3390	1.5037	19	41	1.3390	1.5037	19	41	1.3390	1.5037	19	41			
42	1.2997	1.5655	18	42	1.3190	1.5335	18	42	1.3393	1.5032	18	42	1.3393	1.5032	18	42	1.3393	1.5032	18	42			
43	1.3000	1.5650	17	43	1.3194	1.5330	17	43	1.3397	1.5027	17	43	1.3397	1.5027	17	43	1.3397	1.5027	17	43			
44	1.3003	1.5644	16	44	1.3197	1.5325	16	44	1.3400	1.5023	16	44	1.3400	1.5023	16	44	1.3400	1.5023	16	44			
45	1.3307	1.5639	15	45	1.3200	1.5320	15	45	1.3404	1.5018	15	45	1.3404	1.5018	15	45	1.3404	1.5018	15	45			
46	1.3010	1.5633	14	46	1.3203	1.5314	14	46	1.3407	1.5013	14	46	1.3407	1.5013	14	46	1.3407	1.5013	14	46			
47	1.3013	1.5628	13	47	1.3207	1.5309	13	47	1.3411	1.5008	13	47	1.3411	1.5008	13	47	1.3411	1.5008	13	47			
48	1.3016	1.5622	12	48	1.3210	1.5304	12	48	1.3414	1.5003	12	48	1.3414	1.5003	12	48	1.3414	1.5003	12	48			
49	1.3019	1.5617	11	49	1.3213	1.5299	11	49	1.3418	1.4998	11	49	1.3418	1.4998	11	49	1.3418	1.4998	11	49			
50	1.3022	1.5611	10	50	1.3217	1.5294	10	50	1.3421	1.4993	10	50	1.3421	1.4993	10	50	1.3421	1.4993	10	50			
51	1.3026	1.5606	9	51	1.3220	1.5289	9	51	1.3425	1.4988	9	51	1.3425	1.4988	9	51	1.3425	1.4988	9	51			
52	1.3029	1.5601	8	52	1.3223	1.5283	8	52	1.3428	1.4984	8	52	1.3428	1.4984	8	52	1.3428	1.4984	8	52			
53	1.3032	1.5595	7	53	1.3227	1.5278	7	53	1.3432	1.4979	7	53	1.3432	1.4979	7	53	1.3432	1.4979	7	53			
54	1.3035	1.5590	6	54	1.3230	1.5273	6	54	1.3435	1.4974	6	54	1.3435	1.4974	6	54	1.3435	1.4974	6	54			
55	1.3038	1.5584	5	55	1.3233	1.5268	5	55	1.3439	1.4969	5	55	1.3439	1.4969	5	55	1.3439	1.4969	5	55			
56	1.3041	1.5579	4	56	1.3237	1.5263	4	56	1.3442	1.4964	4	56	1.3442	1.4964	4	56	1.3442	1.4964	4	56			
57	1.3045	1.5573	3	57	1.3240	1.5258	3	57	1.3446	1.4959	3	57	1.3446	1.4959	3	57	1.3446	1.4959	3	57			
58	1.3048	1.5568	2	58	1.3243	1.5253	2	58	1.3449	1.4954	2	58	1.3449	1.4954	2	58	1.3449	1.4954	2	58			
59	1.3051	1.5563	1	59	1.3247	1.5248	1	59	1.3453	1.4950	1	59	1.3453	1.4950	1	59	1.3453	1.4950	1	59			
60	1.3054	1.5557	0	60	1.3250	1.5243	0	60	1.3456	1.4945	0	60	1.3456	1.4945	0	60	1.3456	1.4945	0	60			
/	Csc	Sec	/	/	Csc	Sec	/	/	Csc	Sec	/	/	Csc	Sec	/	/	Csc	Sec	/	/			
129° (309°)				(230°) 50°				130° (310°)				(229°) 49°				131° (311°)				(228°) 48°			

Table 6.5(d) (Cont'd)

NATURAL SECANTS AND COSECANTS OF ANGLES IN DEGREES

42° (222°)				(317°) 137°				43° (223°)				(316°) 136°				44° (224°)				(315°) 135°			
/	Sec	Csc	/	/	Sec	Csc	/	/	Sec	Csc	/	/	Sec	Csc	/	/	Sec	Csc	/	/	Sec	Csc	/
0	1.3456	1.4945	60	0	1.3673	1.4663	60	0	1.3902	1.4396	60	0	1.3902	1.4396	60	0	1.3902	1.4396	60	0	1.3902	1.4396	60
1	1.3460	1.4940	59	1	1.3677	1.4658	59	1	1.3906	1.4391	59	1	1.3906	1.4391	59	1	1.3906	1.4391	59	1	1.3906	1.4391	59
2	1.3463	1.4935	58	2	1.3681	1.4654	58	2	1.3909	1.4387	58	2	1.3909	1.4387	58	2	1.3909	1.4387	58	2	1.3909	1.4387	58
3	1.3467	1.4930	57	3	1.3684	1.4649	57	3	1.3913	1.4383	57	3	1.3913	1.4383	57	3	1.3913	1.4383	57	3	1.3913	1.4383	57
4	1.3470	1.4925	56	4	1.3688	1.4645	56	4	1.3917	1.4378	56	4	1.3917	1.4378	56	4	1.3917	1.4378	56	4	1.3917	1.4378	56
5	1.3474	1.4921	55	5	1.3692	1.4640	55	5	1.3921	1.4374	55	5	1.3921	1.4374	55	5	1.3921	1.4374	55	5	1.3921	1.4374	55
6	1.3478	1.4916	54	6	1.3696	1.4635	54	6	1.3925	1.4370	54	6	1.3925	1.4370	54	6	1.3925	1.4370	54	6	1.3925	1.4370	54
7	1.3481	1.4911	53	7	1.3699	1.4631	53	7	1.3929	1.4365	53	7	1.3929	1.4365	53	7	1.3929	1.4365	53	7	1.3929	1.4365	53
8	1.3485	1.4906	52	8	1.3703	1.4626	52	8	1.3933	1.4361	52	8	1.3933	1.4361	52	8	1.3933	1.4361	52	8	1.3933	1.4361	52
9	1.3488	1.4901	51	9	1.3707	1.4622	51	9	1.3937	1.4357	51	9	1.3937	1.4357	51	9	1.3937	1.4357	51	9	1.3937	1.4357	51
10	1.3492	1.4897	50	10	1.3711	1.4617	50	10	1.3941	1.4352	50	10	1.3941	1.4352	50	10	1.3941	1.4352	50	10	1.3941	1.4352	50
11	1.3495	1.4892	49	11	1.3714	1.4613	49	11	1.3945	1.4348	49	11	1.3945	1.4348	49	11	1.3945	1.4348	49	11	1.3945	1.4348	49
12	1.3499	1.4887	48	12	1.3718	1.4608	48	12	1.3949	1.4344	48	12	1.3949	1.4344	48	12	1.3949	1.4344	48	12	1.3949	1.4344	48
13	1.3502	1.4882	47	13	1.3722	1.4604	47	13	1.3953	1.4340	47	13	1.3953	1.4340	47	13	1.3953	1.4340	47	13	1.3953	1.4340	47
14	1.3506	1.4878	46	14	1.3726	1.4599	46	14	1.3957	1.4335	46	14	1.3957	1.4335	46	14	1.3957	1.4335	46	14	1.3957	1.4335	46
15	1.3510	1.4873	45	15	1.3729	1.4595	45	15	1.3961	1.4331	45	15	1.3961	1.4331	45	15	1.3961	1.4331	45	15	1.3961	1.4331	45
16	1.3513	1.4868	44	16	1.3733	1.4590	44	16	1.3965	1.4327	44	16	1.3965	1.4327	44	16	1.3965	1.4327	44	16	1.3965	1.4327	44
17	1.3517	1.4863	43	17	1.3737	1.4586	43	17	1.3969	1.4322	43	17	1.3969	1.4322	43	17	1.3969	1.4322	43	17	1.3969	1.4322	43
18	1.3520	1.4859	42	18	1.3741	1.4581	42	18	1.3972	1.4318	42	18	1.3972	1.4318	42	18	1.3972	1.4318	42	18	1.3972	1.4318	42
19	1.3524	1.4854	41	19	1.3744	1.4577	41	19	1.3976	1.4314	41	19	1.3976	1.4314	41	19	1.3976	1.4314	41	19	1.3976	1.4314	41
20	1.3527	1.4849	40	20	1.3748	1.4572	40	20	1.3980	1.4310	40	20	1.3980	1.4310	40	20	1.3980	1.4310	40	20	1.3980	1.4310	40
21	1.3531	1.4844	39	21	1.3752	1.4568	39	21	1.3984	1.4305	39	21	1.3984	1.4305	39	21	1.3984	1.4305	39	21	1.3984	1.4305	39
22	1.3535	1.4840	38	22	1.3756	1.4563	38	22	1.3988	1.4301	38	22	1.3988	1.4301	38	22	1.3988	1.4301	38	22	1.3988	1.4301	38
23	1.3538	1.4835	37	23	1.3759	1.4559	37	23	1.3992	1.4297	37	23	1.3992	1.4297	37	23	1.3992	1.4297	37	23	1.3992	1.4297	37
24	1.3542	1.4830	36	24	1.3763	1.4554	36	24	1.3996	1.4293	36	24	1.3996	1.4293	36	24	1.3996	1.4293	36	24	1.3996	1.4293	36
25	1.3545	1.4825	35	25	1.3767	1.4550	35	25	1.4000	1.4288	35	25	1.4000	1.4288	35	25	1.4000	1.4288	35	25	1.4000	1.4288	35
26	1.3549	1.4821	34	26	1.3771	1.4545	34	26	1.4004	1.4284	34	26	1.4004	1.4284	34	26	1.4004	1.4284	34	26	1.4004	1.4284	34
27	1.3553	1.4816	33	27	1.3775	1.4541	33	27	1.4008	1.4280	33	27	1.4008	1.4280	33	27	1.4008	1.4280	33	27	1.4008	1.4280	33
28	1.3556	1.4811	32	28	1.3778	1.4536	32	28	1.4012	1.4276	32	28	1.4012	1.4276	32	28	1.4012	1.4276	32	28	1.4012	1.4276	32
29	1.3560	1.4807	31	29	1.3782	1.4532	31	29	1.4016	1.4271	31	29	1.4016	1.4271	31	29	1.4016	1.4271	31	29	1.4016	1.4271	31
30	1.3563	1.4802	30	30	1.3786	1.4527	30	30	1.4020	1.4267	30	30	1.4020	1.4267	30	30	1.4020	1.4267	30	30	1.4020	1.4267	30
31	1.3567	1.4797	29	31	1.3790	1.4523	29	31	1.4024	1.4263	29	31	1.4024	1.4263	29	31	1.4024	1.4263	29	31	1.4024	1.4263	29
32	1.3571	1.4792	28	32	1.3794	1.4518	28	32	1.4028	1.4259	28	32	1.4028	1.4259	28	32	1.4028	1.4259	28	32	1.4028	1.4259	28
33	1.3574	1.4788	27	33	1.3797	1.4514	27	33	1.4032	1.4255	27	33	1.4032	1.4255	27	33	1.4032	1.4255	27	33	1.4032	1.4255	27
34	1.3578	1.4783	26	34	1.3801	1.4510	26	34	1.4036	1.4250	26	34	1.4036	1.4250	26	34	1.4036	1.4250	26	34	1.4036	1.4250	26
35	1.3582	1.4778	25	35	1.3805	1.4505	25	35	1.4040	1.4246	25	35	1.4040	1.4246	25	35	1.4040	1.4246	25	35	1.4040	1.4246	25
36	1.3585	1.4774	24	36	1.3809	1.4501	24	36	1.4044	1.4242	24	36	1.4044	1.4242	24	36	1.4044	1.4242	24	36	1.4044	1.4242	24
37	1.3589	1.4769	23	37	1.3813	1.4496	23	37	1.4048	1.4238	23	37	1.4048	1.4238	23	37	1.4048	1.4238	23	37	1.4048	1.4238	23
38	1.3592	1.4764	22	38	1.3817	1.4492	22	38	1.4052	1.4234	22	38	1.4052	1.4234	22	38	1.4052	1.4234	22	38	1.4052	1.4234	22
39	1.3596	1.4760	21	39	1.3820	1.4487	21	39	1.4057	1.4229	21	39	1.4057	1.4229	21	39	1.4057	1.4229	21	39	1.4057	1.4229	21
40	1.3600	1.4755	20	40	1.3824	1.4483	20	40	1.4061	1.4225	20	40	1.4061	1.4225	20	40	1.4061	1.4225	20	40	1.4061	1.4225	20
41	1.3603	1.4750	19	41	1.3828	1.4479	19	41	1.4065	1.4221	19	41	1.4065	1.4221	19	41	1.4065	1.4221	19	41	1.4065	1.4221	19
42	1.3607	1.4746	18	42	1.3832	1.4474	18	42	1.4069	1.4217	18	42	1.4069	1.4217	18	42	1.4069	1.4217	18	42	1.4069	1.4217	18
43	1.3611	1.4741	17	43	1.3836	1.4470	17	43	1.4073	1.4213	17	43	1.4073	1.4213	17	43	1.4073	1.4213	17	43	1.4073	1.4213	17
44	1.3614	1.4737	16	44	1.3840	1.4465	16	44	1.4077	1.4208	16	44	1.4077	1.4208	16	44	1.4077	1.4208	16	44	1.4077	1.4208	16
45	1.3618	1.4732	15	45	1.3843	1.4461	15	45	1.4081	1.4204	15	45	1.4081	1.4204	15	45	1.4081	1.4204	15	45	1.4081	1.4204	15
46	1.3622	1.4727	14	46	1.3847	1.4457	14	46	1.4085	1.4200	14	46	1.4085	1.4200	14	46	1.4085	1.4200	14	46	1.4085	1.4200	14
47	1.3625	1.4723	13	47	1.3851	1.4452	13	47	1.4089	1.4196	13	47	1.4089	1.4196	13	47	1.4089	1.4196	13	47	1.4089	1.4196	13
48	1.3629	1.4718	12	48	1.3855	1.4448	12	48	1.4093	1.4192	12	48	1.4093	1.4192	12	48	1.4093	1.4192	12	48	1.4093	1.4192	12
49	1.3633	1.4713	11	49	1.3859	1.4443	11	49	1.4097	1.4188	11	49	1.4097	1.4188	11	49	1.4097	1.4188	11	49	1.4097	1.4188	11
50	1.3636	1.4709	10	50	1.3863	1.4439	10	50	1.4101	1.4183	10	50	1.4101	1.4183	10	50	1.4101	1.4183	10	50	1.410		

6.6 INVERSES OF THE TRIGONOMETRIC FUNCTIONS

Each of the trigonometric functions has the property that to each value of the function there corresponds more than one argument. Their inverses are thus not functions. However, if the domains of the trigonometric functions are restricted in such a way that to each value there is exactly one argument, then the functions defined on these restricted domains have inverses which are functions. In the definitions of the inverse trigonometric functions which follow, the restrictions of domain and range are noted.

6.7 DEFINITION

- | | | |
|-----|---|--|
| (1) | $\arcsin x = \sin^{-1}(x).$ | Domain: $[-1, 1].$
Range: $[-\pi/2, \pi/2].$ |
| (2) | $\arccos x = \cos^{-1}(x).$ | Domain: $[-1, 1].$
Range: $[0, \pi].$ |
| (3) | $\arctan x = \tan^{-1}(x).$ | Domain: $(-\infty, \infty).$
Range: $(-\pi/2, \pi/2).$ |
| (4) | $\operatorname{arccot} x = \cot^{-1}(x).$ | Domain: $(-\infty, \infty).$
Range: $(0, \pi).$ |
| (5) | $\operatorname{arcsec} x = \sec^{-1}(x).$ | Domain: $(-\infty, -1], [1, \infty).$
Range: $[0, \pi/2), [\pi, 3\pi/2).$ |
| (6) | $\operatorname{arccsc} x = \csc^{-1}(x).$ | Domain: $(-\infty, -1], [1, \infty).$
Range: $(0, \pi/2], (\pi, 3\pi/2].$ |

6.8 THE LOGARITHMIC AND THE EXPONENTIAL FUNCTIONS

The natural logarithm function is defined by a differential equation with a boundary condition. If a is a positive number different from 1, then we may define a logarithmic function to the base a in terms of the natural logarithmic function. Since each logarithmic function has the property that for each value there is exactly one argument, each has an inverse which is a function. The exponential functions are the inverses of the logarithmic functions.

6.9 DEFINITION

The function f which satisfies the two equations

$$Df(x) = \frac{1}{x} \quad \text{and} \quad f(1) = 0$$

is called the **natural logarithm function** and is denoted by \ln .

6.10 DEFINITION

The number x for which $\ln x = 1$ is called the **base of natural logarithms** and is denoted by e . The number $e = \lim_{n \rightarrow 0} (1 + n)^{1/n} = 2.7182818$ to seven decimals.

6.11 DEFINITION

If $a > 0$ and $a \neq 1$, the **logarithm function to the base a** is defined by

$$\log_a x = \frac{\ln x}{\ln a}.$$

6.12 PROPERTIES OF LOGARITHMIC FUNCTIONS

- (1) $\ln e = 1.$
- (2) $\log_a a = 1.$
- (3) $\ln x = \log_e x.$

$$(4) \quad \log_a x = \frac{\ln x}{\ln a}.$$

$$(5) \quad \log_a b \log_b a = 1.$$

$$(6) \quad \log_a 1 = 0.$$

$$(7) \quad \log_a x_1 x_2 = \log_a x_1 + \log_a x_2.$$

$$(8) \quad \log_a \frac{x_1}{x_2} = \log_a x_1 - \log_a x_2.$$

$$(9) \quad \log_a x_1^{x_2} = x_2 \log_a x_1.$$

6.13 TABLE OF NATURAL LOGARITHMS

Table 6.13
NATURAL (NAPIERIAN) LOGARITHMS

N	0	1	2	3	4	5	6	7	8	9
1.0	0.0 0000	0995	1980	2956	3922	4879	5827	6766	7696	8618
1.1	9531	*0436	*1333	*2222	*3103	*3976	*4842	*5700	*6551	*7395
1.2	0.1 8232	9062	9885	*0701	*1511	*2314	*3111	*3902	*4686	*5464
1.3	0.2 6236	7003	7763	8518	9267	*0010	*0748	*1481	*2208	*2930
1.4	0.3 3647	4359	5066	5767	6464	7156	7844	8526	9204	9878
1.5	0.4 0547	1211	1871	2527	3178	3825	4469	5108	5742	6373
1.6	7000	7623	8243	8858	9470	*0078	*0672	*1282	*1879	*2473
1.7	0.5 3063	3649	4232	4812	5389	5962	6531	7098	7661	8222
1.8	8779	9333	9884	*0432	*0977	*1519	*2058	*2594	*3127	*3658
1.9	0.6 4185	4710	5233	5752	6269	6783	7294	7803	8310	8813
2.0	9315	9813	*0310	*0804	*1295	*1784	*2271	*2755	*3237	*3716
2.1	0.7 4194	4669	5142	5612	6081	6547	7011	7473	7932	8390
2.2	8846	9299	9751	*0200	*0648	*1093	*1536	*1978	*2418	*2855
2.3	0.8 3291	3725	4157	4587	5015	5442	5866	6289	6710	7129
2.4	7547	7963	8377	8789	9200	9609	*0016	*0422	*0826	*1228
2.5	0.9 1629	2028	2426	2822	3216	3609	4001	4391	4779	5166
2.6	5551	5935	6317	6698	7078	7456	7833	8208	8582	8954
2.7	9325	9695	*0063	*0430	*0796	*1160	*1523	*1885	*2245	*2604
2.8	1.0 2962	3318	3674	4028	4380	4732	5082	5431	5779	6126
2.9	6471	6815	7158	7500	7841	8181	8519	8856	9192	9527
3.0	9861	*0194	*0526	*0856	*1186	*1514	*1841	*2168	*2493	*2817
3.1	1.1 3140	3462	3783	4103	4422	4740	5057	5373	5688	6002
3.2	6315	6627	6938	7248	7557	7865	8173	8479	8784	9089
3.3	9392	9695	9996	*0297	*0597	*0896	*1194	*1491	*1788	*2083
3.4	1.2 2378	2671	2964	3256	3547	3837	4127	4415	4703	4990
3.5	5276	5562	5846	6130	6413	6695	6976	7257	7536	7815
3.6	8093	8371	8647	8923	9198	9473	9746	*0019	*0291	*0563
3.7	1.3 0833	1103	1372	1641	1909	2176	2442	2708	2972	3237
3.8	3500	3763	4025	4286	4547	4807	5067	5325	5584	5841
3.9	6098	6354	6609	6864	7118	7372	7624	7877	8128	8379
4.0	8629	8879	9128	9377	9624	9872	*0118	*0364	*0610	*0854
4.1	1.4 1099	1342	1585	1828	2070	2311	2552	2792	3031	3270
4.2	3508	3746	3984	4220	4456	4692	4927	5161	5395	5629
4.3	5862	6094	6326	6557	6787	7018	7247	7476	7705	7933
4.4	8160	8387	8614	8840	9065	9290	9515	9739	9962	*0185
4.5	1.5 0408	0630	0851	1072	1293	1513	1732	1951	2170	2388
4.6	2606	2823	3039	3256	3471	3687	3902	4116	4330	4543
4.7	4756	4969	5181	5393	5604	5814	6025	6235	6444	6653
4.8	6862	7070	7277	7485	7691	7898	8104	8309	8515	8719
4.9	8924	9127	9331	9534	9737	9939	*0141	*0342	*0543	*0744
5.0	1.6 0944	1144	1343	1542	1741	1939	2137	2334	2531	2728
5.1	2924	3120	3315	3511	3705	3900	4094	4287	4481	4673
5.2	4866	5058	5250	5441	5632	5823	6013	6203	6393	6582
5.3	6771	6959	7147	7335	7523	7710	7896	8083	8269	8455
5.4	8640	8825	9010	9194	9378	9562	9745	9928	*0111	*0293
5.5	1.7 0475	0656	0838	1019	1199	1380	1560	1740	1919	2098
5.6	2277	2455	2633	2811	2988	3166	3342	3519	3695	3871
5.7	4047	4222	4397	4572	4746	4920	5094	5267	5440	5613
5.8	5786	5958	6130	6302	6473	6644	6815	6985	7156	7326
5.9	7495	7665	7834	8002	8171	8339	8507	8675	8842	9009
N	0	1	2	3	4	5	6	7	8	9

Table 6.13 (Cont'd)
NATURAL (NAPIERIAN) LOGARITHMS

N	0	1	2	3	4	5	6	7	8	9
6.0	1.7 9176	9342	9509	9675	9840	*0006	*0171	*0336	*0500	*0665
6.1	1.8 0829	0993	1156	1319	1482	1645	1808	1970	2132	2294
6.2	2455	2616	2777	2938	3098	3258	3418	3578	3737	3896
6.3	4055	4214	4372	4530	4688	4845	5003	5160	5317	5473
6.4	5630	5786	5942	6097	6253	6408	6563	6718	6872	7026
6.5	7180	7334	7487	7641	7794	7947	8099	8251	8403	8555
6.6	8707	8858	9010	9160	9311	9462	9612	9762	9912	*0061
6.7	1.9 0211	0360	0509	0658	0806	0954	1102	1250	1398	1545
6.8	1692	1839	1986	2132	2279	2425	2571	2716	2862	3007
6.9	3152	3297	3442	3586	3730	3874	4018	4162	4305	4448
7.0	4591	4734	4876	5019	5161	5303	5445	5586	5727	5869
7.1	6009	6150	6291	6431	6571	6711	6851	6991	7130	7269
7.2	7408	7547	7685	7824	7962	8100	8238	8376	8513	8650
7.3	8787	8924	9061	9198	9334	9470	9606	9742	9877	*0013
7.4	2.0 0148	0283	0418	0553	0687	0821	0956	1089	1223	1357
7.5	1490	1624	1757	1890	2022	2155	2287	2419	2551	2683
7.6	2815	2946	3078	3209	3340	3471	3601	3732	3862	3992
7.7	4122	4252	4381	4511	4640	4769	4898	5027	5156	5284
7.8	5412	5540	5668	5796	5924	6051	6179	6306	6433	6560
7.9	6686	6813	6939	7065	7191	7317	7443	7568	7694	7819
8.0	7944	8069	8194	8318	8443	8567	8691	8815	8939	9063
8.1	9186	9310	9433	9556	9679	9802	9924	*0047	*0169	*0291
8.2	2.1 0413	0535	0657	0779	0900	1021	1142	1263	1384	1505
8.3	1626	1746	1866	1986	2106	2226	2346	2465	2585	2704
8.4	2823	2942	3061	3180	3298	3417	3535	3653	3771	3889
8.5	4007	4124	4242	4359	4476	4593	4710	4827	4943	5060
8.6	5176	5292	5409	5524	5640	5756	5871	5987	6102	6217
8.7	6332	6447	6562	6677	6791	6905	7020	7134	7248	7361
8.8	7475	7589	7702	7816	7929	8042	8155	8267	8380	8493
8.9	8605	8717	8830	8942	9054	9165	9277	9389	9500	9611
9.0	9722	9834	9944	*0055	*0166	*0276	*0387	*0497	*0607	*0717
9.1	2.2 0827	0937	1047	1157	1266	1375	1485	1594	1703	1812
9.2	1920	2029	2138	2246	2354	2462	2570	2678	2786	2894
9.3	3001	3109	3216	3324	3431	3538	3645	3751	3858	3965
9.4	4071	4177	4284	4390	4496	4601	4707	4813	4918	5024
9.5	5129	5234	5339	5444	5549	5654	5759	5863	5968	6072
9.6	6176	6280	6384	6488	6592	6696	6799	6903	7006	7109
9.7	7213	7316	7419	7521	7624	7727	7829	7932	8034	8136
9.8	8238	8340	8442	8544	8646	8747	8849	8950	9051	9152
9.9	9253	9354	9455	9556	9657	9757	9858	9958	*0058	*0158
10.0	2.3 0259	0358	0458	0558	0658	0757	0857	0956	1055	1154
N	0	1	2	3	4	5	6	7	8	9

6.14 TABLE OF COMMON LOGARITHMS ($\log_{10} x$)

Table 6.14
COMMON LOGARITHMS OF NUMBERS
100 — 150

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts		
											44	43	42
100	00	000	043	087	130	173	217	260	303	346	389		
101		432	475	518	561	604	647	689	732	775	817		
102		860	903	945	988	*030	*072	*115	*157	*199	*242		
103	01	284	326	368	410	452	494	536	578	620	662		
104		703	745	787	828	870	912	953	995	*036	*078		
105	02	119	160	202	243	284	325	366	407	449	490		
106		531	572	612	653	694	735	776	816	857	898		
107		938	979	*019	*060	*100	*141	*181	*222	*262	*302		
108	03	342	383	423	463	503	543	583	623	663	703		
109		743	782	822	862	902	941	981	*021	*060	*100		
110	04	139	179	218	258	297	336	376	415	454	493		
111		532	571	610	650	689	727	766	805	844	883		
112		922	961	999	*038	*077	*115	*154	*192	*231	*269		
113	05	308	346	385	423	461	500	538	576	614	652		
114		690	729	767	805	843	881	918	956	994	*032		
115	06	070	108	145	183	221	258	296	333	371	408		
116		446	483	521	558	595	633	670	707	744	781		
117		819	856	893	930	967	*004	*041	*078	*115	*151		
118	07	188	225	262	298	335	372	408	445	482	518		
119		555	591	628	664	700	737	773	809	846	882		
120		918	954	990	*027	*063	*099	*135	*171	*207	*243		
121	08	279	314	350	386	422	458	493	529	565	600		
122		636	672	707	743	778	814	849	884	920	955		
123		991	*026	*061	*096	*132	*167	*202	*237	*272	*307		
124	09	342	377	412	447	482	517	552	587	621	656		
125		691	726	760	795	830	864	899	934	968	*003		
126	10	037	072	106	140	175	209	243	278	312	346		
127		380	415	449	483	517	551	585	619	653	687		
128		721	755	789	823	857	890	924	958	992	*025		
129	11	059	093	126	160	193	227	261	294	327	361		
130		394	428	461	494	528	561	594	628	661	694		
131		727	760	793	826	860	893	926	959	992	*024		
132	12	057	090	123	156	189	222	254	287	320	352		
133		385	418	450	483	516	548	581	613	646	678		
134		710	743	775	808	840	872	905	937	969	*001		
135	13	033	066	098	130	162	194	226	258	290	322		
136		354	386	418	450	481	513	545	577	609	640		
137		672	704	735	767	799	830	862	893	925	956		
138		988	*019	*051	*082	*114	*145	*176	*208	*239	*270		
139	14	301	333	364	395	426	457	489	520	551	582		
140		613	644	675	706	737	768	799	829	860	891		
141		922	953	983	*014	*045	*076	*106	*137	*168	*198		
142	15	229	259	290	320	351	381	412	442	473	503		
143		534	564	594	625	655	685	715	746	776	806		
144		836	866	897	927	957	987	*017	*047	*077	*107		
145	16	137	167	197	227	256	286	316	346	376	406		
146		435	465	495	524	554	584	613	643	673	702		
147		732	761	791	820	850	879	909	938	967	997		
148	17	026	056	085	114	143	173	202	231	260	289		
149		319	348	377	406	435	464	493	522	551	580		
150		609	638	667	696	725	754	782	811	840	869		
N.		0	1	2	3	4	5	6	7	8	9	Proportional parts	

.00 000 — .17 869

Table 6.14 (Cont'd)
COMMON LOGARITHMS OF NUMBERS
150—200

N.		0	1	2	3	4	5	6	7	8	9	Proportional parts	
150	17	609	638	667	696	725	754	782	811	840	869	29 28	
151		898	926	955	984	*013	*041	*070	*099	*127	*156	1	2.9 2.8
152	18	184	213	241	270	298	327	355	384	412	441	2	5.8 5.6
153		469	498	526	554	583	611	639	667	696	724	3	8.7 8.4
154		752	780	808	837	865	893	921	949	977	*005	4	11.6 11.2
155	19	033	061	089	117	145	173	201	229	257	285	5	14.5 14.0
156		312	340	368	396	424	451	479	507	535	562	6	17.4 16.8
157		590	618	645	673	700	728	756	783	811	838	7	20.3 19.6
158		866	893	921	948	976	*003	*030	*058	*085	*112	8	23.2 22.4
159	20	140	167	194	222	249	276	303	330	358	385	9	26.1 25.2
160		412	439	466	493	520	548	575	602	629	656	27 26	
161		683	710	737	763	790	817	844	871	898	925	1	2.7 2.6
162		952	978	*005	*032	*059	*085	*112	*139	*165	*192	2	5.4 5.2
163	21	219	245	272	299	325	352	378	405	431	458	3	8.1 7.8
164		484	511	537	564	590	617	643	669	696	722	4	10.8 10.4
165		748	775	801	827	854	880	906	932	958	985	5	13.5 13.0
166	22	011	037	063	089	115	141	167	194	220	246	6	16.2 15.6
167		272	298	324	350	376	401	427	453	479	505	7	18.9 18.2
168		531	557	583	608	634	660	686	712	737	763	8	21.6 20.8
169		789	814	840	866	891	917	943	968	994	*019	9	24.3 23.4
170	23	045	070	096	121	147	172	198	223	249	274	25	
171		300	325	350	376	401	426	452	477	502	528	1	2.5
172		553	578	603	629	654	679	704	729	754	779	2	5.0
173		805	830	855	880	905	930	955	980	*005	*030	3	7.5
174	24	055	080	105	130	155	180	204	229	254	279	4	10.0
175		304	329	353	378	403	428	452	477	502	527	5	12.5
176		551	576	601	625	650	674	699	724	748	773	6	15.0
177		797	822	846	871	895	920	944	969	993	*018	7	17.5
178	25	042	066	091	115	139	164	188	212	237	261	8	20.0
179		285	310	334	358	382	406	431	455	479	503	9	22.5
180		527	551	575	600	624	648	672	696	720	744	24 23	
181		768	792	816	840	864	888	912	935	959	983	1	2.4 2.3
182	26	007	031	055	079	102	126	150	174	198	221	2	4.8 4.6
183		245	269	293	316	340	364	387	411	435	458	3	7.2 6.9
184		482	505	529	553	576	600	623	647	670	694	4	9.6 9.2
185		717	741	764	788	811	834	858	881	905	928	5	12.0 11.5
186		951	975	998	*021	*045	*068	*091	*114	*138	*161	6	14.4 13.8
187	27	184	207	231	254	277	300	323	346	370	393	7	16.8 16.1
188		416	439	462	485	508	531	554	577	600	623	8	19.2 18.4
189		646	669	692	715	738	761	784	807	830	852	9	21.6 20.7
190		875	898	921	944	967	989	*012	*035	*058	*081	22 21	
191	28	103	126	149	171	194	217	240	262	285	307	1	2.2 2.1
192		330	353	375	398	421	443	466	488	511	533	2	4.4 4.2
193		556	578	601	623	646	668	691	713	735	758	3	6.6 6.3
194		780	803	825	847	870	892	914	937	959	981	4	8.8 8.4
195	29	003	026	048	070	092	115	137	159	181	203	5	11.0 10.5
196		226	248	270	292	314	336	358	380	403	425	6	13.2 12.6
197		447	469	491	513	535	557	579	601	623	645	7	15.4 14.7
198		667	688	710	732	754	776	798	820	842	863	8	17.6 16.8
199		885	907	929	951	973	994	*016	*038	*060	*081	9	19.8 18.9
200	30	103	125	146	168	190	211	233	255	276	298	Proportional parts	
N.		0	1	2	3	4	5	6	7	8	9		

.17 609 — .30 298

Table 6.14 (Cont'd)
COMMON LOGARITHMS OF NUMBERS
200 — 250

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts		
200	30	103	125	146	168	190	211	233	255	276	298	22	21
201		320	341	363	384	406	428	449	471	492	514	1	2.2
202		535	557	578	600	621	643	664	685	707	728	2	4.4
203		750	771	792	814	835	856	878	899	920	942	3	6.6
204		963	984	*006	*027	*048	*069	*091	*112	*133	*154	4	8.8
205	31	175	197	218	239	260	281	302	323	345	366	5	11.0
206		387	408	429	450	471	492	513	534	555	576	6	13.2
207		597	618	639	660	681	702	723	744	765	785	7	15.4
208		806	827	848	869	890	911	931	952	973	994	8	17.6
209	32	015	035	056	077	098	118	139	160	181	201	9	19.8
210		222	243	263	284	305	325	346	366	387	408	20	
211		428	449	469	490	510	531	552	572	593	613	1	2.0
212		634	654	675	695	715	736	756	777	797	818	2	4.0
213		838	858	879	899	919	940	960	980	*001	*021	3	6.0
214	33	041	062	082	102	122	143	163	183	203	224	4	8.0
215		244	264	284	304	325	345	365	385	405	425	5	10.0
216		445	465	486	506	526	546	566	586	606	626	6	12.0
217		646	666	686	706	726	746	766	786	806	826	7	14.0
218		846	866	885	905	925	945	965	985	*005	*025	8	16.0
219	34	044	064	084	104	124	143	163	183	203	223	9	18.0
220		242	262	282	301	321	341	361	380	400	420	19	
221		439	459	479	498	518	537	557	577	596	616	1	1.9
222		635	655	674	694	713	733	753	772	792	811	2	3.8
223		830	850	869	889	908	928	947	967	986	*005	3	5.7
224	35	025	044	064	083	102	122	141	160	180	199	4	7.6
225		218	238	257	276	295	315	334	353	372	392	5	9.6
226		411	430	449	468	488	507	526	545	564	583	6	11.4
227		603	622	641	660	679	698	717	736	755	774	7	13.3
228		793	813	832	851	870	889	908	927	946	965	8	15.2
229		984	*003	*021	*040	*059	*078	*097	*116	*135	*154	9	17.1
230	36	173	192	211	229	248	267	286	305	324	342	18	
231		361	380	399	418	436	455	474	493	511	530	1	1.8
232		549	568	586	605	624	642	661	680	698	717	2	3.6
233		736	754	773	791	810	829	847	866	884	903	3	5.4
234		922	940	959	977	996	*014	*033	*051	*070	*088	4	7.2
235	37	107	125	144	162	181	199	218	236	254	273	5	9.0
236		291	310	328	346	365	383	401	420	438	457	6	10.8
237		475	493	511	530	548	566	585	603	621	639	7	12.6
238		658	676	694	712	731	749	767	785	803	822	8	14.4
239		840	858	876	894	912	931	949	967	985	*003	9	16.2
240	38	021	039	057	075	093	112	130	148	166	184	17	
241		202	220	238	256	274	292	310	328	346	364	1	1.7
242		382	399	417	435	453	471	489	507	525	543	2	3.4
243		561	578	596	614	632	650	668	686	703	721	3	5.1
244		739	757	775	792	810	828	846	863	881	899	4	6.8
245		917	934	952	970	987	*005	*023	*041	*058	*076	5	8.5
246	39	094	111	129	146	164	182	199	217	235	252	6	10.2
247		270	287	305	322	340	358	375	393	410	428	7	11.9
248		445	463	480	498	515	533	550	568	585	602	8	13.6
249		620	637	655	672	690	707	724	742	759	777	9	15.3
250		794	811	829	846	863	881	898	915	933	950	Proportional parts	
N.		0	1	2	3	4	5	6	7	8	9		

.30 103 — .39 950

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

250 — 300

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts		
250	39	794	811	829	846	863	881	898	915	933	950	18	
251		967	985	*002	*019	*037	*054	*071	*088	*106	*123	1	1.8
252	40	140	157	175	192	209	226	243	261	278	295	2	3.6
253		312	329	346	364	381	398	415	432	449	466	3	5.4
254		483	500	518	535	552	569	586	603	620	637	4	7.2
255		654	671	688	705	722	739	756	773	790	807	5	9.0
256		824	841	858	875	892	909	926	943	960	976	6	10.8
257		993	*010	*027	*044	*061	*078	*095	*111	*128	*145	7	12.6
258	41	162	179	196	212	229	246	263	280	296	313	8	14.4
259		330	347	363	380	397	414	430	447	464	481	9	16.2
260		497	514	531	547	564	581	597	614	631	647	17	
261		664	681	697	714	731	747	764	780	797	814	1	1.7
262		830	847	863	880	896	913	929	946	963	979	2	3.4
263		996	*012	*029	*045	*062	*078	*095	*111	*127	*144	3	5.1
264	42	160	177	193	210	226	243	259	275	292	308	4	6.8
265		325	341	357	374	390	406	423	439	455	472	5	8.5
266		488	504	521	537	553	570	586	602	619	635	6	10.2
267		651	667	684	700	716	732	749	765	781	797	7	11.9
268		813	830	846	862	878	894	911	927	943	959	8	13.6
269		975	991	*008	*024	*040	*056	*072	*088	*104	*120	9	15.3
270	43	136	152	169	185	201	217	233	249	265	281	16	
271		297	313	329	345	361	377	393	409	425	441	1	1.6
272		457	473	489	505	521	537	553	569	584	600	2	3.2
273		616	632	648	664	680	696	712	727	743	759	3	4.8
274		775	791	807	823	838	854	870	886	902	917	4	6.4
275		933	949	965	981	996	*012	*028	*044	*059	*075	5	8.0
276	44	091	107	122	138	154	170	185	201	217	232	6	9.6
277		243	264	279	295	311	326	342	358	373	389	7	11.2
278		404	420	436	451	467	483	498	514	529	545	8	12.8
279		560	576	592	607	623	638	654	669	685	700	9	14.4
280		716	731	747	762	778	793	809	824	840	855	15	
281		871	886	902	917	932	948	963	979	994	*010	1	1.5
282	45	025	040	056	071	086	102	117	133	148	163	2	3.0
283		179	194	209	225	240	255	271	286	301	317	3	4.5
284		332	347	362	378	393	408	423	439	454	469	4	6.0
285		484	500	515	530	545	561	576	591	606	621	5	7.5
286		637	652	667	682	697	712	728	743	758	773	6	9.0
287		788	803	818	834	849	864	879	894	909	924	7	10.5
288		939	954	969	984	*000	*015	*030	*045	*060	*075	8	12.0
289	46	090	105	120	135	150	165	180	195	210	225	9	13.5
290		240	255	270	285	300	315	330	345	359	374	14	
291		389	404	419	434	449	464	479	494	509	523	1	1.4
292		538	553	568	583	598	613	627	642	657	672	2	2.8
293		687	702	716	731	746	761	776	790	805	820	3	4.2
294		835	850	864	879	894	909	923	938	953	967	4	5.6
295		982	997	*012	*026	*041	*056	*070	*085	*100	*114	5	7.0
296	47	129	144	159	173	188	202	217	232	246	261	6	8.4
297		276	290	305	319	334	349	363	378	392	407	7	9.8
298		422	436	451	465	480	494	509	524	538	553	8	11.2
299		567	582	596	611	625	640	654	669	683	698	9	12.6
300		712	727	741	756	770	784	799	813	828	842	log e = 0.43429	
N.	0	1	2	3	4	5	6	7	8	9	Proportional parts		

.39 794 — .47 842

Table 6.14 (Cont'd)
COMMON LOGARITHMS OF NUMBERS
300 — 350

N.		0	1	2	3	4	5	6	7	8	9	Proportional parts
300	47	712	727	741	756	770	784	799	813	828	842	
301		857	871	885	900	914	929	943	958	972	986	
302	48	001	015	029	044	058	073	087	101	116	130	
303		144	159	173	187	202	216	230	244	259	273	
304		287	302	316	330	344	359	373	387	401	416	
305		430	444	458	473	487	501	515	530	544	558	
306		572	586	601	615	629	643	657	671	686	700	
307		714	728	742	756	770	785	799	813	827	841	
308		855	869	883	897	911	926	940	954	968	982	
309		996	*010	*024	*038	*052	*066	*080	*094	*108	*122	
310	49	136	150	164	178	192	206	220	234	248	262	
311		276	290	304	318	332	346	360	374	388	402	
312		415	429	443	457	471	485	499	513	527	541	
313		554	568	582	596	610	624	638	651	665	679	
314		693	707	721	734	748	762	776	790	803	817	
315		831	845	859	872	886	900	914	927	941	955	
316		969	982	996	*010	*024	*037	*051	*065	*079	*092	
317	50	106	120	133	147	161	174	188	202	215	229	
318		243	256	270	284	297	311	325	338	352	365	
319		379	393	406	420	433	447	461	474	488	501	
320		515	529	542	556	569	583	596	610	623	637	
321		651	664	678	691	705	718	732	745	759	772	
322		786	799	813	826	840	853	866	880	893	907	
323		920	934	947	961	974	987	*001	*014	*028	*041	
324	51	055	068	081	095	108	121	135	148	162	175	
325		188	202	215	228	242	255	268	282	295	308	
326		322	335	348	362	375	388	402	415	428	441	
327		455	468	481	495	508	521	534	548	561	574	
328		587	601	614	627	640	654	667	680	693	706	
329		720	733	746	759	772	786	799	812	825	838	
330		851	865	878	891	904	917	930	943	957	970	
331		983	996	*009	*022	*035	*048	*061	*075	*088	*101	
332	52	114	127	140	153	166	179	192	205	218	231	
333		244	257	270	284	297	310	323	336	349	362	
334		375	388	401	414	427	440	453	466	479	492	
335		504	517	530	543	556	569	582	595	608	621	
336		634	647	660	673	686	699	711	724	737	750	
337		763	776	789	802	815	827	840	853	866	879	
338		892	905	917	930	943	956	969	982	994	*007	
339	53	020	033	046	058	071	084	097	110	122	135	
340		148	161	173	186	199	212	224	237	250	263	
341		275	288	301	314	326	339	352	364	377	390	
342		403	415	428	441	453	466	479	491	504	517	
343		529	542	555	567	580	593	605	618	631	643	
344		656	668	681	694	706	719	732	744	757	769	
345		782	794	807	820	832	845	857	870	882	895	
346		908	920	933	945	958	970	983	995	*008	*020	
347	54	033	045	058	070	083	095	108	120	133	145	
348		158	170	183	195	208	220	233	245	258	270	
349		283	295	307	320	332	345	357	370	382	394	
350		407	419	432	444	456	469	481	494	506	518	
N.		0	1	2	3	4	5	6	7	8	9	log π = 0.49715
												Proportional parts

.47 712 — .54 518

Table 6.14 (Cont'd)
COMMON LOGARITHMS OF NUMBERS

350 — 400

N		0	1	2	3	4	5	6	7	8	9	Proportional parts
350	54	407	419	432	444	456	469	481	494	506	518	
351		531	543	555	568	580	593	605	617	630	642	
352		654	667	679	691	704	716	728	741	753	765	
353		777	790	802	814	827	839	851	864	876	888	
354		900	913	925	937	949	962	974	986	998	*011	13
355	55	023	035	047	060	072	084	096	108	121	133	1
356		145	157	169	182	194	206	218	230	242	255	2
357		267	279	291	303	315	328	340	352	364	376	3
358		388	400	413	425	437	449	461	473	485	497	4
359		509	522	534	546	558	570	582	594	606	618	5
360		630	642	654	666	678	691	703	715	727	739	6
361		751	763	775	787	799	811	823	835	847	859	7
362		871	883	895	907	919	931	943	955	967	979	8
363		991	*003	*015	*027	*038	*050	*062	*074	*086	*098	9
364	56	110	122	134	146	158	170	182	194	205	217	
365		229	241	253	265	277	289	301	312	324	336	12
366		348	360	372	384	396	407	419	431	443	455	1
367		467	478	490	502	514	526	538	549	561	573	2
368		585	597	608	620	632	644	656	667	679	691	3
369		703	714	726	738	750	761	773	785	797	808	4
370		820	832	844	855	867	879	891	902	914	926	5
371		937	949	961	972	984	996	*008	*019	*031	*043	6
372	57	054	066	078	089	101	113	124	136	148	159	7
373		171	183	194	206	217	229	241	252	264	276	8
374		287	299	310	322	334	345	357	368	380	392	9
375		403	415	426	438	449	461	473	484	496	507	
376		519	530	542	553	565	576	588	600	611	623	11
377		634	646	657	669	680	692	703	715	726	738	1
378		749	761	772	784	795	807	818	830	841	852	2
379		864	875	887	898	910	921	933	944	955	967	3
380		978	990	*001	*013	*024	*035	*047	*058	*070	*081	4
381	58	092	104	115	127	138	149	161	172	184	195	5
382		206	218	229	240	252	263	274	286	297	309	6
383		320	331	343	354	365	377	388	399	410	422	7
384		433	444	456	467	478	490	501	512	524	535	8
385		546	557	569	580	591	602	614	625	636	647	9
386		659	670	681	692	704	715	726	737	749	760	
387		771	782	794	805	816	827	838	850	861	872	
388		883	894	906	917	928	939	950	961	973	984	
389		995	*006	*017	*028	*040	*051	*062	*073	*084	*095	
390	59	106	118	129	140	151	162	173	184	195	207	10
391		218	229	240	251	262	273	284	295	306	318	1
392		329	340	351	362	373	384	395	406	417	428	2
393		439	450	461	472	483	494	506	517	528	539	3
394		550	561	572	583	594	605	616	627	638	649	4
395		660	671	682	693	704	715	726	737	748	759	5
396		770	780	791	802	813	824	835	846	857	868	6
397		879	890	901	912	923	934	945	956	966	977	7
398		988	999	*010	*021	*032	*043	*054	*065	*076	*086	8
399	60	097	108	119	130	141	152	163	173	184	195	9
400		206	217	228	239	249	260	271	282	293	304	
N		0	1	2	3	4	5	6	7	8	9	Proportional parts

.54 407 — .60 304

Table 6.14 (Cont'd)
COMMON LOGARITHMS OF NUMBERS
400 — 450

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts
400	206	217	228	239	249	260	271	282	293	304	
401	314	325	336	347	358	369	379	390	401	412	
402	423	433	444	455	466	477	487	498	509	520	
403	531	541	552	563	574	584	595	606	617	627	
404	638	649	660	670	681	692	703	713	724	735	
405	746	756	767	778	788	799	810	821	831	842	
406	853	863	874	885	895	906	917	927	938	949	
407	959	970	981	991	*002	*013	*023	*034	*045	*055	11
408	066	077	087	098	109	119	130	140	151	162	1 1.1
409	172	183	194	204	215	225	236	247	257	268	2 2.2
410	278	289	300	310	321	331	342	352	363	374	3 3.3
411	384	395	405	416	426	437	448	458	469	479	4 4.4
412	490	500	511	521	532	542	553	563	574	584	5 5.5
413	595	606	616	627	637	648	658	669	679	690	6 6.6
414	700	711	721	731	742	752	763	773	784	794	7 7.7
415	805	815	826	836	847	857	868	878	888	899	8 8.8
416	909	920	930	941	951	962	972	982	993	*003	9 9.9
417	014	024	034	045	055	066	076	086	097	107	
418	118	128	138	149	159	170	180	190	201	211	
419	221	232	242	252	263	273	284	294	304	315	
420	325	335	346	356	366	377	387	397	408	418	10
421	428	439	449	459	469	480	490	500	511	521	1 1.0
422	531	542	552	562	572	583	593	603	613	624	2 2.0
423	634	644	655	665	675	685	696	706	716	726	3 3.0
424	737	747	757	767	778	788	798	808	818	829	4 4.0
425	839	849	859	870	880	890	900	910	921	931	5 5.0
426	941	951	961	972	982	992	*002	*012	*022	*033	6 6.0
427	043	053	063	073	083	094	104	114	124	134	7 7.0
428	144	155	165	175	185	195	205	215	225	236	8 8.0
429	246	256	266	276	286	296	306	317	327	337	9 9.0
430	347	357	367	377	387	397	407	417	428	438	
431	448	458	468	478	488	498	508	518	528	538	
432	548	558	568	579	589	599	609	619	629	639	
433	649	659	669	679	689	699	709	719	729	739	
434	749	759	769	779	789	799	809	819	829	839	
435	849	859	869	879	889	899	909	919	929	939	9
436	949	959	969	979	988	998	*008	*018	*028	*038	1 0.9
437	048	058	068	078	088	098	108	118	128	137	2 1.8
438	147	157	167	177	187	197	207	217	227	237	3 2.7
439	246	256	266	276	286	296	306	316	326	335	4 3.6
440	345	355	365	375	385	395	404	414	424	434	5 4.5
441	444	454	464	473	483	493	503	513	523	532	6 5.4
442	542	552	562	572	582	591	601	611	621	631	7 6.3
443	640	650	660	670	680	689	699	709	719	729	8 7.2
444	738	748	758	768	777	787	797	807	816	826	9 8.1
445	836	846	856	865	875	885	895	904	914	924	
446	933	943	953	963	972	982	992	*002	*011	*021	
447	031	040	050	060	070	079	089	099	108	118	
448	128	137	147	157	167	176	186	196	205	215	
449	225	234	244	254	263	273	283	292	302	312	
450	321	331	341	350	360	369	379	389	398	408	
N.	0	1	2	3	4	5	6	7	8	9	Proportional parts

.60 206 — .65 408

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

450 500

N	0	1	2	3	4	5	6	7	8	9	Proportional parts
450	321	331	341	350	360	369	379	389	398	408	
451	418	427	437	447	456	466	475	485	495	504	
452	514	523	533	543	552	562	571	581	591	600	
453	610	619	629	639	648	658	667	677	686	696	
454	706	715	725	734	744	753	763	772	782	792	
455	801	811	820	830	839	849	858	868	877	887	
456	896	906	916	925	935	944	954	963	973	982	
457	992	*001	*011	*020	*030	*039	*049	*058	*068	*077	10
458	66	087	096	106	115	124	134	143	153	162	1
459	181	191	200	210	219	229	238	247	257	266	2
460	276	285	295	304	314	323	332	342	351	361	3
461	370	380	389	398	408	417	427	436	445	455	4
462	464	474	483	492	502	511	521	530	539	549	5
463	558	567	577	586	596	605	614	624	633	642	6
464	652	661	671	680	689	699	708	717	727	736	7
465	745	755	764	773	783	792	801	811	820	829	8
466	839	848	857	867	876	885	894	904	913	922	9
467	932	941	950	960	969	978	987	997	*006	*015	
468	67	025	034	043	052	062	071	080	089	099	1
469	117	127	136	145	154	164	173	182	191	201	2
470	210	219	228	237	247	256	265	274	284	293	3
471	302	311	321	330	339	348	357	367	376	385	4
472	394	403	413	422	431	440	449	459	468	477	5
473	486	495	504	514	523	532	541	550	560	569	6
474	578	587	596	605	614	624	633	642	651	660	7
475	669	679	688	697	706	715	724	733	742	752	8
476	761	770	779	788	797	806	815	825	834	843	9
477	852	861	870	879	888	897	906	916	925	934	
478	943	952	961	970	979	988	997	*006	*015	*024	
479	68	034	043	052	061	070	079	088	097	106	1
480	124	133	142	151	160	169	178	187	196	205	2
481	215	224	233	242	251	260	269	278	287	296	3
482	305	314	323	332	341	350	359	368	377	386	4
483	395	404	413	422	431	440	449	458	467	476	5
484	485	494	502	511	520	529	538	547	556	565	6
485	574	583	592	601	610	619	728	637	646	655	7
486	664	673	681	690	699	708	717	726	735	744	8
487	753	762	771	780	789	797	806	815	824	833	9
488	842	851	860	869	878	886	895	904	913	922	
489	931	940	949	958	966	975	984	993	*002	*011	
490	69	020	028	037	046	055	064	073	082	090	1
491	108	117	126	135	144	152	161	170	179	188	2
492	197	205	214	223	232	241	249	258	267	276	3
493	285	294	302	311	320	329	338	346	355	364	4
494	373	381	390	399	408	417	425	434	443	452	5
495	461	469	478	487	496	504	513	522	531	539	6
496	548	557	566	574	583	592	601	609	618	627	7
497	636	644	653	662	671	679	688	697	705	714	8
498	723	732	740	749	758	767	775	784	793	801	9
499	810	819	827	836	845	854	862	871	880	888	
500	897	906	914	923	932	940	949	958	966	975	
N	0	1	2	3	4	5	6	7	8	9	Proportional parts

.65 321 .69 975

Table 6.14 (Cont'd)
COMMON LOGARITHMS OF NUMBERS
500 — 550

N.		0	1	2	3	4	5	6	7	8	9	Proportional parts
500	69	897	906	914	923	932	940	949	958	966	975	
501		984	992	*001	*010	*018	*027	*036	*044	*053	*062	
502	70	070	079	088	096	105	114	122	131	140	148	
503		157	165	174	183	191	200	209	217	226	234	
504		243	252	260	269	278	286	295	303	312	321	
505		329	338	346	355	364	372	381	389	398	406	
506		415	424	432	441	449	458	467	475	484	492	
507		501	509	518	526	535	544	552	561	569	578	
508		586	595	603	612	621	629	638	646	655	663	
509		672	680	689	697	706	714	723	731	740	749	
510		757	766	774	783	791	800	808	817	825	834	
511		842	851	859	868	876	885	893	902	910	919	
512		927	935	944	952	961	969	978	986	995	*003	
513	71	012	020	029	037	046	054	063	071	079	088	
514		096	105	113	122	130	139	147	155	164	172	
515		181	189	198	206	214	223	231	240	248	257	
516		265	273	282	290	299	307	315	324	332	341	
517		349	357	366	374	383	391	399	408	416	425	
518		433	441	450	458	466	475	483	492	500	508	
519		517	525	533	542	550	559	567	575	584	592	
520		600	609	617	625	634	642	650	659	667	675	
521		684	692	700	709	717	725	734	742	750	759	
522		767	775	784	792	800	809	817	825	834	842	
523		850	858	867	875	883	892	900	908	917	925	
524		933	941	950	958	966	975	983	991	999	*008	
525	72	016	024	032	041	049	057	066	074	082	090	
526		099	107	115	123	132	140	148	156	165	173	
527		181	189	198	206	214	222	230	239	247	255	
528		263	272	280	288	296	304	313	321	329	337	
529		346	354	362	370	378	387	395	403	411	419	
530		428	436	444	452	460	469	477	485	493	501	
531		509	518	526	534	542	550	558	567	575	583	
532		591	599	607	616	624	632	640	648	656	665	
533		673	681	689	697	705	713	722	730	738	746	
534		754	762	770	779	787	795	803	811	819	827	
535		835	843	852	860	868	876	884	892	900	908	
536		916	925	933	941	949	957	965	973	981	989	
537		997	*006	*014	*022	*030	*038	*046	*054	*062	*070	
538	73	078	086	094	102	111	119	127	135	143	151	
539		159	167	175	183	191	199	207	215	223	231	
540		239	247	255	263	272	280	288	296	304	312	
541		320	328	336	344	352	360	368	376	384	392	
542		400	408	416	424	432	440	448	456	464	472	
543		480	488	496	504	512	520	528	536	544	552	
544		560	568	576	584	592	600	608	616	624	632	
545		640	648	656	664	672	679	687	695	703	711	
546		719	727	735	743	751	759	767	775	783	791	
547		799	807	815	823	830	838	846	854	862	870	
548		878	886	894	902	910	918	926	933	941	949	
549		957	965	973	981	989	997	*005	*013	*020	*028	
550	74	036	044	052	060	068	076	084	092	099	107	
N.		0	1	2	3	4	5	6	7	8	9	Proportional parts

.69 897 — .74 107

Table 6.14 (Cont'd)
COMMON LOGARITHMS OF NUMBERS
550 — 600

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts
550	74	036	044	052	060	068	076	084	092	099	107
551		115	123	131	139	147	155	162	170	178	186
552		194	202	210	218	225	233	241	249	257	265
553		273	280	288	296	304	312	320	327	335	343
554		351	359	367	374	382	390	398	406	414	421
555		429	437	445	453	461	468	476	484	492	500
556		507	515	523	531	539	547	554	562	570	578
557		586	593	601	609	617	624	632	640	648	656
558		663	671	679	687	695	702	710	718	726	733
559		741	749	757	764	772	780	788	796	803	811
560		819	827	834	842	850	858	865	873	881	889
561		896	904	912	920	927	935	943	950	958	966
562		974	981	989	997	*005	*012	*020	*028	*035	*043
563	75	051	059	066	074	082	089	097	105	113	120
564		128	136	143	151	159	166	174	182	189	197
565		205	213	220	228	236	243	251	259	266	274
566		282	289	297	305	312	320	328	335	343	351
567		358	366	374	381	389	397	404	412	420	427
568		435	442	450	458	465	473	481	488	496	504
569		511	519	526	534	542	549	557	565	572	580
570		587	595	603	610	618	626	633	641	648	656
571		664	671	679	686	694	702	709	717	724	732
572		740	747	755	762	770	778	785	793	800	808
573		815	823	831	838	846	853	861	868	876	884
574		891	899	906	914	921	929	937	944	952	959
575		967	974	982	989	997	*005	*012	*020	*027	*035
576	76	042	050	057	065	072	080	087	095	103	110
577		118	125	133	140	148	155	163	170	178	185
578		193	200	208	215	223	230	238	245	253	260
579		268	275	283	290	298	305	313	320	328	335
580		343	350	358	365	373	380	388	395	403	410
581		418	425	433	440	448	455	462	470	477	485
582		492	500	507	515	522	530	537	545	552	559
583		567	574	582	589	597	604	612	619	626	634
584		641	649	656	664	671	678	686	693	701	708
585		716	723	730	738	745	753	760	768	775	782
586		790	797	805	812	819	827	834	842	849	856
587		864	871	879	886	893	901	908	916	923	930
588		938	945	953	960	967	975	982	989	997	*004
589	77	012	019	026	034	041	048	056	063	070	078
590		085	093	100	107	115	122	129	137	144	151
591		159	166	173	181	188	195	203	210	217	225
592		232	240	247	254	262	269	276	283	291	298
593		305	313	320	327	335	342	349	357	364	371
594		379	386	393	401	408	415	422	430	437	444
595		452	459	466	474	481	488	495	503	510	517
596		525	532	539	546	554	561	568	576	583	590
597		597	605	612	619	627	634	641	648	656	663
598		670	677	685	692	699	706	714	721	728	735
599		743	750	757	764	772	779	786	793	801	808
600		815	822	830	837	844	851	859	866	873	880
N.		0	1	2	3	4	5	6	7	8	9
											Proportional parts

8
1 0.8
2 1.6
3 2.4
4 3.2
5 4.0
6 4.8
7 5.6
8 6.4
9 7.2

7
1 0.7
2 1.4
3 2.1
4 2.8
5 3.5
6 4.2
7 4.9
8 5.6
9 6.3

.74 036 — .77 880

Table 6.14 (Cont'd)
COMMON LOGARITHMS OF NUMBERS
600 — 650

N.		0	1	2	3	4	5	6	7	8	9	Proportional parts	
600	77	815	822	830	837	844	851	859	866	873	880	<div>10.8 21.6 32.4 43.2 54.0 64.8 75.6 86.4 97.2</div>	
601		887	895	902	909	916	924	931	938	945	952		
602		960	967	974	981	988	996	*003	*010	*017	*025		
603	78	032	039	046	053	061	068	075	082	089	097		
604		104	111	118	125	132	140	147	154	161	168		
605		176	183	190	197	204	211	219	226	233	240		
606		247	254	262	269	276	283	290	297	305	312		
607		319	326	333	340	347	355	362	369	376	383		
608		390	398	405	412	419	426	433	440	447	455		
609		462	469	476	483	490	497	504	512	519	526		
610		533	540	547	554	561	569	576	583	590	597		
611		604	611	618	625	633	640	647	654	661	668		
612		675	682	689	696	704	711	718	725	732	739		
613		746	753	760	767	774	781	789	796	803	810		
614		817	824	831	838	845	852	859	866	873	880		
615		888	895	902	909	916	923	930	937	944	951		
616		958	965	972	979	986	993	*000	*007	*014	*021		
617	79	029	036	043	050	057	064	071	078	085	092		
618		099	106	113	120	127	134	141	148	155	162		
619		169	176	183	190	197	204	211	218	225	232		
620		239	246	253	260	267	274	281	288	295	302		
621		309	316	323	330	337	344	351	358	365	372		
622		379	386	393	400	407	414	421	428	435	442		
623		449	456	463	470	477	484	491	498	505	511		
624		518	525	532	539	546	553	560	567	574	581		
625		588	595	602	609	616	623	630	637	644	650		
626		657	664	671	678	685	692	699	706	713	720		
627		727	734	741	748	754	761	768	775	782	789		
628		796	803	810	817	824	831	837	844	851	858		
629		865	872	879	886	893	900	906	913	920	927		
630		934	941	948	955	962	969	975	982	989	996		
631	80	003	010	017	024	030	037	044	051	058	065		
632		072	079	085	092	099	106	113	120	127	134		
633		140	147	154	161	168	175	182	188	195	202		
634		209	216	223	229	236	243	250	257	264	271		
635		277	284	291	298	305	312	318	325	332	339		
636		346	353	359	366	373	380	387	393	400	407		
637		414	421	428	434	441	448	455	462	468	475		
638		482	489	496	502	509	516	523	530	536	543		
639		550	557	564	570	577	584	591	598	604	611		
640		618	625	632	638	645	652	659	665	672	679		
641		686	693	699	706	713	720	726	733	740	747		
642		754	760	767	774	781	787	794	801	808	814		
643		821	828	835	841	848	855	862	868	875	882		
644		889	895	902	909	916	922	929	936	943	949		
645		956	963	969	976	983	990	996	*003	*010	*017		
646	81	023	030	037	043	050	057	064	070	077	084		
647		090	097	104	111	117	124	131	137	144	151		
648		158	164	171	178	184	191	198	204	211	218		
649		224	231	238	245	251	258	265	271	278	285		
650		291	298	305	311	318	325	331	338	345	351		
N.		0	1	2	3	4	5	6	7	8	9	Proportional parts	

.77 815 — .81 351

Table 6.14 (Cont'd).
COMMON LOGARITHMS OF NUMBERS
650 — 700

N.		0	1	2	3	4	5	6	7	8	9	Proportional parts
650	81	291	298	305	311	318	325	331	338	345	351	
651		358	365	371	378	385	391	398	405	411	418	
652		425	431	438	445	451	458	465	471	478	485	
653		491	498	505	511	518	525	531	538	544	551	
654		558	564	571	578	584	591	598	604	611	617	
655		624	631	637	644	651	657	664	671	677	684	
656		690	697	704	710	717	723	730	737	743	750	
657		757	763	770	776	783	790	796	803	809	816	
658		823	829	836	842	849	856	862	869	875	882	
659		889	895	902	908	915	921	928	935	941	948	
660		954	961	968	974	981	987	994	*000	*007	*014	
661	82	020	027	033	040	046	053	060	066	073	079	7
662		086	092	099	105	112	119	125	132	138	145	1
663		151	158	164	171	178	184	191	197	204	210	2
664		217	223	230	236	243	249	256	263	269	276	3
665		282	289	295	302	308	315	321	328	334	341	4
666		347	354	360	367	373	380	387	393	400	406	5
667		413	419	426	432	439	445	452	458	465	471	6
668		478	484	491	497	504	510	517	523	530	536	7
669		543	549	556	562	569	575	582	588	595	601	8
670		607	614	620	627	633	640	646	653	659	666	9
671		672	679	685	692	698	705	711	718	724	730	
672		737	743	750	756	763	769	776	782	789	795	
673		802	808	814	821	827	834	840	847	853	860	
674		866	872	879	885	892	898	905	911	918	924	
675		930	937	943	950	956	963	969	975	982	988	
676		995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
677	83	059	065	072	078	085	091	097	104	110	117	
678		123	129	136	142	149	155	161	168	174	181	
679		187	193	200	206	213	219	225	232	238	245	
680		251	257	264	270	276	283	289	296	302	308	
681		315	321	327	334	340	347	353	359	366	372	
682		378	385	391	398	404	410	417	423	429	436	
683		442	448	455	461	467	474	480	487	493	499	
684		506	512	518	525	531	537	544	550	556	563	
685		569	575	582	588	594	601	607	613	620	626	
686		632	639	645	651	658	664	670	677	683	689	
687		696	702	708	715	721	727	734	740	746	753	
688		759	765	771	778	784	790	797	803	809	816	
689		822	828	835	841	847	853	860	866	872	879	
690		885	891	897	904	910	916	923	929	935	942	
691		948	954	960	967	973	979	985	992	998	*004	
692	84	011	017	023	029	036	042	048	055	061	067	
693		073	080	086	092	098	105	111	117	123	130	
694		136	142	148	155	161	167	173	180	186	192	
695		198	205	211	217	223	230	236	242	248	255	
696		261	267	273	280	286	292	298	305	311	317	
697		323	330	336	342	348	354	361	367	373	379	
698		386	392	398	404	410	417	423	429	435	442	
699		448	454	460	466	473	479	485	491	497	504	
700		510	516	522	528	535	541	547	553	559	566	
N.		0	1	2	3	4	5	6	7	8	9	Proportional parts

.81 291 — .84 566

Table 6.14 (Cont'd)
COMMON LOGARITHMS OF NUMBERS
700 — 750

N.		0	1	2	3	4	5	6	7	8	9	Proportional parts
700	84	510	516	522	528	535	541	547	553	559	566	
701		572	578	584	590	597	603	609	615	621	628	
702		634	640	646	652	658	665	671	677	683	689	
703		696	702	708	714	720	726	733	739	745	751	
704		757	763	770	776	782	788	794	800	807	813	
705		819	825	831	837	844	850	856	862	868	874	
706		880	887	893	899	905	911	917	924	930	936	
707		942	948	954	960	967	973	979	985	991	997	
708	85	003	009	016	022	028	034	040	046	052	058	
709		065	071	077	083	089	095	101	107	114	120	
710		126	132	138	144	150	156	163	169	175	181	
711		187	193	199	205	211	217	224	230	236	242	
712		248	254	260	266	272	278	285	291	297	303	
713		309	315	321	327	333	339	345	352	358	364	
714		370	376	382	388	394	400	406	412	418	425	
715		431	437	443	449	455	461	467	473	479	485	
716		491	497	503	509	516	522	528	534	540	546	
717		552	558	564	570	576	582	588	594	600	606	
718		612	618	625	631	637	643	649	655	661	667	
719		673	679	685	691	697	703	709	715	721	727	
720		733	739	745	751	757	763	769	775	781	788	
721		794	800	806	812	818	824	830	836	842	848	
722		854	860	866	872	878	884	890	896	902	908	
723		914	920	926	932	938	944	950	956	962	968	
724		974	980	986	992	998	*004	*010	*016	*022	*028	
725	86	034	040	046	052	058	064	070	076	082	088	
726		094	100	106	112	118	124	130	136	141	147	
727		153	159	165	171	177	183	189	195	201	207	
728		213	219	225	231	237	243	249	255	261	267	
729		273	279	285	291	297	303	308	314	320	326	
730		332	338	344	350	356	362	368	374	380	386	
731		392	398	404	410	415	421	427	433	439	445	
732		451	457	463	469	475	481	487	493	499	504	
733		510	516	522	528	534	540	546	552	558	564	
734		570	576	581	587	593	599	605	611	617	623	
735		629	635	641	646	652	658	664	670	676	682	
736		688	694	700	705	711	717	723	729	735	741	
737		747	753	759	764	770	776	782	788	794	800	
738		806	812	817	823	829	835	841	847	853	859	
739		864	870	876	882	888	894	900	906	911	917	
740		923	929	935	941	947	953	958	964	970	976	
741		982	988	994	999	*005	*011	*017	*023	*029	*035	
742	87	040	046	052	058	064	070	075	081	087	093	
743		099	105	111	116	122	128	134	140	146	151	
744		157	163	169	175	181	186	192	198	204	210	
745		216	221	227	233	239	245	251	256	262	268	
746		274	280	286	291	297	303	309	315	320	326	
747		332	338	344	349	355	361	367	373	379	384	
748		390	396	402	408	413	419	425	431	437	442	
749		448	454	460	466	471	477	483	489	495	500	
750		506	512	518	523	529	535	541	547	552	558	
N.		0	1	2	3	4	5	6	7	8	9	Proportional parts

.84 510 — .87 558

Table 6.14 (Cont'd)
COMMON LOGARITHMS OF NUMBERS
750 — 800

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts	
750	87	506	512	518	523	529	535	541	547	552	558	
751		564	570	576	581	587	593	599	604	610	616	
752		622	628	633	639	645	651	656	662	668	674	
753		679	685	691	697	703	708	714	720	726	731	
754		737	743	749	754	760	766	772	777	783	789	
755		795	800	806	812	818	823	829	835	841	846	
756		852	858	864	869	875	881	887	892	898	904	
757		910	915	921	927	933	938	944	950	955	961	
758		967	973	978	984	990	996	*001	*007	*013	*018	
759	88	024	030	036	041	047	053	058	064	070	076	
760		081	087	093	098	104	110	116	121	127	133	
761		138	144	150	156	161	167	173	178	184	190	
762		195	201	207	213	218	224	230	235	241	247	
763		252	258	264	270	275	281	287	292	298	304	
764		309	315	321	326	332	338	343	349	355	360	
765		366	372	377	383	389	395	400	406	412	417	
766		423	429	434	440	446	451	457	463	468	474	
767		480	485	491	497	502	508	513	519	525	530	
768		536	542	547	553	559	564	570	576	581	587	
769		593	598	604	610	615	621	627	632	638	643	
770		649	655	660	666	672	677	683	689	694	700	
771		705	711	717	722	728	734	739	745	750	756	
772		762	767	773	779	784	790	795	801	807	812	
773		818	824	829	835	840	846	852	857	863	868	
774		874	880	885	891	897	902	908	913	919	925	
775		930	936	941	947	953	958	964	969	975	981	
776		986	992	997	*003	*009	*014	*020	*025	*031	*037	
777	89	042	048	053	059	064	070	076	081	087	092	
778		098	104	109	115	120	126	131	137	143	148	
779		154	159	165	170	176	182	187	193	198	204	
780		209	215	221	226	232	237	243	248	254	260	
781		265	271	276	282	287	293	298	304	310	315	
782		321	326	332	337	343	348	354	360	365	371	
783		376	382	387	393	398	404	409	415	421	426	
784		432	437	443	448	454	459	465	470	476	481	
785		487	492	498	504	509	515	520	526	531	537	
786		542	548	553	559	564	570	575	581	586	592	
787		597	603	609	614	620	625	631	636	642	647	
788		653	658	664	669	675	680	686	691	697	702	
789		708	713	719	724	730	735	741	746	752	757	
790		763	768	774	779	785	790	796	801	807	812	
791		818	823	829	834	840	845	851	856	862	867	
792		873	878	883	889	894	900	905	911	916	922	
793		927	933	938	944	949	955	960	966	971	977	
794		982	988	993	998	*004	*009	*015	*020	*026	*031	
795	90	037	042	048	053	059	064	069	075	080	086	
796		091	097	102	108	113	119	124	129	135	140	
797		146	151	157	162	168	173	179	184	189	195	
798		200	206	211	217	222	227	233	238	244	249	
799		255	260	266	271	276	282	287	293	298	304	
800		309	314	320	325	331	336	342	347	352	358	
N.	0	1	2	3	4	5	6	7	8	9	Proportional parts	

.87 506 — .90 358

Table 6.14 (Cont'd)
COMMON LOGARITHMS OF NUMBERS
800 — 850

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts	
800	90	309	314	320	325	331	336	342	347	352	358	
801		363	369	374	380	385	390	396	401	407	412	
802		417	423	428	434	439	445	450	455	461	466	
803		472	477	482	488	493	499	504	509	515	520	
804		526	531	536	542	547	553	558	563	569	574	
805		580	585	590	596	601	607	612	617	623	628	
806		634	639	644	650	655	660	666	671	677	682	
807		687	693	698	703	709	714	720	725	730	736	
808		741	747	752	757	763	768	773	779	784	789	
809		795	800	806	811	816	822	827	832	838	843	
810		849	854	859	865	870	875	881	886	891	897	6
811		902	907	913	918	924	929	934	940	945	950	1 0.6
812		956	961	966	972	977	982	988	993	998	*004	2 1.2
813	91	009	014	020	025	030	036	041	046	052	057	3 1.8
814		062	068	073	078	084	089	094	100	105	110	4 2.4
815		116	121	126	132	137	142	148	153	158	164	5 3.0
816		169	174	180	185	190	196	201	206	212	217	6 3.6
817		222	228	233	238	243	249	254	259	265	270	7 4.2
818		275	281	286	291	297	302	307	312	318	323	8 4.8
819		328	334	339	344	350	355	360	365	371	376	9 5.4
820		381	387	392	397	403	408	413	418	424	429	
821		434	440	445	450	455	461	466	471	477	482	
822		487	492	498	503	508	514	519	524	529	535	
823		540	545	551	556	561	566	572	577	582	587	
824		593	598	603	609	614	619	624	630	635	640	
825		645	651	656	661	666	672	677	682	687	693	
826		698	703	709	714	719	724	730	735	740	745	
827		751	756	761	766	772	777	782	787	793	798	
828		803	808	814	819	824	829	834	840	845	850	
829		855	861	866	871	876	882	887	892	897	903	
830		908	913	918	924	929	934	939	944	950	955	5
831		960	965	971	976	981	986	991	997	*002	*007	1 0.5
832	92	012	018	023	028	033	038	044	049	054	059	2 1.0
833		065	070	075	080	085	091	096	101	106	111	3 1.5
834		117	122	127	132	137	143	148	153	158	163	4 2.0
835		169	174	179	184	189	195	200	205	210	215	5 2.5
836		221	226	231	236	241	247	252	257	262	267	6 3.0
837		273	278	283	288	293	298	304	309	314	319	7 3.5
838		324	330	335	340	345	350	355	361	366	371	8 4.0
839		376	381	387	392	397	402	407	412	418	423	9 4.5
840		428	433	438	443	449	454	459	464	469	474	
841		480	485	490	495	500	505	511	516	521	526	
842		531	536	542	547	552	557	562	567	572	578	
843		583	588	593	598	603	609	614	619	624	629	
844		634	639	645	650	655	660	665	670	675	681	
845		686	691	696	701	706	711	716	722	727	732	
846		737	742	747	752	758	763	768	773	778	783	
847		788	793	799	804	809	814	819	824	829	834	
848		840	845	850	855	860	865	870	875	881	886	
849		891	896	901	906	911	916	921	927	932	937	
850		942	947	952	957	962	967	973	978	983	988	
N.	0	1	2	3	4	5	6	7	8	9	Proportional parts	

.90 309 — .92 988

Table 6.14 (Cont'd)
COMMON LOGARITHMS OF NUMBERS
850 — 900

N	0	1	2	3	4	5	6	7	8	9	Proportional parts	
850	92	942	947	952	957	962	967	973	978	983	988	
851		993	998	*003	*008	*013	*018	*024	*029	*034	*039	
852	93	044	049	054	059	064	069	075	080	085	090	
853		095	100	105	110	115	120	125	131	136	141	
854		146	151	156	161	166	171	176	181	186	192	
855		197	202	207	212	217	222	227	232	237	242	
856		247	252	258	263	268	273	278	283	288	293	
857		298	303	308	313	318	323	328	334	339	344	
858		349	354	359	364	369	374	379	384	389	394	
859		399	404	409	414	420	425	430	435	440	445	
860		450	455	460	465	470	475	480	485	490	495	
861		500	505	510	515	520	526	531	536	541	546	
862		551	556	561	566	571	576	581	586	591	596	
863		601	606	611	616	621	626	631	636	641	646	
864		651	656	661	666	671	676	682	687	692	697	
865		702	707	712	717	722	727	732	737	742	747	
866		752	757	762	767	772	777	782	787	792	797	
867		802	807	812	817	822	827	832	837	842	847	
868		852	857	862	867	872	877	882	887	892	897	
869		902	907	912	917	922	927	932	937	942	947	
870		952	957	962	967	972	977	982	987	992	997	
871	94	002	007	012	017	022	027	032	037	042	047	
872		052	057	062	067	072	077	082	086	091	096	
873		101	106	111	116	121	126	131	136	141	146	
874		151	156	161	166	171	176	181	186	191	196	
875		201	206	211	216	221	226	231	236	240	245	
876		250	255	260	265	270	275	280	285	290	295	
877		300	305	310	315	320	325	330	335	340	345	
878		349	354	359	364	369	374	379	384	389	394	
879		399	404	409	414	419	424	429	433	438	443	
880		448	453	458	463	468	473	478	483	488	493	
881		498	503	507	512	517	522	527	532	537	542	
882		547	552	557	562	567	571	576	581	586	591	
883		596	601	606	611	616	621	626	630	635	640	
884		645	650	655	660	665	670	675	680	685	689	
885		694	699	704	709	714	719	724	729	734	738	
886		743	748	753	758	763	768	773	778	783	787	
887		792	797	802	807	812	817	822	827	832	836	
888		841	846	851	856	861	866	871	876	880	885	
889		890	895	900	905	910	915	919	924	929	934	
890		939	944	949	954	959	963	968	973	978	983	
891		988	993	998	*002	*007	*012	*017	*022	*027	*032	
892	95	036	041	046	051	056	061	066	071	075	080	
893		085	090	095	100	105	109	114	119	124	129	
894		134	139	143	148	153	158	163	168	173	177	
895		182	187	192	197	202	207	211	216	221	226	
896		231	236	240	245	250	255	260	265	270	274	
897		279	284	289	294	299	303	308	313	318	323	
898		328	332	337	342	347	352	357	361	366	371	
899		376	381	386	390	395	400	405	410	415	419	
900		424	429	434	439	444	448	453	458	463	468	
N		0	1	2	3	4	5	6	7	8	9	Proportional parts

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3 1.2
4 1.6
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.92 942 — .95 468

Table 6.14 (Cont'd)
COMMON LOGARITHMS OF NUMBERS
900 — 950

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts	
95	424	429	434	439	444	448	453	458	463	468	<div>5</div> <div>1 0.5</div> <div>2 1.0</div> <div>3 1.5</div> <div>4 2.0</div> <div>5 2.5</div> <div>6 3.0</div> <div>7 3.5</div> <div>8 4.0</div> <div>9 4.5</div>	
901	472	477	482	487	492	497	501	506	511	516		
902	521	525	530	535	540	545	550	554	559	564		
903	569	574	578	583	588	593	598	602	607	612		
904	617	622	626	631	636	641	646	650	655	660		
905	665	670	674	679	684	689	694	698	703	708		
906	713	718	722	727	732	737	742	746	751	756		
907	761	766	770	775	780	785	789	794	799	804		
908	809	813	818	823	828	832	837	842	847	852		
909	856	861	866	871	875	880	885	890	895	899		
910	904	909	914	918	923	928	933	938	942	947		
911	952	957	961	966	971	976	980	985	990	995		
912	999	*004	*009	*014	*019	*023	*028	*033	*038	*042		
913	96	047	052	057	061	066	071	076	080	085	090	
914	095	099	104	109	114	118	123	128	133	137		
915	142	147	152	156	161	166	171	175	180	185		
916	190	194	199	204	209	213	218	223	227	232		
917	237	242	246	251	256	261	265	270	275	280		
918	284	289	294	298	303	308	313	317	322	327		
919	332	336	341	346	350	355	360	365	369	374		
920	379	384	388	393	398	402	407	412	417	421		
921	426	431	435	440	445	450	454	459	464	468		
922	473	478	483	487	492	497	501	506	511	515		
923	520	525	530	534	539	544	548	553	558	562		
924	567	572	577	581	586	591	595	600	605	609		
925	614	619	624	628	633	638	642	647	652	656		
926	661	666	670	675	680	685	689	694	699	703		
927	708	713	717	722	727	731	736	741	745	750		
928	755	759	764	769	774	778	783	788	792	797		
929	802	806	811	816	820	825	830	834	839	844		
930	848	853	858	862	867	872	876	881	886	890		
931	895	900	904	909	914	918	923	928	932	937		
932	942	946	951	956	960	965	970	974	979	984		
933	988	993	997	*002	*007	*011	*016	*021	*025	*030		
934	97	035	039	044	049	053	058	063	067	072	077	
935	081	086	090	095	100	104	109	114	118	123		
936	128	132	137	142	146	151	155	160	165	169		
937	174	179	183	188	192	197	202	206	211	216		
938	220	225	230	234	239	243	248	253	257	262		
939	267	271	276	280	285	290	294	299	304	308		
940	313	317	322	327	331	336	340	345	350	354		
941	359	364	368	373	377	382	387	391	396	400		
942	405	410	414	419	424	428	433	437	442	447		
943	451	456	460	465	470	474	479	483	488	493		
944	497	502	506	511	516	520	525	529	534	539		
945	543	548	552	557	562	566	571	575	580	585		
946	589	594	598	603	607	612	617	621	626	630		
947	635	640	644	649	653	658	663	667	672	676		
948	681	685	690	695	699	704	708	713	717	722		
949	727	731	736	740	745	749	754	759	763	768		
950	772	777	782	786	791	795	800	804	809	813		
N.	0	1	2	3	4	5	6	7	8	9	Proportional parts	

.95 424 — .97 813

Table 6.14 (Cont'd)
COMMON LOGARITHMS OF NUMBERS
950 — 1000

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts
950	772	777	782	786	791	795	800	804	809	813	
951	818	823	827	832	836	841	845	850	855	859	
952	864	868	873	877	882	886	891	896	900	905	
953	909	914	918	923	928	932	937	941	946	950	
954	955	959	964	968	973	978	982	987	991	996	
955	000	005	009	014	019	023	028	032	037	041	
956	046	050	055	059	064	068	073	078	082	087	
957	091	096	100	105	109	114	118	123	127	132	
958	137	141	146	150	155	159	164	168	173	177	
959	182	186	191	195	200	204	209	214	218	223	
960	227	232	236	241	245	250	254	259	263	268	
961	272	277	281	286	290	295	299	304	308	313	
962	318	322	327	331	336	340	345	349	354	358	
963	363	367	372	376	381	385	390	394	399	403	
964	408	412	417	421	426	430	435	439	444	448	
965	453	457	462	466	471	475	480	484	489	493	
966	498	502	507	511	516	520	525	529	534	538	
967	543	547	552	556	561	565	570	574	579	583	
968	588	592	597	601	605	610	614	619	623	628	
969	632	637	641	646	650	655	659	664	668	673	
970	677	682	686	691	695	700	704	709	713	717	
971	722	726	731	735	740	744	749	753	758	762	
972	767	771	776	780	784	789	793	798	802	807	
973	811	816	820	825	829	834	838	843	847	851	
974	856	860	865	869	874	878	883	887	892	896	
975	900	905	909	914	918	923	927	932	936	941	
976	945	949	954	958	963	967	972	976	981	985	
977	989	994	998	*003	*007	*012	*016	*021	*025	*029	
978	034	038	043	047	052	056	061	065	069	074	
979	078	083	087	092	096	100	105	109	114	118	
980	123	127	131	136	140	145	149	154	158	162	
981	167	171	176	180	185	189	193	198	202	207	
982	211	216	220	224	229	233	238	242	247	251	
983	255	260	264	269	273	277	282	286	291	295	
984	300	304	308	313	317	322	326	330	335	339	
985	344	348	352	357	361	366	370	374	379	383	
986	388	392	396	401	405	410	414	419	423	427	
987	432	436	441	445	449	454	458	463	467	471	
988	476	480	484	489	493	498	502	506	511	515	
989	520	524	528	533	537	542	546	550	555	559	
990	564	568	572	577	581	585	590	594	599	603	
991	607	612	616	621	625	629	634	638	642	647	
992	651	656	660	664	669	673	677	682	686	691	
993	695	699	704	708	712	717	721	726	730	734	
994	739	743	747	752	756	760	765	769	774	778	
995	782	787	791	795	800	804	808	813	817	822	
996	826	830	835	839	843	848	852	856	861	865	
997	870	874	878	883	887	891	896	900	904	909	
998	913	917	922	926	930	935	939	944	948	952	
999	957	961	965	970	974	978	983	987	991	996	
1000	00	000	004	009	013	017	022	026	030	035	
N.	0	1	2	3	4	5	6	7	8	9	Proportional parts

.97 772 — .99 996

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

1000 — 1050

N.		0	1	2	3	4	5	6	7	8	9	d.
1000	000	0000	0434	0869	1303	1737	2171	2605	3039	3473	3907	434
1001		4341	4775	5208	5642	6076	6510	6943	7377	7810	8244	434
1002		8677	9111	9544	9977	*0411	*0844	*1277	*1710	*2143	*2576	433
1003	001	3009	3442	3875	4308	4741	5174	5607	6039	6472	6905	433
1004		7337	7770	8202	8635	9067	9499	9932	*0364	*0796	*1228	432
1005												
1006	002	1661	2093	2525	2957	3389	3821	4253	4685	5116	5548	432
1007		5980	6411	6843	7275	7706	8138	8569	9001	9432	9863	431
1008	003	0295	0726	1157	1588	2019	2451	2882	3313	3744	4174	431
1009		4605	5036	5467	5898	6328	6759	7190	7620	8051	8481	431
		8912	9342	9772	*0203	*0633	*1063	*1493	*1924	*2354	*2784	430
1010												
1011	004	3214	3644	4074	4504	4933	5363	5793	6223	6652	7082	430
1012		7512	7941	8371	8800	9229	9659	*0088	*0517	*0947	*1376	429
1013	005	1805	2234	2663	3092	3521	3950	4379	4808	5237	5666	429
1014		6084	6523	6952	7380	7809	8238	8666	9094	9523	9951	429
	006	0380	0808	1236	1664	2092	2521	2949	3377	3805	4233	428
1015												
1016		4660	5088	5516	5944	6372	6799	7227	7655	8082	8510	428
1017		8937	9365	9792	*0219	*0647	*1074	*1501	*1928	*2355	*2782	427
1018	007	3210	3637	4064	4490	4917	5344	5771	6198	6624	7051	427
1019		7478	7904	8331	8757	9184	9610	*0037	*0463	*0889	*1316	426
	008	1742	2168	2594	3020	3446	3872	4298	4724	5150	5576	426
1020												
1021		6002	6427	6853	7279	7704	8130	8556	8981	9407	9832	426
1022	009	0257	0683	1108	1533	1959	2384	2809	3234	3659	4084	425
1023		4509	4934	5359	5784	6208	6633	7058	7483	7907	8332	425
1024		8756	9181	9605	*0030	*0454	*0878	*1303	*1727	*2151	*2575	424
	010	3000	3424	3848	4272	4696	5120	5544	5967	6391	6815	424
1025												
1026		7239	7662	8086	8510	8933	9357	9780	*0204	*0627	*1050	424
1027	011	1474	1897	2320	2743	3166	3590	4013	4436	4859	5282	423
1028		5704	6127	6550	6973	7396	7818	8241	8664	9086	9509	423
1029		9931	*0354	*0776	*1198	*1621	*2043	*2465	*2887	*3310	*3732	422
	012	4154	4576	4998	5420	5842	6264	6685	7107	7529	7951	422
1030												
1031		8372	8794	9215	9637	*0059	*0480	*0901	*1323	*1744	*2165	422
1032	013	2587	3008	3429	3850	4271	4692	5113	5534	5955	6376	421
1033		6797	7218	7639	8059	8480	8901	9321	9742	*0162	*0583	421
1034	014	1003	1424	1844	2264	2685	3105	3525	3945	4365	4785	420
		5205	5625	6045	6465	6885	7305	7725	8144	8564	8984	420
1035												
1036		9403	9823	*0243	*0662	*1082	*1501	*1920	*2340	*2759	*3178	420
1037	015	3598	4017	4436	4855	5274	5693	6112	6531	6950	7369	419
1038		7788	8206	8625	9044	9462	9881	*0300	*0718	*1137	*1555	419
1039	016	1974	2392	2810	3229	3647	4065	4483	4901	5319	5737	418
		6155	6573	6991	7409	7827	8245	8663	9080	9498	9916	418
1040												
1041	017	0333	0751	1168	1586	2003	2421	2838	3256	3673	4090	417
1042		4507	4924	5342	5759	6176	6593	7010	7427	7844	8260	417
1043		8677	9094	9511	9927	*0344	*0761	*1177	*1594	*2010	*2427	417
1044	018	2843	3259	3676	4092	4508	4925	5341	5757	6173	6589	416
		7005	7421	7837	8253	8669	9084	9500	9916	*0332	*0747	416
1045												
1046	019	1163	1578	1994	2410	2825	3240	3656	4071	4486	4902	415
1047		5317	5732	6147	6562	6977	7392	7807	8222	8637	9052	415
1048		9467	9882	*0296	*0711	*1126	*1540	*1955	*2369	*2784	*3198	415
1049	020	3613	4027	4442	4856	5270	5684	6099	6513	6927	7341	414
		7755	8169	8583	8997	9411	9824	*0238	*0652	*1066	*1479	414
1050	021	1893	2307	2720	3134	3547	3961	4374	4787	5201	5614	413
N.		0	1	2	3	4	5	6	7	8	9	d.

.000 0000 — .021 5614

Table 6.14 (Cont'd)

COMMON LOGARITHMS OF NUMBERS

1050 — 1100

N.		0	1	2	3	4	5	6	7	8	9	d.
1050	021	1893	2307	2720	3134	3547	3961	4374	4787	5201	5614	413
1051		6027	6440	6854	7267	7680	8093	8506	8919	9332	9745	413
1052	022	0157	0570	0983	1396	1808	2221	2634	3046	3459	3871	413
1053		4284	4696	5109	5521	5933	6345	6758	7170	7582	7994	412
1054		8406	8818	9230	9642	*0054	*0466	*0878	*1289	*1701	*2113	412
1055	023	2525	2936	3348	3759	4171	4582	4994	5405	5817	6228	411
1056		6639	7050	7462	7873	8284	8695	9106	9517	9928	*0339	411
1057	024	0750	1161	1572	1982	2393	2804	3214	3625	4036	4446	411
1058		4857	5267	5678	6088	6498	6909	7319	7729	8139	8549	410
1059		8960	9370	9780	*0190	*0600	*1010	*1419	*1829	*2239	*2649	410
1060	025	3059	3468	3878	4288	4697	5107	5516	5926	6335	6744	410
1061		6639	7050	7462	7873	8284	8695	9106	*0018	*0427	*0836	409
1062	026	1245	1654	2063	2472	2881	3289	3698	4107	4515	4924	409
1063		5333	5741	6150	6558	6967	7375	7783	8192	8600	9008	408
1064		9416	9824	*0233	*0641	*1049	*1457	*1865	*2273	*2680	*3088	408
1065	027	3496	3904	4312	4719	5127	5535	5942	6350	6757	7165	408
1066		7572	7979	8387	8794	9201	9609	*0016	*0423	*0830	*1237	407
1067	028	1644	2051	2458	2865	3272	3679	4086	4492	4899	5306	407
1068		5713	6119	6526	6932	7339	7745	8152	8558	8964	9371	406
1069		9777	*0183	*0590	*0996	*1402	*1808	*2214	*2620	*3026	*3432	406
1070	029	3838	4244	4649	5055	5461	5867	6272	6678	7084	7489	406
1071		7895	8300	8706	9111	9516	9922	*0327	*0732	*1138	*1543	405
1072	030	1948	2353	2758	3163	3568	3973	4378	4783	5188	5592	405
1073		5997	6402	6807	7211	7616	8020	8425	8830	9234	9638	405
1074	031	0043	0447	0851	1256	1660	2064	2468	2872	3277	3681	404
1075		4085	4489	4893	5296	5700	6104	6508	6912	7315	7719	404
1076		8123	8526	8930	9333	9737	*0140	*0544	*0947	*1350	*1754	403
1077	032	2157	2560	2963	3367	3770	4173	4576	4979	5382	5785	403
1078		6188	6590	6993	7396	7799	8201	8604	9007	9409	9812	403
1079	033	0214	0617	1019	1422	1824	2226	2629	3031	3433	3835	402
1080		4238	4640	5042	5444	5846	6248	6650	7052	7453	7855	402
1081		8257	8659	9060	9462	9864	*0265	*0667	*1068	*1470	*1871	402
1082	034	2273	2674	3075	3477	3878	4279	4680	5081	5482	5884	401
1083		6285	6686	7087	7487	7888	8289	8690	9091	9491	9892	401
1084	035	0293	0693	1094	1495	1895	2296	2696	3096	3497	3897	400
1085		4297	4698	5098	5498	5898	6298	6698	7098	7498	7898	400
1086		8298	8698	9098	9498	9898	*0297	*0697	*1097	*1496	*1896	400
1087	036	2295	2695	3094	3494	3893	4293	4692	5091	5491	5890	399
1088		6299	6698	7097	7496	7895	8284	8683	9082	9481	9880	399
1089	037	0279	0678	1076	1475	1874	2272	2671	3070	3468	3867	399
1090		4265	4663	5062	5460	5858	6257	6655	7053	7451	7849	398
1091		8243	8646	9044	9442	9839	*0237	*0635	*1033	*1431	*1829	398
1092	038	2226	2624	3022	3419	3817	4214	4612	5009	5407	5804	398
1093		6202	6599	6996	7393	7791	8188	8585	8982	9379	9776	397
1094	039	0173	0570	0967	1364	1761	2158	2554	2951	3348	3745	397
1095		4141	4538	4934	5331	5727	6124	6520	6917	7313	7709	397
1096		8106	8502	8898	9294	9690	*0086	*0482	*0878	*1274	*1670	396
1097	040	2066	2462	2858	3254	3650	4045	4441	4837	5232	5628	396
1098		6023	6419	6814	7210	7605	8001	8396	8791	9187	9582	395
1099		9977	*0372	*0767	*1162	*1557	*1952	*2347	*2742	*3137	*3532	395
1100	041	3927	4322	4716	5111	5506	5900	6295	6690	7084	7479	395
N.		0	1	2	3	4	5	6	7	8	9	d.

.021 1893 — .041 7479

Table 6.14 (Cont'd)
COMMON LOGARITHMS OF NUMBERS
1100 — 1150

N.		0	1	2	3	4	5	6	7	8	9	d.
1100	041	3927	4322	4716	5111	5506	5900	6295	6690	7084	7479	395
1101		7873	8268	8662	9056	9451	9845	*0239	*0633	*1028	*1422	394
1102	042	1816	2210	2604	2998	3392	3786	4180	4574	4968	5361	394
1103		5755	6149	6543	6936	7330	7723	8117	8510	8904	9297	394
1104		9691	*0084	*0477	*0871	*1264	*1657	*2050	*2444	*2837	*3230	393
1105	043	3623	4016	4409	4802	5195	5587	5980	6373	6766	7159	393
1106		7551	7944	8337	8729	9122	9514	9907	*0299	*0692	*1084	393
1107	044	1476	1869	2261	2653	3045	3437	3829	4222	4614	5006	392
1108		5398	5790	6181	6573	6965	7357	7749	8140	8532	8924	392
1109		9315	9707	*0099	*0490	*0882	*1273	*1664	*2056	*2447	*2839	392
1110	045	3230	3621	4012	4403	4795	5186	5577	5968	6359	6750	391
1111		7141	7531	7922	8313	8704	9095	9485	9876	*0267	*0657	391
1112	046	1048	1438	1829	2219	2610	3000	3391	3781	4171	4561	390
1113		4952	5342	5732	6122	6512	6902	7292	7682	8072	8462	390
1114		8852	9242	9632	*0021	*0411	*0801	*1190	*1580	*1970	*2359	390
1115	047	2749	3138	3528	3917	4306	4696	5085	5474	5864	6253	389
1116		6642	7031	7420	7809	8198	8587	8976	9365	9754	*0143	389
1117	048	0532	0921	1309	1698	2087	2475	2864	3253	3641	4030	389
1118		4418	4806	5195	5583	5972	6360	6748	7136	7525	7913	388
1119		8301	8689	9077	9465	9853	*0241	*0629	*1017	*1405	*1792	388
1120	049	2180	2568	2956	3343	3731	4119	4506	4894	5281	5669	388
1121		6056	6444	6831	7218	7606	7993	8380	8767	9154	9541	387
1122		9929	*0316	*0703	*1090	*1477	*1863	*2250	*2637	*3024	*3411	387
1123	050	3798	4184	4571	4958	5344	5731	6117	6504	6890	7277	387
1124		7663	8049	8436	8822	9208	9595	9981	*0367	*0753	*1139	386
1125	051	1525	1911	2297	2683	3069	3455	3841	4227	4612	4998	386
1126		5384	5770	6155	6541	6926	7312	7697	8083	8468	8854	386
1127		9239	9624	*0010	*0395	*0780	*1166	*1551	*1936	*2321	*2706	385
1128	052	3091	3476	3861	4246	4631	5016	5400	5785	6170	6555	385
1129		6939	7324	7709	8093	8478	8862	9247	9631	*0016	*0400	385
1130	053	0784	1169	1553	1937	2321	2706	3090	3474	3858	4242	384
1131		4626	5010	5394	5778	6162	6546	6929	7313	7697	8081	384
1132		8464	8848	9232	9615	9999	*0382	*0766	*1149	*1532	*1916	384
1133	054	2299	2682	3066	3449	3832	4215	4598	4981	5365	5748	383
1134		6131	6514	6896	7279	7662	8045	8428	8811	9193	9576	383
1135		9959	*0341	*0724	*1106	*1489	*1871	*2254	*2636	*3019	*3401	382
1136	055	3783	4166	4548	4930	5312	5694	6077	6459	6841	7223	382
1137		7605	7987	8369	8750	9132	9514	9896	*0278	*0659	*1041	382
1138	056	1423	1804	2186	2567	2949	3330	3712	4093	4475	4856	381
1139		5237	5619	6000	6381	6762	7143	7524	7905	8287	8668	381
1140		9049	9429	9810	*0191	*0572	*0953	*1334	*1714	*2095	*2476	381
1141	057	2856	3237	3618	3998	4379	4759	5140	5520	5900	6281	381
1142		6661	7041	7422	7802	8182	8562	8942	9322	9702	*0082	380
1143	058	0462	0842	1222	1602	1982	2362	2741	3121	3501	3881	380
1144		4260	4640	5019	5399	5778	6158	6537	6917	7296	7676	380
1145		8055	8434	8813	9193	9572	9951	*0330	*0709	*1088	*1467	379
1146	059	1846	2225	2604	2983	3362	3741	4119	4498	4877	5256	379
1147		5634	6013	6391	6770	7148	7527	7905	8284	8662	9041	379
1148		9419	9797	*0175	*0554	*0932	*1310	*1688	*2066	*2444	*2822	378
1149	060	3200	3578	3956	4334	4712	5090	5468	5845	6223	6601	378
1150		6978	7356	7734	8111	8489	8866	9244	9621	9999	*0376	378
N.		0	1	2	3	4	5	6	7	8	9	d.

.041 3927 — .061 0376

Table 6.14 (Cont'd)
COMMON LOGARITHMS OF NUMBERS
1150 — 1200

N.	0	1	2	3	4	5	6	7	8	9	d.
1150	060	6978	7356	7734	8111	8489	8866	9244	9621	*0376	378
1151	061	0753	1131	1508	1885	2262	2639	3017	3394	3771	377
1152		4525	4902	5279	5656	6032	6409	6786	7163	7540	377
1153		8293	8670	9046	9423	9799	*0176	*0552	*0929	*1305	377
1154	062	2058	2434	2811	3187	3563	3939	4316	4692	5068	376
1155		5820	6196	6572	6948	7324	7699	8075	8451	8827	376
1156		9578	9954	*0330	*0705	*1081	*1456	*1832	*2207	*2583	376
1157	063	3334	3709	4084	4460	4835	5210	5585	5960	6335	375
1158		7086	7461	7836	8211	8585	8960	9335	9710	*0085	375
1159	064	0834	1209	1584	1958	2333	2708	3082	3457	3831	375
1160		4580	4954	5329	5703	6077	6451	6826	7200	7574	374
1161		8322	8696	9070	9444	9818	*0192	*0566	*0940	*1314	374
1162	065	2061	2435	2809	3182	3556	3930	4303	4677	5050	374
1163		5797	6171	6544	6917	7291	7664	8037	8410	8784	373
1164		9530	9903	*0276	*0649	*1022	*1395	*1768	*2141	*2514	373
1165	066	3259	3632	4005	4377	4750	5123	5495	5868	6241	373
1166		6986	7358	7730	8103	8475	8847	9220	9592	9964	372
1167	067	0709	1081	1453	1825	2197	2569	2941	3313	3685	372
1168		4428	4800	5172	5544	5915	6287	6659	7030	7402	372
1169		8145	8517	8888	9259	9631	*0002	*0374	*0745	*1116	371
1170	068	1859	2230	2601	2972	3343	3714	4085	4456	4827	371
1171		5569	5940	6311	6681	7052	7423	7794	8164	8535	371
1172		9276	9647	*0017	*0388	*0758	*1129	*1499	*1869	*2240	370
1173	069	2980	3350	3721	4091	4461	4831	5201	5571	5941	370
1174		6681	7051	7421	7791	8160	8530	8900	9270	9639	370
1175	070	0379	0748	1118	1487	1857	2226	2596	2965	3335	369
1176		4073	4442	4812	5181	5550	5919	6288	6658	7027	369
1177		7765	8134	8503	8871	9240	9609	9978	*0347	*0715	369
1178	071	1453	1822	2190	2559	2927	3296	3664	4033	4401	369
1179		5138	5506	5875	6243	6611	6979	7348	7716	8084	368
1180		8820	9188	9556	9924	*0292	*0660	*1028	*1396	*1763	368
1181	072	2499	2867	3234	3602	3970	4337	4705	5072	5440	368
1182		6175	6542	6910	7277	7644	8011	8379	8746	9113	367
1183		9847	*0215	*0582	*0949	*1316	*1683	*2050	*2416	*2783	367
1184	073	3517	3884	4251	4617	4984	5351	5717	6084	6450	367
1185		7184	7550	7916	8283	8649	9016	9382	9748	*0114	366
1186	074	0847	1213	1579	1945	2311	2677	3043	3409	3775	366
1187		4507	4873	5239	5605	5970	6336	6702	7068	7433	366
1188		8164	8530	8895	9261	9626	9992	*0357	*0723	*1088	365
1189	075	1819	2184	2549	2914	3279	3644	4010	4375	4740	365
1190		5470	5835	6199	6564	6929	7294	7659	8024	8388	365
1191		9118	9482	9847	*0211	*0576	*0940	*1305	*1669	*2034	364
1192	076	2763	3127	3491	3855	4220	4584	4948	5312	5676	364
1193		6404	6768	7132	7496	7860	8224	8588	8952	9316	364
1194	077	0043	0407	0771	1134	1498	1862	2225	2589	2952	364
1195		3679	4042	4406	4769	5133	5496	5859	6222	6585	363
1196		7312	7675	8038	8401	8764	9127	9490	9853	*0216	363
1197	078	0942	1304	1667	2030	2393	2755	3118	3480	3843	363
1198		4568	4931	5293	5656	6018	6380	6743	7105	7467	362
1199		8192	8554	8916	9278	9640	*0003	*0365	*0727	*1089	362
1200	079	1812	2174	2536	2898	3260	3622	3983	4345	4707	362
N.	0	1	2	3	4	5	6	7	8	9	d.

.060 6978 — .079 5068

6.15 TABLE OF COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

Table 6.15

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

0° (180°)				(359°) 179°				
//	/	L Sin	d	L Tan	c d	L Ctn	L Cos	/
0	0						10.00 000	60
60	1	6.46 373	30103	6.46 373	30103	13.53 627	10.00 000	59
120	2	6.76 476	17609	6.76 476	17609	13.23 524	10.00 000	58
180	3	6.94 085	12494	6.94 085	12494	13.05 915	10.00 000	57
240	4	7.06 579	9691	7.06 579	9691	12.93 421	10.00 000	56
300	5	7.16 270	7918	7.16 270	7918	12.83 730	10.00 000	55
360	6	7.24 188	6694	7.24 188	6694	12.75 812	10.00 000	54
420	7	7.30 882	5800	7.30 882	5800	12.69 118	10.00 000	53
480	8	7.36 682	5115	7.36 682	5115	12.63 318	10.00 000	52
540	9	7.41 797	4576	7.41 797	4576	12.58 203	10.00 000	51
600	10	7.46 373	4139	7.46 373	4139	12.53 627	10.00 000	50
660	11	7.50 512	3779	7.50 512	3779	12.49 488	10.00 000	49
720	12	7.54 291	3476	7.54 291	3476	12.45 709	10.00 000	48
780	13	7.57 767	3218	7.57 767	3219	12.42 233	10.00 000	47
840	14	7.60 985	2997	7.60 986	2996	12.39 014	10.00 000	46
900	15	7.63 982	2802	7.63 982	2803	12.36 018	10.00 000	45
960	16	7.66 784	2633	7.66 785	2633	12.33 215	10.00 000	44
1020	17	7.69 417	2483	7.69 418	2482	12.30 582	9.99 999	43
1080	18	7.71 900	2348	7.71 900	2348	12.28 100	9.99 999	42
1140	19	7.74 248	2227	7.74 248	2228	12.25 752	9.99 999	41
1200	20	7.76 475	2119	7.76 476	2119	12.23 524	9.99 999	40
1260	21	7.78 594	2021	7.78 595	2020	12.21 405	9.99 999	39
1320	22	7.80 615	1930	7.80 615	1931	12.19 385	9.99 999	38
1380	23	7.82 545	1848	7.82 546	1848	12.17 454	9.99 999	37
1440	24	7.84 393	1773	7.84 394	1773	12.15 606	9.99 999	36
1500	25	7.86 166	1704	7.86 167	1704	12.13 833	9.99 999	35
1560	26	7.87 870	1639	7.87 871	1639	12.12 129	9.99 999	34
1620	27	7.89 509	1579	7.89 510	1579	12.10 490	9.99 999	33
1680	28	7.91 088	1524	7.91 089	1524	12.08 911	9.99 999	32
1740	29	7.92 612	1472	7.92 613	1473	12.07 387	9.99 998	31
1800	30	7.94 084	1424	7.94 086	1424	12.05 914	9.99 998	30
1860	31	7.95 508	1379	7.95 510	1379	12.04 490	9.99 998	29
1920	32	7.96 887	1336	7.96 889	1336	12.03 111	9.99 998	28
1980	33	7.98 223	1297	7.98 225	1297	12.01 775	9.99 998	27
2040	34	7.99 520	1259	7.99 522	1259	12.00 478	9.99 998	26
2100	35	8.00 779	1223	8.00 781	1223	11.99 219	9.99 998	25
2160	36	8.02 002	1190	8.02 004	1190	11.97 996	9.99 998	24
2220	37	8.03 192	1158	8.03 194	1159	11.96 806	9.99 997	23
2280	38	8.04 350	1128	8.04 353	1128	11.95 647	9.99 997	22
2340	39	8.05 478	1100	8.05 481	1100	11.94 519	9.99 997	21
2400	40	8.06 578	1072	8.06 581	1072	11.93 419	9.99 997	20
2460	41	8.07 650	1046	8.07 653	1047	11.92 347	9.99 997	19
2520	42	8.08 696	1022	8.08 700	1022	11.91 300	9.99 997	18
2580	43	8.09 718	999	8.09 722	998	11.90 278	9.99 997	17
2640	44	8.10 717	976	8.10 720	976	11.89 280	9.99 996	16
2700	45	8.11 693	954	8.11 696	955	11.88 304	9.99 996	15
2760	46	8.12 647	934	8.12 651	934	11.87 349	9.99 996	14
2820	47	8.13 581	914	8.13 585	915	11.86 415	9.99 996	13
2880	48	8.14 495	896	8.14 500	895	11.85 500	9.99 996	12
2940	49	8.15 391	877	8.15 395	878	11.84 605	9.99 996	11
3000	50	8.16 268	860	8.16 273	860	11.83 727	9.99 995	10
3060	51	8.17 128	843	8.17 133	843	11.82 867	9.99 995	9
3120	52	8.17 971	827	8.17 976	828	11.82 024	9.99 995	8
3180	53	8.18 798	812	8.18 804	812	11.81 196	9.99 995	7
3240	54	8.19 610	797	8.19 616	797	11.80 384	9.99 995	6
3300	55	8.20 407	782	8.20 413	782	11.79 587	9.99 994	5
3360	56	8.21 189	769	8.21 195	769	11.78 805	9.99 994	4
3420	57	8.21 958	755	8.21 964	756	11.78 036	9.99 994	3
3480	58	8.22 713	743	8.22 720	742	11.77 280	9.99 994	2
3540	59	8.23 456	730	8.23 462	730	11.76 538	9.99 994	1
3600	60	8.24 186		8.24 192		11.75 808	9.99 993	0
//	/	L Cos	d	L Ctn	c d	L Tan	L Sin	/
90° (270°)				(269°) 89°				

90° (270°)

(269°) 89°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

1° (181°)				(358°) 178°				
//	'	L Sin	d	L Tan	c d	L Ctn	L Cos	'
3600	0	8.24 186	717	8.24 192	718	11.75 808	9.99 993	60
3660	1	8.24 903	706	8.24 910	706	11.75 090	9.99 993	59
3720	2	8.25 609	695	8.25 616	696	11.74 384	9.99 993	58
3780	3	8.26 304	684	8.26 312	684	11.73 688	9.99 993	57
3840	4	8.26 988	673	8.26 996	673	11.73 004	9.99 992	56
3900	5	8.27 661	663	8.27 669	663	11.72 331	9.99 992	55
3960	6	8.28 324	653	8.28 332	654	11.71 668	9.99 992	54
4020	7	8.28 997	644	8.28 986	643	11.71 014	9.99 992	53
4080	8	8.29 621	634	8.29 629	634	11.70 371	9.99 992	52
4140	9	8.30 255	624	8.30 263	625	11.69 737	9.99 991	51
4200	10	8.30 879	616	8.30 888	617	11.69 112	9.99 991	50
4260	11	8.31 495	608	8.31 505	607	11.68 495	9.99 991	49
4320	12	8.32 103	599	8.32 112	599	11.67 888	9.99 990	48
4380	13	8.32 702	590	8.32 711	591	11.67 289	9.99 990	47
4440	14	8.33 292	583	8.33 302	584	11.66 698	9.99 990	46
4500	15	8.33 875	575	8.33 886	575	11.66 114	9.99 990	45
4560	16	8.34 450	568	8.34 461	568	11.65 539	9.99 989	44
4620	17	8.35 018	560	8.35 029	561	11.64 971	9.99 989	43
4680	18	8.35 578	553	8.35 590	553	11.64 410	9.99 989	42
4740	19	8.36 131	547	8.36 143	546	11.63 857	9.99 989	41
4800	20	8.36 678	539	8.36 689	540	11.63 311	9.99 988	40
4860	21	8.37 217	533	8.37 229	533	11.62 771	9.99 988	39
4920	22	8.37 750	526	8.37 762	527	11.62 238	9.99 988	38
4980	23	8.38 276	520	8.38 289	520	11.61 711	9.99 987	37
5040	24	8.38 796	514	8.38 809	514	11.61 191	9.99 987	36
5100	25	8.39 310	508	8.39 323	509	11.60 677	9.99 987	35
5160	26	8.39 818	502	8.39 832	502	11.60 168	9.99 986	34
5220	27	8.40 320	496	8.40 334	496	11.59 666	9.99 986	33
5280	28	8.40 816	491	8.40 830	491	11.59 170	9.99 986	32
5340	29	8.41 307	485	8.41 321	486	11.58 679	9.99 985	31
5400	30	8.41 792	480	8.41 807	480	11.58 193	9.99 985	30
5460	31	8.42 272	474	8.42 287	475	11.57 713	9.99 985	29
5520	32	8.42 746	470	8.42 762	470	11.57 238	9.99 984	28
5580	33	8.43 216	464	8.43 232	464	11.56 768	9.99 984	27
5640	34	8.43 680	459	8.43 696	460	11.56 304	9.99 984	26
5700	35	8.44 139	455	8.44 156	455	11.55 844	9.99 983	25
5760	36	8.44 594	450	8.44 611	450	11.55 389	9.99 983	24
5820	37	8.45 044	445	8.45 061	446	11.54 939	9.99 983	23
5880	38	8.45 489	441	8.45 507	441	11.54 493	9.99 982	22
5940	39	8.45 930	436	8.45 948	437	11.54 052	9.99 982	21
6000	40	8.46 366	433	8.46 385	432	11.53 615	9.99 982	20
6060	41	8.46 799	427	8.46 817	428	11.53 183	9.99 981	19
6120	42	8.47 226	424	8.47 245	424	11.52 755	9.99 981	18
6180	43	8.47 650	419	8.47 669	420	11.52 331	9.99 981	17
6240	44	8.48 069	416	8.48 089	416	11.51 911	9.99 980	16
6300	45	8.48 485	411	8.48 505	412	11.51 495	9.99 980	15
6360	46	8.48 896	408	8.48 917	408	11.51 083	9.99 979	14
6420	47	8.49 304	404	8.49 325	404	11.50 675	9.99 979	13
6480	48	8.49 708	400	8.49 729	401	11.50 271	9.99 979	12
6540	49	8.50 108	396	8.50 130	397	11.49 870	9.99 978	11
6600	50	8.50 504	393	8.50 527	393	11.49 473	9.99 978	10
6660	51	8.50 897	390	8.50 920	390	11.49 080	9.99 977	9
6720	52	8.51 287	386	8.51 310	386	11.48 690	9.99 977	8
6780	53	8.51 673	382	8.51 696	383	11.48 304	9.99 977	7
6840	54	8.52 055	379	8.52 079	380	11.47 921	9.99 976	6
6900	55	8.52 434	376	8.52 459	376	11.47 541	9.99 976	5
6960	56	8.52 810	373	8.52 835	373	11.47 165	9.99 975	4
7020	57	8.53 183	369	8.53 208	370	11.46 792	9.99 975	3
7080	58	8.53 552	367	8.53 578	367	11.46 422	9.99 974	2
7140	59	8.53 919	363	8.53 945	363	11.46 055	9.99 974	1
7200	60	8.54 282		8.54 308		11.45 692	9.99 974	0
//	'	L Cos	d	L Ctn	c d	L Tan	L Sin	'
91° (271°)				(268°) 88°				

91° (271°)

(268°) 88°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

2° (182°)				(357°) 177°				
//	/	L Sin	d	L Tan	c d	L Ctn	L Cos	/
7200	0	8.54 282	360	8.54 308	361	11.45 692	9.99 974	60
7260	1	8.54 642	357	8.54 669	358	11.45 331	9.99 973	59
7320	2	8.54 999	355	8.55 027	355	11.44 973	9.99 973	58
7380	3	8.55 354	351	8.55 382	352	11.44 618	9.99 972	57
7440	4	8.55 705	349	8.55 734	349	11.44 266	9.99 972	56
7500	5	8.56 054	346	8.56 083	346	11.43 917	9.99 971	55
7560	6	8.56 400	343	8.56 429	344	11.43 571	9.99 971	54
7620	7	8.56 743	341	8.56 773	341	11.43 227	9.99 970	53
7680	8	8.57 084	337	8.57 114	338	11.42 886	9.99 970	52
7740	9	8.57 421	336	8.57 452	336	11.42 548	9.99 969	51
7800	10	8.57 757	332	8.57 788	333	11.42 212	9.99 969	50
7860	11	8.58 089	330	8.58 121	330	11.41 879	9.99 968	49
7920	12	8.58 419	328	8.58 451	328	11.41 549	9.99 968	48
7980	13	8.58 747	325	8.58 779	326	11.41 221	9.99 967	47
8040	14	8.59 072	323	8.59 105	323	11.40 895	9.99 967	46
8100	15	8.59 395	320	8.59 428	321	11.40 572	9.99 967	45
8160	16	8.59 715	318	8.59 749	319	11.40 251	9.99 966	44
8220	17	8.60 033	316	8.60 068	316	11.39 932	9.99 966	43
8280	18	8.60 349	313	8.60 384	314	11.39 616	9.99 965	42
8340	19	8.60 662	311	8.60 698	311	11.39 302	9.99 964	41
8400	20	8.60 973	309	8.61 009	310	11.38 991	9.99 964	40
8460	21	8.61 282	307	8.61 319	307	11.38 681	9.99 963	39
8520	22	8.61 589	305	8.61 626	305	11.38 374	9.99 963	38
8580	23	8.61 894	302	8.61 931	303	11.38 069	9.99 962	37
8640	24	8.62 196	301	8.62 234	301	11.37 766	9.99 962	36
8700	25	8.62 497	298	8.62 535	299	11.37 465	9.99 961	35
8760	26	8.62 795	296	8.62 834	297	11.37 166	9.99 961	34
8820	27	8.63 091	294	8.63 131	295	11.36 869	9.99 960	33
8880	28	8.63 385	293	8.63 426	292	11.36 574	9.99 960	32
8940	29	8.63 678	290	8.63 718	291	11.36 282	9.99 959	31
9000	30	8.63 968	288	8.64 009	289	11.35 991	9.99 959	30
9060	31	8.64 256	287	8.64 298	287	11.35 702	9.99 958	29
9120	32	8.64 543	284	8.64 585	285	11.35 415	9.99 958	28
9180	33	8.64 827	283	8.64 870	284	11.35 130	9.99 957	27
9240	34	8.65 110	281	8.65 154	281	11.34 846	9.99 956	26
9300	35	8.65 391	279	8.65 435	280	11.34 565	9.99 956	25
9360	36	8.65 670	277	8.65 715	278	11.34 285	9.99 955	24
9420	37	8.65 947	276	8.65 993	276	11.34 007	9.99 955	23
9480	38	8.66 223	274	8.66 269	274	11.33 731	9.99 954	22
9540	39	8.66 497	272	8.66 543	273	11.33 457	9.99 954	21
9600	40	8.66 769	270	8.66 816	271	11.33 184	9.99 953	20
9660	41	8.67 039	269	8.67 087	269	11.32 913	9.99 952	19
9720	42	8.67 308	267	8.67 356	268	11.32 644	9.99 952	18
9780	43	8.67 575	266	8.67 624	266	11.32 376	9.99 951	17
9840	44	8.67 841	263	8.67 890	264	11.32 110	9.99 951	16
9900	45	8.68 104	263	8.68 154	263	11.31 846	9.99 950	15
9960	46	8.68 367	260	8.68 417	261	11.31 583	9.99 949	14
10020	47	8.68 627	259	8.68 678	260	11.31 322	9.99 949	13
10080	48	8.68 886	258	8.68 938	258	11.31 062	9.99 948	12
10140	49	8.69 144	256	8.69 196	257	11.30 804	9.99 948	11
10200	50	8.69 400	254	8.69 453	255	11.30 547	9.99 947	10
10260	51	8.69 654	253	8.69 708	254	11.30 292	9.99 946	9
10320	52	8.69 907	252	8.69 962	252	11.30 038	9.99 946	8
10380	53	8.70 159	250	8.70 214	251	11.29 786	9.99 945	7
10440	54	8.70 409	249	8.70 465	249	11.29 535	9.99 944	6
10500	55	8.70 658	247	8.70 714	248	11.29 286	9.99 944	5
10560	56	8.70 905	246	8.70 962	246	11.29 038	9.99 943	4
10620	57	8.71 151	244	8.71 208	245	11.28 792	9.99 942	3
10680	58	8.71 395	243	8.71 453	244	11.28 547	9.99 942	2
10740	59	8.71 638	242	8.71 697	243	11.28 303	9.99 941	1
10800	60	8.71 880		8.71 940		11.28 060	9.99 940	0
//	/	L Cos	d	L Ctn	c d	L Tan	L Sin	/

32° (272°)

(267°) 87°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

3° (183°)

(356°) 176°

°	L Sin			d	L Tan			c d	L Ctn			L Cos	°	Proportional parts						
															//					
0	8.71	880	240		8.71	940	241		11.28	060		9.99	940	60	//	241	239	237	235	234
1	8.72	120	239		8.72	181	239		11.27	819		9.99	940	59	1	4.0	4.0	4.0	3.9	3.9
2	8.72	359	238		8.72	420	239		11.27	580		9.99	939	58	2	8.0	8.0	7.9	7.8	7.8
3	8.72	597	237		8.72	659	237		11.27	341		9.99	938	57	3	12.0	12.0	11.8	11.8	11.7
4	8.72	834	236		8.72	896	236		11.27	104		9.99	938	56	4	16.1	15.9	15.8	15.7	15.6
5	8.73	069	234		8.73	132	234		11.26	868		9.99	937	55	5	20.1	19.9	19.8	19.6	19.5
6	8.73	303	232		8.73	366	234		11.26	634		9.99	936	54	6	24.1	23.9	23.7	23.5	23.4
7	8.73	535	232		8.73	600	232		11.26	400		9.99	936	53	7	28.1	27.9	27.6	27.4	27.3
8	8.73	767	230		8.73	832	231		11.26	168		9.99	935	52	8	32.1	31.9	31.6	31.3	31.2
9	8.73	997	229		8.74	063	229		11.25	937		9.99	934	51	9	36.2	35.8	35.6	35.2	35.1
10	8.74	226	228		8.74	292	229		11.25	708		9.99	934	50	//	232	229	227	225	223
11	8.74	454	226		8.74	521	227		11.25	479		9.99	933	49	1	3.9	3.8	3.8	3.8	3.7
12	8.74	680	226		8.74	743	226		11.25	252		9.99	932	48	2	7.7	7.6	7.6	7.5	7.4
13	8.74	906	224		8.74	974	225		11.25	026		9.99	932	47	3	11.6	11.4	11.4	11.2	11.2
14	8.75	130	223		8.75	199	224		11.24	801		9.99	931	46	4	15.5	15.3	15.1	15.0	14.9
15	8.75	353	222		8.75	423	222		11.24	577		9.99	930	45	5	19.3	19.1	18.9	18.8	18.6
16	8.75	575	220		8.75	645	222		11.24	355		9.99	929	44	6	23.2	22.9	22.7	22.5	22.3
17	8.75	795	220		8.75	867	220		11.24	133		9.99	929	43	7	27.1	26.7	26.5	26.2	26.0
18	8.76	015	219		8.76	087	219		11.23	913		9.99	928	42	8	30.9	30.5	30.3	30.0	29.7
19	8.76	234	217		8.76	306	219		11.23	694		9.99	927	41	9	34.8	34.4	34.0	33.8	33.4
20	8.76	451	216		8.76	525	217		11.23	475		9.99	926	40	//	222	220	217	215	213
21	8.76	667	216		8.76	742	216		11.23	258		9.99	926	39	1	3.7	3.7	3.6	3.6	3.6
22	8.76	883	214		8.76	958	215		11.23	042		9.99	925	38	2	7.4	7.3	7.2	7.2	7.1
23	8.77	097	213		8.77	173	214		11.22	827		9.99	924	37	3	11.1	11.0	10.8	10.8	10.6
24	8.77	310	212		8.77	387	213		11.22	613		9.99	923	36	4	14.8	14.7	14.5	14.3	14.2
25	8.77	522	211		8.77	600	211		11.22	400		9.99	923	35	5	18.5	18.3	18.1	17.9	17.8
26	8.77	733	210		8.77	811	211		11.22	189		9.99	922	34	6	22.2	22.0	21.7	21.5	21.3
27	8.77	943	209		8.78	022	210		11.21	978		9.99	921	33	7	25.9	25.7	25.3	25.1	24.8
28	8.78	152	208		8.78	232	209		11.21	768		9.99	920	32	8	29.6	29.3	28.9	28.7	28.4
29	8.78	360	208		8.78	441	208		11.21	559		9.99	920	31	9	33.3	33.0	32.6	32.2	32.0
30	8.78	568	206		8.78	649	206		11.21	351		9.99	919	30	//	211	208	206	203	201
31	8.78	774	205		8.78	855	206		11.21	145		9.99	918	29	1	3.5	3.5	3.4	3.4	3.4
32	8.78	979	204		8.79	061	205		11.20	939		9.99	917	28	2	7.0	6.9	6.9	6.8	6.7
33	8.79	183	203		8.79	266	204		11.20	734		9.99	917	27	3	10.6	10.4	10.3	10.2	10.0
34	8.79	386	202		8.79	470	203		11.20	530		9.99	916	26	4	14.1	13.9	13.7	13.5	13.4
35	8.79	588	201		8.79	673	202		11.20	327		9.99	915	25	5	17.6	17.3	17.2	16.9	16.8
36	8.79	789	201		8.79	875	201		11.20	125		9.99	914	24	6	21.1	20.8	20.6	20.3	20.1
37	8.79	990	199		8.80	076	201		11.19	924		9.99	913	23	7	24.6	24.3	24.0	23.7	23.4
38	8.80	189	199		8.80	277	199		11.19	723		9.99	913	22	8	28.1	27.7	27.5	27.1	26.8
39	8.80	388	197		8.80	476	198		11.19	524		9.99	912	21	9	31.6	31.2	30.9	30.4	30.2
40	8.80	585	197		8.80	674	198		11.19	326		9.99	911	20	//	199	197	195	193	192
41	8.80	782	196		8.80	872	196		11.19	128		9.99	910	19	1	3.3	3.3	3.2	3.2	3.2
42	8.80	978	195		8.81	068	196		11.18	932		9.99	909	18	2	6.6	6.6	6.5	6.4	6.4
43	8.81	173	194		8.81	264	195		11.18	736		9.99	909	17	3	10.0	9.8	9.8	9.6	9.6
44	8.81	367	193		8.81	459	194		11.18	541		9.99	908	16	4	13.3	13.1	13.0	12.9	12.8
45	8.81	560	192		8.81	653	193		11.18	347		9.99	907	15	5	16.6	16.4	16.2	16.1	16.0
46	8.81	752	192		8.81	846	192		11.18	154		9.99	906	14	6	19.9	19.7	19.5	19.3	19.2
47	8.81	944	190		8.82	038	192		11.17	962		9.99	905	13	7	23.2	23.0	22.8	22.5	22.4
48	8.82	134	190		8.82	230	190		11.17	770		9.99	904	12	8	26.5	26.3	26.0	25.7	25.6
49	8.82	324	189		8.82	420	190		11.17	580		9.99	904	11	9	29.8	29.6	29.2	29.0	28.8
50	8.82	513	188		8.82	610	189		11.17	390		9.99	903	10	//	189	187	185	183	181
51	8.82	701	187		8.82	799	188		11.17	201		9.99	902	9	1	3.2	3.1	3.1	3.0	3.0
52	8.82	888	187		8.82	987	188		11.17	013		9.99	901	8	2	6.3	6.2	6.2	6.1	6.0
53	8.83	075	186		8.83	175	186		11.16	825		9.99	900	7	3	9.4	9.4	9.2	9.2	9.0
54	8.83	261	185		8.83	361	186		11.16	639		9.99	899	6	4	12.6	12.5	12.3	12.2	12.1
55	8.83	446	184		8.83	547	185		11.16	453		9.99	898	5	5	15.8	15.6	15.4	15.2	15.1
56	8.83	630	183		8.83	732	184		11.16	268		9.99	898	4	6	18.9	18.7	18.5	18.3	18.1
57	8.83	813	183		8.83	916	184		11.16	084		9.99	897	3	7	22.0	21.8	21.6	21.4	21.1
58	8.83	996	181		8.84	100	182		11.15	900		9.99	896	2	8	25.2	24.9	24.7	24.4	24.1
59	8.84	177	181		8.84	282	182		11.15	718		9.99	895	1	9	28.4	28.0	27.8	27.4	27.2
60	8.84	358			8.84	464			11.15	536		9.99	894	0						
°	L Cos			d	L Ctn			c d	L Tan			L Sin	°	Proportional parts						

93° (273°)

(266°) 86°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

4° (184°)					(355°) 175°								
/	L Sin	d	L Tan	c d	L Ctn	L Cos	/	Proportional parts					
0	8.84 358	181	8.84 464	182	11.15 536	9.99 894	60	//	182	181	179	178	177
1	8.84 539	179	8.84 646	180	11.15 354	9.99 893	59	1	3.0	3.0	3.0	3.0	3.0
2	8.84 718	179	8.84 826	180	11.15 174	9.99 892	58	2	6.1	6.0	6.0	5.9	5.9
3	8.84 897	178	8.85 006	179	11.14 994	9.99 891	57	3	9.1	9.0	9.0	8.9	8.8
4	8.85 075	177	8.85 185	178	11.14 815	9.99 891	56	4	12.1	12.1	11.9	11.9	11.8
5	8.85 252	177	8.85 363	177	11.14 637	9.99 890	55	5	15.2	15.1	14.9	14.8	14.8
6	8.85 429	176	8.85 540	177	11.14 460	9.99 889	54	6	18.2	18.1	17.9	17.8	17.7
7	8.85 605	175	8.85 717	176	11.14 283	9.99 888	53	7	21.2	21.1	20.9	20.8	20.6
8	8.85 780	175	8.85 893	176	11.14 107	9.99 887	52	8	24.3	24.1	23.9	23.7	23.6
9	8.85 955	173	8.86 069	174	11.13 931	9.99 886	51	9	27.3	27.2	26.8	26.7	26.6
10	8.86 128	173	8.86 243	174	11.13 757	9.99 885	50	//	176	175	174	173	172
11	8.86 301	173	8.86 417	174	11.13 583	9.99 884	49	1	2.9	2.9	2.9	2.9	2.9
12	8.86 474	171	8.86 591	172	11.13 409	9.99 883	48	2	5.9	5.8	5.8	5.8	5.7
13	8.86 645	171	8.86 763	172	11.13 237	9.99 882	47	3	8.8	8.8	8.7	8.6	8.6
14	8.86 816	171	8.86 935	171	11.13 065	9.99 881	46	4	11.7	11.7	11.6	11.5	11.5
15	8.86 987	169	8.87 106	171	11.12 894	9.99 880	45	5	14.7	14.6	14.5	14.4	14.3
16	8.87 156	169	8.87 277	170	11.12 723	9.99 879	44	6	17.6	17.5	17.4	17.3	17.2
17	8.87 325	169	8.87 447	169	11.12 553	9.99 879	43	7	20.5	20.4	20.3	20.2	20.1
18	8.87 494	167	8.87 616	169	11.12 384	9.99 878	42	8	23.5	23.3	23.2	23.1	22.9
19	8.87 661	168	8.87 785	168	11.12 215	9.99 877	41	9	26.4	26.2	26.1	26.0	25.8
20	8.87 829	166	8.87 953	167	11.12 047	9.99 876	40	//	171	170	169	168	167
21	8.87 995	166	8.88 120	167	11.11 880	9.99 875	39	1	2.8	2.8	2.8	2.8	2.8
22	8.88 161	165	8.88 287	166	11.11 713	9.99 874	38	2	5.7	5.7	5.6	5.6	5.6
23	8.88 326	164	8.88 453	165	11.11 547	9.99 873	37	3	8.6	8.5	8.4	8.4	8.4
24	8.88 490	164	8.88 618	165	11.11 382	9.99 872	36	4	11.4	11.3	11.3	11.2	11.1
25	8.88 654	163	8.88 783	165	11.11 217	9.99 871	35	5	14.2	14.2	14.1	14.0	13.9
26	8.88 817	163	8.88 948	163	11.11 052	9.99 870	34	6	17.1	17.0	16.9	16.8	16.7
27	8.88 980	162	8.89 111	163	11.10 889	9.99 869	33	7	20.0	19.8	19.7	19.6	19.5
28	8.89 142	162	8.89 274	163	11.10 726	9.99 868	32	8	22.8	22.7	22.5	22.4	22.3
29	8.89 304	160	8.89 437	161	11.10 563	9.99 867	31	9	25.6	25.5	25.4	25.2	25.0
30	8.89 464	161	8.89 598	162	11.10 402	9.99 866	30	//	166	165	164	163	162
31	8.89 625	159	8.89 760	160	11.10 240	9.99 865	29	1	2.8	2.8	2.7	2.7	2.7
32	8.89 784	159	8.89 920	160	11.10 080	9.99 864	28	2	5.5	5.5	5.5	5.4	5.4
33	8.89 943	159	8.90 080	160	11.09 920	9.99 863	27	3	8.3	8.2	8.2	8.2	8.1
34	8.90 102	158	8.90 240	159	11.09 760	9.99 862	26	4	11.1	11.0	10.9	10.9	10.8
35	8.90 260	157	8.90 399	158	11.09 601	9.99 861	25	5	13.8	13.8	13.7	13.6	13.5
36	8.90 417	157	8.90 557	158	11.09 443	9.99 860	24	6	16.6	16.5	16.4	16.3	16.2
37	8.90 574	156	8.90 715	157	11.09 285	9.99 859	23	7	19.4	19.2	19.1	19.0	18.9
38	8.90 730	155	8.90 872	157	11.09 128	9.99 858	22	8	22.1	22.0	21.9	21.7	21.6
39	8.90 885	155	8.91 029	156	11.08 971	9.99 857	21	9	24.9	24.8	24.6	24.4	24.3
40	8.91 040	155	8.91 185	155	11.08 815	9.99 856	20	//	161	160	159	158	157
41	8.91 195	154	8.91 340	155	11.08 660	9.99 855	19	1	2.7	2.7	2.6	2.6	2.6
42	8.91 349	153	8.91 495	155	11.08 505	9.99 854	18	2	5.4	5.3	5.3	5.3	5.2
43	8.91 502	153	8.91 650	153	11.08 350	9.99 853	17	3	8.0	8.0	8.0	7.9	7.8
44	8.91 655	152	8.91 803	154	11.08 197	9.99 852	16	4	10.7	10.7	10.6	10.5	10.5
45	8.91 807	152	8.91 957	153	11.08 043	9.99 851	15	5	13.4	13.3	13.2	13.2	13.1
46	8.91 959	151	8.92 110	152	11.07 890	9.99 850	14	6	16.1	16.0	15.9	15.8	15.7
47	8.92 110	151	8.92 262	152	11.07 738	9.99 848	13	7	18.8	18.7	18.6	18.4	18.3
48	8.92 261	150	8.92 414	151	11.07 586	9.99 847	12	8	21.5	21.3	21.2	21.1	20.9
49	8.92 411	150	8.92 565	151	11.07 435	9.99 846	11	9	24.2	24.0	23.8	23.7	23.6
50	8.92 561	149	8.92 716	150	11.07 284	9.99 845	10	//	156	155	154	153	152
51	8.92 710	149	8.92 866	150	11.07 134	9.99 844	9	1	2.6	2.6	2.6	2.6	2.5
52	8.92 859	148	8.93 016	149	11.06 984	9.99 843	8	2	5.2	5.2	5.1	5.1	5.1
53	8.93 007	147	8.93 165	148	11.06 835	9.99 842	7	3	7.8	7.8	7.7	7.6	7.6
54	8.93 154	147	8.93 313	149	11.06 687	9.99 841	6	4	10.4	10.3	10.3	10.2	10.1
55	8.93 301	147	8.93 462	147	11.06 538	9.99 840	5	5	13.0	12.9	12.8	12.8	12.7
56	8.93 448	146	8.93 609	147	11.06 391	9.99 839	4	6	15.6	15.5	15.4	15.3	15.2
57	8.93 594	146	8.93 756	147	11.06 244	9.99 838	3	7	18.2	18.1	18.0	17.8	17.7
58	8.93 740	145	8.93 903	146	11.06 097	9.99 837	2	8	20.8	20.7	20.5	20.4	20.3
59	8.93 885	145	8.94 049	146	11.05 951	9.99 836	1	9	23.4	23.2	23.1	23.0	22.8
60	8.94 030		8.94 195		11.05 805	9.99 834	0	Proportional parts					
/	L Cos	d	L Ctn	c d	L Tan	L Sin	/						

94° (274°)

(265°) 85°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

5° (185°)

(354°) 174°

/	L Sin	d	L Tan	c d	L Ctn	L Cos	/	Proportional parts					
0	8.94 030	144	8.94 195	145	11.05 805	9.99 834	60	//	151	149	148	147	146
1	8.94 174	143	8.94 340	145	11.05 660	9.99 833	59	1	2.5	2.5	2.5	2.4	2.4
2	8.94 317	144	8.94 485	145	11.05 515	9.99 832	58	2	5.0	5.0	4.9	4.9	4.9
3	8.94 461	142	8.94 630	143	11.05 370	9.99 831	57	3	7.6	7.4	7.4	7.4	7.3
4	8.94 603	143	8.94 773	144	11.05 227	9.99 830	56	4	10.1	9.9	9.9	9.8	9.7
5	8.94 746	141	8.94 917	143	11.05 083	9.99 829	55	5	12.6	12.4	12.3	12.2	12.2
6	8.94 887	142	8.95 060	142	11.04 940	9.99 828	54	6	15.1	14.9	14.8	14.7	14.6
7	8.95 029	141	8.95 202	142	11.04 798	9.99 827	53	7	17.6	17.4	17.3	17.2	17.0
8	8.95 170	140	8.95 344	142	11.04 656	9.99 825	52	8	20.1	19.9	19.7	19.6	19.5
9	8.95 310	140	8.95 486	141	11.04 514	9.99 824	51	9	22.6	22.4	22.2	22.0	21.9
10	8.95 450	139	8.95 627	140	11.04 373	9.99 823	50	//	145	144	143	142	141
11	8.95 589	139	8.95 767	141	11.04 233	9.99 822	49	1	2.4	2.4	2.4	2.4	2.4
12	8.95 728	139	8.95 908	139	11.04 092	9.99 821	48	2	4.8	4.8	4.8	4.7	4.7
13	8.95 867	138	8.96 047	140	11.03 953	9.99 820	47	3	7.2	7.2	7.2	7.1	7.0
14	8.96 005	138	8.96 187	138	11.03 813	9.99 819	46	4	9.7	9.6	9.5	9.5	9.4
15	8.96 143	137	8.96 325	139	11.03 675	9.99 817	45	5	12.1	12.0	11.9	11.8	11.8
16	8.96 280	137	8.96 464	138	11.03 536	9.99 816	44	6	14.5	14.4	14.3	14.2	14.1
17	8.96 417	136	8.96 602	137	11.03 398	9.99 815	43	7	16.9	16.8	16.7	16.6	16.4
18	8.96 553	136	8.96 739	138	11.03 261	9.99 814	42	8	19.3	19.2	19.1	18.9	18.8
19	8.96 689	136	8.96 877	136	11.03 123	9.99 813	41	9	21.8	21.6	21.4	21.3	21.2
20	8.96 825	135	8.97 013	137	11.02 987	9.99 812	40	//	140	139	138	137	136
21	8.96 960	135	8.97 150	135	11.02 850	9.99 810	39	1	2.3	2.3	2.3	2.3	2.3
22	8.97 095	134	8.97 285	136	11.02 715	9.99 809	38	2	4.7	4.6	4.6	4.6	4.5
23	8.97 229	134	8.97 421	135	11.02 579	9.99 808	37	3	7.0	7.0	6.9	6.8	6.8
24	8.97 363	133	8.97 556	135	11.02 444	9.99 807	36	4	9.3	9.3	9.2	9.1	9.1
25	8.97 496	133	8.97 691	134	11.02 309	9.99 806	35	5	11.7	11.6	11.5	11.4	11.3
26	8.97 629	133	8.97 825	134	11.02 175	9.99 804	34	6	14.0	13.9	13.8	13.7	13.6
27	8.97 762	132	8.97 959	133	11.02 041	9.99 803	33	7	16.3	16.2	16.1	16.0	15.9
28	8.97 894	132	8.98 092	133	11.01 908	9.99 802	32	8	18.7	18.5	18.4	18.3	18.1
29	8.98 026	131	8.98 225	133	11.01 775	9.99 801	31	9	21.0	20.8	20.7	20.6	20.4
30	8.98 157	131	8.98 358	132	11.01 642	9.99 800	30	//	135	134	133	132	131
31	8.98 288	131	8.98 490	132	11.01 510	9.99 798	29	1	2.2	2.2	2.2	2.2	2.2
32	8.98 419	130	8.98 622	131	11.01 378	9.99 797	28	2	4.5	4.5	4.4	4.4	4.4
33	8.98 549	130	8.98 753	131	11.01 247	9.99 796	27	3	6.8	6.7	6.6	6.6	6.6
34	8.98 679	129	8.98 884	131	11.01 116	9.99 795	26	4	9.0	8.9	8.9	8.8	8.7
35	8.98 808	129	8.99 015	130	11.00 985	9.99 793	25	5	11.2	11.2	11.1	11.0	10.9
36	8.98 937	129	8.99 145	130	11.00 855	9.99 792	24	6	13.5	13.4	13.3	13.2	13.1
37	8.99 066	128	8.99 275	130	11.00 725	9.99 791	23	7	15.8	15.6	15.5	15.4	15.3
38	8.99 194	128	8.99 405	129	11.00 595	9.99 790	22	8	18.0	17.9	17.7	17.6	17.5
39	8.99 322	128	8.99 534	128	11.00 466	9.99 788	21	9	20.2	20.1	20.0	19.8	19.6
40	8.99 450	127	8.99 662	129	11.00 338	9.99 787	20	//	130	129	128	127	126
41	8.99 577	127	8.99 791	128	11.00 209	9.99 786	19	1	2.2	2.2	2.1	2.1	2.1
42	8.99 704	126	8.99 919	127	11.00 081	9.99 785	18	2	4.3	4.3	4.3	4.2	4.2
43	8.99 830	126	9.00 046	128	10.99 954	9.99 783	17	3	6.5	6.4	6.4	6.4	6.3
44	8.99 956	126	9.00 174	127	10.99 826	9.99 782	16	4	8.7	8.6	8.5	8.5	8.4
45	9.00 082	125	9.00 301	126	10.99 699	9.99 781	15	5	10.8	10.8	10.7	10.6	10.5
46	9.00 207	125	9.00 427	126	10.99 573	9.99 780	14	6	13.0	12.9	12.8	12.7	12.6
47	9.00 332	124	9.00 553	126	10.99 447	9.99 778	13	7	15.2	15.0	14.9	14.8	14.7
48	9.00 456	125	9.00 679	126	10.99 321	9.99 777	12	8	17.3	17.2	17.1	16.9	16.8
49	9.00 581	123	9.00 805	125	10.99 195	9.99 776	11	9	19.5	19.4	19.2	19.0	18.9
50	9.00 704	124	9.00 930	125	10.99 070	9.99 775	10	//	125	124	123	122	121
51	9.00 828	123	9.01 055	124	10.98 945	9.99 773	9	1	2.1	2.1	2.0	2.0	2.0
52	9.00 951	123	9.01 179	124	10.98 821	9.99 772	8	2	4.2	4.1	4.1	4.1	4.0
53	9.01 074	122	9.01 303	124	10.98 697	9.99 771	7	3	6.2	6.2	6.2	6.1	6.0
54	9.01 196	122	9.01 427	123	10.98 573	9.99 769	6	4	8.3	8.3	8.2	8.1	8.1
55	9.01 318	122	9.01 550	123	10.98 450	9.99 768	5	5	10.4	10.3	10.2	10.2	10.1
56	9.01 440	121	9.01 673	123	10.98 327	9.99 767	4	6	12.5	12.4	12.3	12.2	12.1
57	9.01 561	121	9.01 796	122	10.98 204	9.99 765	3	7	14.6	14.5	14.4	14.2	14.1
58	9.01 682	121	9.01 918	122	10.98 082	9.99 764	2	8	16.7	16.5	16.4	16.3	16.1
59	9.01 803	120	9.02 040	122	10.97 960	9.99 763	1	9	18.8	18.6	18.4	18.3	18.2
60	9.01 923		9.02 162		10.97 838	9.99 761	0						
/	L Cos	d	L Ctn	c d	L Tan	L Sin	/	Proportional parts					

95° (275°)

(264°) 84°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

6° (186°)						(353°) 173°						
/	L Sin	d	L Tan	c d	L Ctn	L Cos	/	Proportional parts				
0	9.01 923	120	9.02 162	121	10.97 838	9.99 761	60	//	121	120	119	118
1	9.02 043	120	9.02 283	121	10.97 717	9.99 760	59	1	2.0	2.0	2.0	2.0
2	9.02 163	120	9.02 404	121	10.97 596	9.99 759	58	2	4.0	4.0	4.0	3.9
3	9.02 283	119	9.02 525	120	10.97 475	9.99 757	57	3	6.0	6.0	6.0	5.9
4	9.02 402	118	9.02 645	121	10.97 355	9.99 756	56	4	8.1	8.0	7.9	7.9
5	9.02 520	119	9.02 766	119	10.97 234	9.99 755	55	5	10.1	10.0	9.9	9.8
6	9.02 639	118	9.02 885	120	10.97 115	9.99 753	54	6	12.1	12.0	11.9	11.8
7	9.02 757	117	9.03 005	119	10.96 995	9.99 752	53	7	14.1	14.0	13.9	13.8
8	9.02 874	118	9.03 124	118	10.96 876	9.99 751	52	8	16.1	16.0	15.9	15.7
9	9.02 992	117	9.03 242	119	10.96 758	9.99 749	51	9	18.2	18.0	17.8	17.7
10	9.03 109	117	9.03 361	118	10.96 639	9.99 748	50	10	20.2	20.0	19.8	19.7
11	9.03 226	116	9.03 479	118	10.96 521	9.99 747	49	20	40.3	40.0	39.7	39.3
12	9.03 342	116	9.03 597	117	10.96 403	9.99 745	48	30	60.5	60.0	59.5	59.0
13	9.03 458	116	9.03 714	118	10.96 286	9.99 744	47	40	80.7	80.0	79.3	78.7
14	9.03 574	116	9.03 832	116	10.96 168	9.99 742	46	50	100.8	100.0	99.2	98.3
15	9.03 690	115	9.03 948	117	10.96 052	9.99 741	45	//	117	116	115	114
16	9.03 805	115	9.04 065	116	10.95 935	9.99 740	44	1	2.0	1.9	1.9	1.9
17	9.03 920	114	9.04 181	116	10.95 819	9.99 738	43	2	3.9	3.9	3.8	3.8
18	9.04 034	115	9.04 297	116	10.95 703	9.99 737	42	3	5.8	5.8	5.8	5.7
19	9.04 149	113	9.04 413	115	10.95 587	9.99 736	41	4	7.8	7.7	7.7	7.6
20	9.04 262	114	9.04 528	115	10.95 472	9.99 734	40	5	9.8	9.7	9.6	9.5
21	9.04 376	114	9.04 643	115	10.95 357	9.99 733	39	6	11.7	11.6	11.5	11.4
22	9.04 490	113	9.04 758	115	10.95 242	9.99 731	38	7	13.6	13.5	13.4	13.3
23	9.04 603	112	9.04 873	114	10.95 127	9.99 730	37	8	15.6	15.5	15.3	15.2
24	9.04 715	113	9.04 987	114	10.95 013	9.99 728	36	9	17.6	17.4	17.2	17.1
25	9.04 828	112	9.05 101	113	10.94 899	9.99 727	35	10	19.5	19.3	19.2	19.0
26	9.04 940	112	9.05 214	114	10.94 786	9.99 726	34	20	39.0	38.7	38.3	38.0
27	9.05 052	112	9.05 328	113	10.94 672	9.99 724	33	30	58.5	58.0	57.5	57.0
28	9.05 164	111	9.05 441	112	10.94 559	9.99 723	32	40	78.0	77.3	76.7	76.0
29	9.05 275	111	9.05 553	113	10.94 447	9.99 721	31	50	97.5	96.7	95.8	95.0
30	9.05 386	111	9.05 666	112	10.94 334	9.99 720	30	//	113	112	111	110
31	9.05 497	110	9.05 778	112	10.94 222	9.99 718	29	1	1.9	1.9	1.8	1.8
32	9.05 607	110	9.05 890	112	10.94 110	9.99 717	28	2	3.8	3.7	3.7	3.7
33	9.05 717	110	9.06 002	111	10.93 998	9.99 716	27	3	5.6	5.6	5.6	5.5
34	9.05 827	110	9.06 113	111	10.93 887	9.99 714	26	4	7.5	7.5	7.4	7.3
35	9.05 937	109	9.06 224	111	10.93 776	9.99 713	25	5	9.4	9.3	9.2	9.2
36	9.06 046	109	9.06 335	110	10.93 665	9.99 711	24	6	11.3	11.2	11.1	11.0
37	9.06 155	109	9.06 445	111	10.93 555	9.99 710	23	7	13.2	13.1	13.0	12.8
38	9.06 264	108	9.06 556	110	10.93 444	9.99 708	22	8	15.1	14.9	14.8	14.7
39	9.06 372	109	9.06 666	109	10.93 334	9.99 707	21	9	17.0	16.8	16.6	16.5
40	9.06 481	103	9.06 775	110	10.93 225	9.99 705	20	10	18.8	18.7	18.5	18.3
41	9.06 589	107	9.06 885	109	10.93 115	9.99 704	19	20	37.7	37.3	37.0	36.7
42	9.06 696	108	9.06 994	109	10.93 006	9.99 702	18	30	56.5	56.0	55.5	55.0
43	9.06 804	107	9.07 103	108	10.92 897	9.99 701	17	40	75.3	74.7	74.0	73.3
44	9.06 911	107	9.07 211	109	10.92 789	9.99 699	16	50	94.2	93.3	92.5	91.7
45	9.07 018	106	9.07 320	108	10.92 680	9.99 698	15	//	109	108	107	106
46	9.07 124	107	9.07 428	108	10.92 572	9.99 696	14	1	1.8	1.8	1.8	1.8
47	9.07 231	106	9.07 536	107	10.92 464	9.99 695	13	2	3.6	3.6	3.6	3.5
48	9.07 337	105	9.07 643	108	10.92 357	9.99 693	12	3	5.4	5.4	5.4	5.3
49	9.07 442	106	9.07 751	107	10.92 249	9.99 692	11	4	7.3	7.2	7.1	7.1
50	9.07 548	105	9.07 858	106	10.92 142	9.99 690	10	5	9.1	9.0	8.9	8.8
51	9.07 653	105	9.07 964	107	10.92 036	9.99 689	9	6	10.9	10.8	10.7	10.6
52	9.07 758	105	9.08 071	106	10.91 929	9.99 687	8	7	12.7	12.6	12.5	12.4
53	9.07 863	105	9.08 177	106	10.91 823	9.99 686	7	8	14.5	14.4	14.3	14.1
54	9.07 968	104	9.08 283	106	10.91 717	9.99 684	6	9	16.4	16.2	16.0	15.9
55	9.08 072	104	9.08 389	106	10.91 611	9.99 683	5	10	18.2	18.0	17.8	17.7
56	9.08 176	104	9.08 495	105	10.91 505	9.99 681	4	20	36.3	36.0	35.7	35.3
57	9.08 280	103	9.08 600	105	10.91 400	9.99 680	3	30	54.5	54.0	53.5	53.0
58	9.08 383	103	9.08 705	105	10.91 295	9.99 678	2	40	72.7	72.0	71.3	70.7
59	9.08 486	103	9.08 810	104	10.91 190	9.99 677	1	50	90.8	90.0	89.2	88.3
60	9.08 589	—	9.08 914	—	10.91 086	9.99 675	0					
/	L Cos	d	L Ctn	c d	L Tan	L Sin	/	Proportional parts				

36° (276°)

(263°) 83°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

7° (187°)

(352°) 172°

/	L Sin	d	L Tan	c d	L Ctn	L Cos	/	Proportional parts				
0	9.08 589	103	9.08 914	105	10.91 086	9.99 675	60	//	105	104	103	102
1	9.08 692	103	9.09 019	104	10.90 981	9.99 674	59	1	1.8	1.7	1.7	1.7
2	9.08 795	102	9.09 123	104	10.90 877	9.99 672	58	2	3.5	3.5	3.4	3.4
3	9.08 897	102	9.09 227	103	10.90 773	9.99 670	57	3	5.2	5.2	5.2	5.1
4	9.08 999	102	9.09 330	104	10.90 670	9.99 669	56	4	7.0	6.9	6.9	6.8
5	9.09 101	101	9.09 434	103	10.90 566	9.99 667	55	5	8.8	8.7	8.6	8.5
6	9.09 202	102	9.09 537	103	10.90 463	9.99 666	54	6	10.5	10.4	10.3	10.2
7	9.09 304	101	9.09 640	102	10.90 360	9.99 664	53	7	12.2	12.1	12.0	11.9
8	9.09 405	101	9.09 742	103	10.90 258	9.99 663	52	8	14.0	13.9	13.7	13.6
9	9.09 506	100	9.09 845	102	10.90 155	9.99 661	51	9	15.8	15.6	15.4	15.3
10	9.09 606	101	9.09 947	102	10.90 053	9.99 659	50	10	17.5	17.3	17.2	17.0
11	9.09 707	100	9.10 049	101	10.89 951	9.99 658	49	20	35.0	34.7	34.3	34.0
12	9.09 807	100	9.10 150	102	10.89 850	9.99 656	48	30	52.5	52.0	51.5	51.0
13	9.09 907	99	9.10 252	101	10.89 748	9.99 655	47	40	70.0	69.3	68.7	68.0
14	9.10 006	100	9.10 353	101	10.89 647	9.99 653	46	50	87.5	86.7	85.8	85.0
15	9.10 106	99	9.10 454	101	10.89 546	9.99 651	45	//	101	100	99	98
16	9.10 205	99	9.10 555	101	10.89 445	9.99 650	44	1	1.7	1.7	1.6	1.6
17	9.10 304	98	9.10 656	100	10.89 344	9.99 648	43	2	3.4	3.3	3.3	3.3
18	9.10 402	99	9.10 756	100	10.89 244	9.99 647	42	3	5.0	5.0	5.0	4.9
19	9.10 501	98	9.10 856	100	10.89 144	9.99 645	41	4	6.7	6.7	6.6	6.5
20	9.10 599	98	9.10 956	100	10.89 044	9.99 643	40	5	8.4	8.3	8.2	8.2
21	9.10 697	98	9.11 056	99	10.88 944	9.99 642	39	6	10.1	10.0	9.9	9.8
22	9.10 795	98	9.11 155	99	10.88 845	9.99 640	38	7	11.8	11.7	11.6	11.4
23	9.10 893	97	9.11 254	99	10.88 746	9.99 638	37	8	13.5	13.3	13.2	13.1
24	9.10 990	97	9.11 353	99	10.88 647	9.99 637	36	9	15.2	15.0	14.8	14.7
25	9.11 087	97	9.11 452	99	10.88 548	9.99 635	35	10	16.8	16.7	16.5	16.3
26	9.11 184	97	9.11 551	98	10.88 449	9.99 633	34	20	33.7	33.3	33.0	32.7
27	9.11 281	96	9.11 649	98	10.88 351	9.99 632	33	30	50.5	50.0	49.5	49.0
28	9.11 377	97	9.11 747	98	10.88 253	9.99 630	32	40	67.3	66.7	66.0	65.3
29	9.11 474	96	9.11 845	98	10.88 155	9.99 629	31	50	84.2	83.3	82.5	81.7
30	9.11 570	96	9.11 943	97	10.88 057	9.99 627	30	//	97	96	95	94
31	9.11 666	95	9.12 040	98	10.87 960	9.99 625	29	1	1.6	1.6	1.6	1.6
32	9.11 761	96	9.12 138	97	10.87 862	9.99 624	28	2	3.2	3.2	3.2	3.1
33	9.11 857	95	9.12 235	97	10.87 765	9.99 622	27	3	4.8	4.8	4.8	4.7
34	9.11 952	95	9.12 332	96	10.87 668	9.99 620	26	4	6.5	6.4	6.3	6.3
35	9.12 047	95	9.12 428	97	10.87 572	9.99 618	25	5	8.1	8.0	7.9	7.8
36	9.12 142	94	9.12 525	96	10.87 475	9.99 617	24	6	9.7	9.6	9.5	9.4
37	9.12 236	95	9.12 621	96	10.87 379	9.99 615	23	7	11.3	11.2	11.1	11.0
38	9.12 331	94	9.12 717	96	10.87 283	9.99 613	22	8	12.9	12.8	12.7	12.5
39	9.12 425	94	9.12 813	96	10.87 187	9.99 612	21	9	14.6	14.4	14.2	14.1
40	9.12 519	93	9.12 909	95	10.87 091	9.99 610	20	10	16.2	16.0	15.8	15.7
41	9.12 612	94	9.13 004	95	10.86 996	9.99 608	19	20	32.3	32.0	31.7	31.3
42	9.12 706	93	9.13 099	95	10.86 901	9.99 607	18	30	48.5	48.0	47.5	47.0
43	9.12 799	93	9.13 194	95	10.86 806	9.99 605	17	40	64.7	64.0	63.3	62.7
44	9.12 892	93	9.13 289	95	10.86 711	9.99 603	16	50	80.8	80.0	79.2	78.3
45	9.12 985	93	9.13 384	94	10.86 616	9.99 601	15	//	93	92	91	90
46	9.13 078	93	9.13 478	95	10.86 522	9.99 600	14	1	1.6	1.5	1.5	1.5
47	9.13 171	92	9.13 573	94	10.86 427	9.99 598	13	2	3.1	3.1	3.0	3.0
48	9.13 263	92	9.13 667	94	10.86 333	9.99 596	12	3	4.6	4.6	4.6	4.5
49	9.13 355	92	9.13 761	93	10.86 239	9.99 595	11	4	6.2	6.1	6.1	6.0
50	9.13 447	92	9.13 854	94	10.86 146	9.99 593	10	5	7.8	7.7	7.6	7.5
51	9.13 539	91	9.13 948	93	10.86 052	9.99 591	9	6	9.3	9.2	9.1	9.0
52	9.13 630	92	9.14 041	93	10.85 959	9.99 589	8	7	10.8	10.7	10.6	10.5
53	9.13 722	91	9.14 134	93	10.85 866	9.99 588	7	8	12.4	12.3	12.1	12.0
54	9.13 813	91	9.14 227	93	10.85 773	9.99 586	6	9	14.0	13.8	13.6	13.5
55	9.13 904	90	9.14 320	92	10.85 680	9.99 584	5	10	15.5	15.3	15.2	15.0
56	9.13 994	91	9.14 412	92	10.85 588	9.99 582	4	20	31.0	30.7	30.3	30.0
57	9.14 085	90	9.14 504	93	10.85 496	9.99 581	3	30	46.5	46.0	45.5	45.0
58	9.14 175	91	9.14 597	91	10.85 403	9.99 579	2	40	62.0	61.3	60.7	60.0
59	9.14 266	90	9.14 688	92	10.85 312	9.99 577	1	50	77.5	76.7	75.8	75.0
60	9.14 356		9.14 780		10.85 220	9.99 575	0	Proportional parts				
/	L Cos	d	L Ctn	c d	L Tan	L Sin	/					

97° (277°)

(262°) 82°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

8° (188°)						(351°) 171°					
/	L Sin	d	L Tan	c d	L Ctn	L Cos	/	Proportional parts			
0	9.14 356	89	9.14 780	92	10.85 220	9.99 575	60	//	92	91	90
1	9.14 455	90	9.14 872	91	10.85 128	9.99 574	59	1	1.5	1.5	1.5
2	9.14 535	89	9.14 963	91	10.85 037	9.99 572	58	2	3.1	3.0	3.0
3	9.14 624	90	9.15 054	91	10.84 946	9.99 570	57	3	4.6	4.6	4.5
4	9.14 714	89	9.15 145	91	10.84 855	9.99 568	56	4	6.1	6.1	6.0
5	9.14 803	88	9.15 236	91	10.84 764	9.99 566	55	5	7.7	7.6	7.5
6	9.14 891	89	9.15 327	90	10.84 673	9.99 565	54	6	9.2	9.1	9.0
7	9.14 980	89	9.15 417	91	10.84 583	9.99 563	53	7	10.7	10.6	10.5
8	9.15 069	88	9.15 508	90	10.84 492	9.99 561	52	8	12.3	12.1	12.0
9	9.15 157	88	9.15 598	90	10.84 402	9.99 559	51	9	13.8	13.6	13.5
10	9.15 245	88	9.15 688	89	10.84 312	9.99 557	50	10	15.3	15.2	15.0
11	9.15 333	88	9.15 777	90	10.84 223	9.99 556	49	20	30.7	30.3	30.0
12	9.15 421	87	9.15 867	89	10.84 133	9.99 554	48	30	46.0	45.5	45.0
13	9.15 508	88	9.15 956	90	10.84 044	9.99 552	47	40	61.3	60.7	60.0
14	9.15 596	87	9.16 046	89	10.83 954	9.99 550	46	50	76.7	75.8	75.0
15	9.15 683	87	9.16 135	89	10.83 865	9.99 548	45	//	89	88	87
16	9.15 770	87	9.16 224	88	10.83 776	9.99 546	44	1	1.5	1.5	1.4
17	9.15 857	87	9.16 312	89	10.83 688	9.99 545	43	2	3.0	2.9	2.9
18	9.15 944	86	9.16 401	88	10.83 599	9.99 543	42	3	4.4	4.4	4.4
19	9.16 030	86	9.16 489	88	10.83 511	9.99 541	41	4	5.9	5.9	5.8
20	9.16 116	87	9.16 577	88	10.83 423	9.99 539	40	5	7.4	7.3	7.2
21	9.16 203	86	9.16 665	88	10.83 335	9.99 537	39	6	8.9	8.8	8.7
22	9.16 289	85	9.16 753	88	10.83 247	9.99 535	38	7	10.4	10.3	10.2
23	9.16 374	86	9.16 841	87	10.83 159	9.99 533	37	8	11.9	11.7	11.6
24	9.16 460	85	9.16 928	88	10.83 072	9.99 532	36	9	13.4	13.2	13.0
25	9.16 545	86	9.17 016	87	10.82 984	9.99 530	35	10	14.8	14.7	14.5
26	9.16 631	85	9.17 103	87	10.82 897	9.99 528	34	20	29.7	29.3	29.0
27	9.16 716	85	9.17 190	87	10.82 810	9.99 526	33	30	44.5	44.0	43.5
28	9.16 801	85	9.17 277	86	10.82 723	9.99 524	32	40	59.3	58.7	58.0
29	9.16 886	84	9.17 363	87	10.82 637	9.99 522	31	50	74.2	73.3	72.5
30	9.16 970	85	9.17 450	86	10.82 550	9.99 520	30	//	86	85	84
31	9.17 055	84	9.17 536	86	10.82 464	9.99 518	29	1	1.4	1.4	1.4
32	9.17 139	84	9.17 622	86	10.82 378	9.99 517	28	2	2.9	2.8	2.8
33	9.17 223	84	9.17 708	86	10.82 292	9.99 515	27	3	4.3	4.2	4.2
34	9.17 307	84	9.17 794	86	10.82 206	9.99 513	26	4	5.7	5.7	5.6
35	9.17 391	83	9.17 880	85	10.82 120	9.99 511	25	5	7.2	7.1	7.0
36	9.17 474	84	9.17 965	86	10.82 035	9.99 509	24	6	8.6	8.5	8.4
37	9.17 558	83	9.18 051	85	10.81 949	9.99 507	23	7	10.0	9.9	9.8
38	9.17 641	83	9.18 136	85	10.81 864	9.99 505	22	8	11.5	11.3	11.2
39	9.17 724	83	9.18 221	85	10.81 779	9.99 503	21	9	12.9	12.8	12.6
40	9.17 807	83	9.18 306	85	10.81 694	9.99 501	20	10	14.3*	14.2	14.0
41	9.17 890	83	9.18 391	84	10.81 609	9.99 499	19	20	28.7	28.3	28.0
42	9.17 973	82	9.18 475	85	10.81 525	9.99 497	18	30	43.0	42.5	42.0
43	9.18 055	82	9.18 560	84	10.81 440	9.99 495	17	40	57.3	56.7	56.0
44	9.18 137	83	9.18 644	84	10.81 356	9.99 494	16	50	71.7	70.8	70.0
45	9.18 220	82	9.18 728	84	10.81 272	9.99 492	15	//	83	82	81
46	9.18 302	81	9.18 812	84	10.81 188	9.99 490	14	1	1.4	1.4	1.4
47	9.18 383	82	9.18 896	83	10.81 104	9.99 488	13	2	2.8	2.7	2.7
48	9.18 465	82	9.18 979	84	10.81 021	9.99 486	12	3	4.2	4.1	4.0
49	9.18 547	81	9.19 063	83	10.80 937	9.99 484	11	4	5.5	5.5	5.4
50	9.18 628	81	9.19 146	83	10.80 854	9.99 482	10	5	6.9	6.8	6.8
51	9.18 709	81	9.19 229	83	10.80 771	9.99 480	9	6	8.3	8.2	8.1
52	9.18 790	81	9.19 312	83	10.80 688	9.99 478	8	7	9.7	9.6	9.4
53	9.18 871	81	9.19 395	83	10.80 605	9.99 476	7	8	11.1	10.9	10.8
54	9.18 952	81	9.19 478	83	10.80 522	9.99 474	6	9	12.4	12.3	12.2
55	9.19 033	80	9.19 561	82	10.80 439	9.99 472	5	10	13.8	13.7	13.5
56	9.19 113	80	9.19 643	82	10.80 357	9.99 470	4	20	27.7	27.3	27.0
57	9.19 193	80	9.19 725	82	10.80 275	9.99 468	3	30	41.5	41.0	40.5
58	9.19 273	80	9.19 807	82	10.80 193	9.99 466	2	40	55.3	54.7	54.0
59	9.19 353	80	9.19 889	82	10.80 111	9.99 464	1	50	69.2	68.3	67.5
60	9.19 433		9.19 971		10.80 029	9.99 462	0				
/	L Cos	d	L Ctn	c d	L Tan	L Sin	/	Proportional parts			

98° (278°)

(261°) 81°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

9° (189°)

(350°) 170°

/	L Sin	d	L Tan	c d	L Ctn	L Cos	/	Proportional parts				
0	9.19 433	80	9.19 971	82	10.80 029	9.99 462	60	//	80	79	78	77
1	9.19 513	79	9.20 053	81	10.79 947	9.99 460	59		1	1.3	1.3	1.3
2	9.19 592	80	9.20 134	82	10.79 866	9.99 458	58		2	2.7	2.6	2.6
3	9.19 672	79	9.20 216	81	10.79 784	9.99 456	57		3	4.0	4.0	3.9
4	9.19 751	79	9.20 297	81	10.79 703	9.99 454	56	4	5.3	5.3	5.2	6.1
5	9.19 830	79	9.20 378	81	10.79 622	9.99 452	55	5	6.7	6.6	6.5	6.4
6	9.19 909	79	9.20 459	81	10.79 541	9.99 450	54	6	8.0	7.9	7.8	7.7
7	9.19 988	79	9.20 540	81	10.79 460	9.99 448	53	7	9.3	9.2	9.1	9.0
8	9.20 067	78	9.20 621	80	10.79 379	9.99 446	52	8	10.7	10.5	10.4	10.3
9	9.20 145	78	9.20 701	81	10.79 299	9.99 444	51	9	12.0	11.8	11.7	11.6
10	9.20 223	79	9.20 782	80	10.79 218	9.99 442	50	10	13.3	13.2	13.0	12.8
11	9.20 302	78	9.20 862	80	10.79 138	9.99 440	49	20	26.7	26.3	26.0	25.7
12	9.20 380	78	9.20 942	80	10.79 058	9.99 438	48	30	40.0	39.5	39.0	38.5
13	9.20 458	77	9.21 022	80	10.78 978	9.99 436	47	40	53.3	52.7	52.0	51.3
14	9.20 535	78	9.21 102	80	10.78 898	9.99 434	46	50	66.7	65.8	65.0	64.2
15	9.20 613	78	9.21 182	79	10.78 818	9.99 432	45	//	76	75	74	73
16	9.20 691	77	9.21 261	80	10.78 739	9.99 429	44		1	1.3	1.2	1.2
17	9.20 768	77	9.21 341	79	10.78 659	9.99 427	43		2	2.5	2.5	2.5
18	9.20 845	77	9.21 420	79	10.78 580	9.99 425	42		3	3.8	3.8	3.7
19	9.20 922	77	9.21 499	79	10.78 501	9.99 423	41	4	5.1	5.0	4.9	4.9
20	9.20 999	77	9.21 578	79	10.78 422	9.99 421	40	5	6.3	6.2	6.2	6.1
21	9.21 076	77	9.21 657	79	10.78 343	9.99 419	39	6	7.6	7.5	7.4	7.3
22	9.21 153	76	9.21 736	78	10.78 264	9.99 417	38	7	8.9	8.8	8.6	8.5
23	9.21 229	77	9.21 814	79	10.78 186	9.99 415	37	8	10.1	10.0	9.9	9.7
24	9.21 306	76	9.21 893	78	10.78 107	9.99 413	36	9	11.4	11.2	11.1	11.0
25	9.21 382	76	9.21 971	78	10.78 029	9.99 411	35	10	12.7	12.5	12.3	12.2
26	9.21 458	76	9.22 049	78	10.77 951	9.99 409	34	20	25.3	25.0	24.7	24.3
27	9.21 534	76	9.22 127	78	10.77 873	9.99 407	33	30	38.0	37.5	37.0	36.5
28	9.21 610	75	9.22 205	78	10.77 795	9.99 404	32	40	50.7	50.0	49.3	48.7
29	9.21 685	76	9.22 283	78	10.77 717	9.99 402	31	50	63.3	62.5	61.7	60.8
30	9.21 761	75	9.22 361	77	10.77 639	9.99 400	30	//	72	71	3	2
31	9.21 836	76	9.22 438	78	10.77 562	9.99 398	29		1	1.2	1.2	0.0
32	9.21 912	75	9.22 516	77	10.77 484	9.99 396	28		2	2.4	2.4	0.1
33	9.21 987	75	9.22 593	77	10.77 407	9.99 394	27		3	3.6	3.6	0.2
34	9.22 062	75	9.22 670	77	10.77 330	9.99 392	26	4	4.8	4.7	0.2	0.1
35	9.22 137	74	9.22 747	77	10.77 253	9.99 390	25	5	6.0	5.9	0.2	0.2
36	9.22 211	75	9.22 824	77	10.77 176	9.99 388	24	6	7.2	7.1	0.3	0.2
37	9.22 286	75	9.22 901	76	10.77 099	9.99 385	23	7	8.4	8.3	0.4	0.2
38	9.22 361	74	9.22 977	77	10.77 023	9.99 383	22	8	9.6	9.5	0.4	0.3
39	9.22 435	74	9.23 054	76	10.76 946	9.99 381	21	9	10.8	10.6	0.4	0.3
40	9.22 509	74	9.23 130	76	10.76 870	9.99 379	20	10	12.0	11.8	0.5	0.3
41	9.22 583	74	9.23 206	77	10.76 794	9.99 377	19	20	24.0	23.7	1.0	0.7
42	9.22 657	74	9.23 283	76	10.76 717	9.99 375	18	30	36.0	35.5	1.5	1.0
43	9.22 731	74	9.23 359	76	10.76 641	9.99 372	17	40	48.0	47.3	2.0	1.3
44	9.22 805	73	9.23 435	75	10.76 565	9.99 370	16	50	60.0	59.2	2.5	1.7
45	9.22 878	74	9.23 510	76	10.76 490	9.99 368	15	//	72	71	3	2
46	9.22 952	73	9.23 586	75	10.76 414	9.99 366	14		1	1.2	1.2	0.0
47	9.23 025	73	9.23 661	76	10.76 339	9.99 364	13		2	2.4	2.4	0.1
48	9.23 098	73	9.23 737	75	10.76 263	9.99 362	12		3	3.6	3.6	0.2
49	9.23 171	73	9.23 812	75	10.76 188	9.99 359	11	4	4.8	4.7	0.2	0.1
50	9.23 244	73	9.23 887	75	10.76 113	9.99 357	10	5	6.0	5.9	0.2	0.2
51	9.23 317	73	9.23 962	75	10.76 038	9.99 355	9	6	7.2	7.1	0.3	0.2
52	9.23 390	72	9.24 037	75	10.75 963	9.99 353	8	7	8.4	8.3	0.4	0.2
53	9.23 462	73	9.24 112	74	10.75 888	9.99 351	7	8	9.6	9.5	0.4	0.3
54	9.23 535	72	9.24 186	75	10.75 814	9.99 348	6	9	10.8	10.6	0.4	0.3
55	9.23 607	72	9.24 261	74	10.75 739	9.99 346	5	10	12.0	11.8	0.5	0.3
56	9.23 679	73	9.24 335	75	10.75 665	9.99 344	4	20	24.0	23.7	1.0	0.7
57	9.23 752	71	9.24 410	74	10.75 590	9.99 342	3	30	36.0	35.5	1.5	1.0
58	9.23 823	72	9.24 484	74	10.75 516	9.99 340	2	40	48.0	47.3	2.0	1.3
59	9.23 895	72	9.24 558	74	10.75 442	9.99 337	1	50	60.0	59.2	2.5	1.7
60	9.23 967		9.24 632		10.75 368	9.99 335	0	//	72	71	3	2
/	L Cos	d	L Ctn	c d	L Tan	L Sin	/		1	1.2	1.2	0.0
									2	2.4	2.4	0.1
									3	3.6	3.6	0.2

99° (279°)

(260°) 80°

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

(259°) 79°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

11° (191°)						(348°) 168°						
/	L Sin	d	L Tan	c d	L Ctn	L Cos	d	/	Proportional parts			
0	9.28 060	65	9.28 865	68	10.71 135	9.99 195	3	60	//	65	64	63
1	9.28 125	65	9.28 933	67	10.71 067	9.99 192	2	59				
2	9.28 190	64	9.29 000	67	10.71 000	9.99 190	3	58				
3	9.28 254	65	9.29 067	67	10.70 933	9.99 187	2	57				
4	9.28 319	65	9.29 134	67	10.70 866	9.99 185	3	56				
5	9.28 384	64	9.29 201	67	10.70 799	9.99 182	2	55	1	1.1	1.1	1.0
6	9.28 448	64	9.29 268	67	10.70 732	9.99 180	3	54				
7	9.28 512	65	9.29 335	67	10.70 665	9.99 177	2	53				
8	9.28 577	64	9.29 402	66	10.70 598	9.99 175	3	52				
9	9.28 641	64	9.29 468	67	10.70 532	9.99 172	2	51				
10	9.28 705	64	9.29 535	66	10.70 465	9.99 170	3	50	5	5.4	5.3	5.2
11	9.28 769	64	9.29 601	67	10.70 399	9.99 167	2	49				
12	9.28 833	63	9.29 668	66	10.70 332	9.99 165	3	48				
13	9.28 896	64	9.29 734	66	10.70 266	9.99 162	2	47				
14	9.28 960	64	9.29 800	66	10.70 200	9.99 160	3	46				
15	9.29 024	63	9.29 866	66	10.70 134	9.99 157	2	45	10	10.8	10.7	10.5
16	9.29 087	63	9.29 932	66	10.70 068	9.99 155	3	44				
17	9.29 150	64	9.29 998	66	10.70 002	9.99 152	2	43				
18	9.29 214	63	9.30 064	66	10.69 936	9.99 150	3	42				
19	9.29 277	63	9.30 130	65	10.69 870	9.99 147	2	41				
20	9.29 340	63	9.30 195	66	10.69 805	9.99 145	3	40	20	21.7	21.3	21.0
21	9.29 403	63	9.30 261	65	10.69 739	9.99 142	2	39				
22	9.29 466	63	9.30 326	65	10.69 674	9.99 140	3	38				
23	9.29 529	62	9.30 391	66	10.69 609	9.99 137	2	37				
24	9.29 591	63	9.30 457	65	10.69 543	9.99 135	3	36				
25	9.29 654	62	9.30 522	65	10.69 478	9.99 132	2	35	//	62	61	60
26	9.29 716	63	9.30 587	65	10.69 413	9.99 130	3	34				
27	9.29 779	62	9.30 652	65	10.69 348	9.99 127	3	33				
28	9.29 841	62	9.30 717	65	10.69 283	9.99 124	2	32				
29	9.29 903	63	9.30 782	64	10.69 218	9.99 122	3	31				
30	9.29 966	62	9.30 846	65	10.69 154	9.99 119	2	30	5	5.2	5.1	5.0
31	9.30 028	62	9.30 911	64	10.69 089	9.99 117	3	29				
32	9.30 090	61	9.30 975	65	10.69 025	9.99 114	2	28				
33	9.30 151	62	9.31 040	64	10.68 960	9.99 112	3	27				
34	9.30 213	62	9.31 104	64	10.68 896	9.99 109	3	26				
35	9.30 275	61	9.31 168	65	10.68 832	9.99 106	2	25	10	10.3	10.2	10.0
36	9.30 336	62	9.31 233	64	10.68 767	9.99 104	3	24				
37	9.30 398	61	9.31 297	64	10.68 703	9.99 101	2	23				
38	9.30 459	62	9.31 361	64	10.68 639	9.99 099	3	22				
39	9.30 521	61	9.31 425	64	10.68 575	9.99 096	3	21				
40	9.30 582	61	9.31 489	63	10.68 511	9.99 093	2	20	//	59	3	2
41	9.30 643	61	9.31 552	64	10.68 448	9.99 091	3	19				
42	9.30 704	61	9.31 616	63	10.68 384	9.99 088	2	18				
43	9.30 765	61	9.31 679	64	10.68 321	9.99 086	3	17				
44	9.30 826	61	9.31 743	63	10.68 257	9.99 083	3	16				
45	9.30 887	60	9.31 806	64	10.68 194	9.99 080	2	15	5	4.9	0.2	0.2
46	9.30 947	61	9.31 870	63	10.68 130	9.99 078	3	14				
47	9.31 008	60	9.31 933	63	10.68 067	9.99 075	3	13				
48	9.31 068	61	9.31 996	63	10.68 004	9.99 072	2	12				
49	9.31 129	60	9.32 059	63	10.67 941	9.99 070	3	11				
50	9.31 189	61	9.32 122	63	10.67 878	9.99 067	3	10	10	9.8	0.5	0.3
51	9.31 250	60	9.32 185	63	10.67 815	9.99 064	2	9				
52	9.31 310	60	9.32 248	63	10.67 752	9.99 062	3	8				
53	9.31 370	60	9.32 311	62	10.67 689	9.99 059	3	7				
54	9.31 430	60	9.32 373	63	10.67 627	9.99 056	2	6				
55	9.31 490	59	9.32 436	62	10.67 564	9.99 054	3	5	50	49.2	2.5	1.7
56	9.31 549	60	9.32 498	63	10.67 502	9.99 051	3	4				
57	9.31 609	60	9.32 561	62	10.67 439	9.99 048	2	3				
58	9.31 669	59	9.32 623	62	10.67 377	9.99 046	3	2				
59	9.31 728	60	9.32 685	62	10.67 315	9.99 043	3	1				
60	9.31 788	—	9.32 747	—	10.67 253	9.99 040	—	0				
/	L Cos	d	L Ctn	c d	L Tan	L Sin	d	/	Proportional parts			

101° (281°)

(253°) 78°

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

(257°) 77°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

13° (193°)					(346°) 166°				
/	L Sin	d	L Tan	c d	L Ctn	L Cos	d	/	Proportional parts
0	9.35 209	54	9.36 336	58	10.63 664	9.98 872	3	60	
1	9.35 263	55	9.36 394	58	10.63 606	9.98 869	2	59	
2	9.35 318	55	9.36 452	57	10.63 548	9.98 867	3	58	
3	9.35 373	54	9.36 509	57	10.63 491	9.98 864	3	57	
4	9.35 427	54	9.36 566	58	10.63 434	9.98 861	3	56	
5	9.35 481	55	9.36 624	57	10.63 376	9.98 858	3	55	
6	9.35 536	54	9.36 681	57	10.63 319	9.98 855	3	54	
7	9.35 590	54	9.36 738	57	10.63 262	9.98 852	3	53	
8	9.35 644	54	9.36 795	57	10.63 205	9.98 849	3	52	
9	9.35 698	54	9.36 852	57	10.63 148	9.98 846	3	51	
10	9.35 752	54	9.36 909	57	10.63 091	9.98 843	3	50	
11	9.35 806	54	9.36 966	57	10.63 034	9.98 840	3	49	
12	9.35 860	54	9.37 023	57	10.62 977	9.98 837	3	48	
13	9.35 914	54	9.37 080	57	10.62 920	9.98 834	3	47	
14	9.35 968	54	9.37 137	56	10.62 863	9.98 831	3	46	
15	9.36 022	53	9.37 193	57	10.62 807	9.98 828	3	45	
16	9.36 075	54	9.37 250	56	10.62 750	9.98 825	3	44	
17	9.36 129	53	9.37 306	57	10.62 694	9.98 822	3	43	
18	9.36 182	54	9.37 363	56	10.62 637	9.98 819	3	42	
19	9.36 236	53	9.37 419	57	10.62 581	9.98 816	3	41	
20	9.36 289	53	9.37 476	56	10.62 524	9.98 813	3	40	
21	9.36 342	53	9.37 532	56	10.62 468	9.98 810	3	39	
22	9.36 395	54	9.37 588	56	10.62 412	9.98 807	3	38	
23	9.36 449	53	9.37 644	56	10.62 356	9.98 804	3	37	
24	9.36 502	53	9.37 700	56	10.62 300	9.98 801	3	36	
25	9.36 555	53	9.37 756	56	10.62 244	9.98 798	3	35	
26	9.36 608	52	9.37 812	56	10.62 188	9.98 795	3	34	
27	9.36 660	53	9.37 868	56	10.62 132	9.98 792	3	33	
28	9.36 713	53	9.37 924	56	10.62 076	9.98 789	3	32	
29	9.36 766	53	9.37 980	55	10.62 020	9.98 786	3	31	
30	9.36 819	52	9.38 035	56	10.61 965	9.98 783	3	30	
31	9.36 871	53	9.38 091	56	10.61 909	9.98 780	3	29	
32	9.36 924	52	9.38 147	55	10.61 853	9.98 777	3	28	
33	9.36 976	52	9.38 202	55	10.61 798	9.98 774	3	27	
34	9.37 028	53	9.38 257	56	10.61 743	9.98 771	3	26	
35	9.37 081	52	9.38 313	55	10.61 687	9.98 768	3	25	
36	9.37 133	52	9.38 368	55	10.61 632	9.98 765	3	24	
37	9.37 185	52	9.38 423	56	10.61 577	9.98 762	3	23	
38	9.37 237	52	9.38 479	55	10.61 521	9.98 759	3	22	
39	9.37 289	52	9.38 534	55	10.61 466	9.98 756	3	21	
40	9.37 341	52	9.38 589	55	10.61 411	9.98 753	3	20	
41	9.37 393	52	9.38 644	55	10.61 356	9.98 750	4	19	
42	9.37 445	52	9.38 699	55	10.61 301	9.98 746	3	18	
43	9.37 497	52	9.38 754	54	10.61 246	9.98 743	3	17	
44	9.37 549	51	9.38 808	55	10.61 192	9.98 740	3	16	
45	9.37 600	52	9.38 863	55	10.61 137	9.98 737	3	15	
46	9.37 652	51	9.38 918	54	10.61 082	9.98 734	3	14	
47	9.37 703	52	9.38 972	55	10.61 028	9.98 731	3	13	
48	9.37 755	51	9.39 027	55	10.60 973	9.98 728	3	12	
49	9.37 806	52	9.39 082	54	10.60 918	9.98 725	3	11	
50	9.37 858	51	9.39 136	54	10.60 864	9.98 722	3	10	
51	9.37 909	51	9.39 190	55	10.60 810	9.98 719	4	9	
52	9.37 960	51	9.39 245	54	10.60 755	9.98 715	3	8	
53	9.38 011	51	9.39 299	54	10.60 701	9.98 712	3	7	
54	9.38 062	51	9.39 353	54	10.60 647	9.98 709	3	6	
55	9.38 113	51	9.39 407	54	10.60 593	9.98 706	3	5	
56	9.38 164	51	9.39 461	54	10.60 539	9.98 703	3	4	
57	9.38 215	51	9.39 515	54	10.60 485	9.98 700	3	3	
58	9.38 266	51	9.39 569	54	10.60 431	9.98 697	3	2	
59	9.38 317	51	9.39 623	54	10.60 377	9.98 694	4	1	
60	9.38 368	—	9.39 677	—	10.60 323	9.98 690	—	0	
/	L Cos	d	L Ctn	c d	L Tan	L Sin	d	/	Proportional parts

103 (283°)

(256°) 76°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

14° (194°)					(345°) 165°				
/	L Sin	d	L Tan	c d	L Ctn	L Cos	d	/	Proportional parts
0	9.38 368	50	9.39 677	54	10.60 323	9.98 690	3	60	
1	9.38 418	51	9.39 731	54	10.60 269	9.98 687	3	59	
2	9.38 469	50	9.39 785	53	10.60 215	9.98 684	3	58	
3	9.38 519	51	9.39 838	54	10.60 162	9.98 681	3	57	
4	9.38 570	50	9.39 892	53	10.60 108	9.98 678	3	56	
5	9.38 620	50	9.39 945	54	10.60 055	9.98 675	4	55	
6	9.38 670	51	9.39 999	53	10.60 001	9.98 671	3	54	
7	9.38 721	50	9.40 052	54	10.59 948	9.98 668	3	53	
8	9.38 771	50	9.40 106	53	10.59 894	9.98 665	3	52	//
9	9.38 821	50	9.40 159	53	10.59 841	9.98 662	3	51	1 0.9 0.9 0.9
10	9.38 871	50	9.40 212	54	10.59 788	9.98 659	3	50	2 1.8 1.8 1.7
11	9.38 921	50	9.40 266	53	10.59 734	9.98 656	4	49	3 2.7 2.6 2.6
12	9.38 971	50	9.40 319	53	10.59 681	9.98 652	3	48	4 3.6 3.5 3.5
13	9.39 021	50	9.40 372	53	10.59 628	9.98 649	3	47	5 4.5 4.4 4.3
14	9.39 071	50	9.40 425	53	10.59 575	9.98 646	3	46	6 5.4 5.3 5.2
15	9.39 121	49	9.40 478	53	10.59 522	9.98 643	3	45	7 6.3 6.2 6.1
16	9.39 170	50	9.40 531	53	10.59 469	9.98 640	4	44	8 7.2 7.1 6.9
17	9.39 220	50	9.40 584	52	10.59 416	9.98 636	3	43	9 8.1 8.0 7.8
18	9.39 270	49	9.40 636	53	10.59 364	9.98 633	3	42	10 9.0 8.8 8.7
19	9.39 319	50	9.40 689	53	10.59 311	9.98 630	3	41	20 18.0 17.7 17.3
20	9.39 369	49	9.40 742	53	10.59 258	9.98 627	4	40	30 27.0 26.5 26.0
21	9.39 418	49	9.40 795	52	10.59 205	9.98 623	3	39	40 36.0 35.3 34.7
22	9.39 467	50	9.40 847	53	10.59 153	9.98 620	3	38	50 45.0 44.2 43.3
23	9.39 517	49	9.40 900	52	10.59 100	9.98 617	3	37	
24	9.39 566	49	9.40 952	53	10.59 048	9.98 614	4	36	//
25	9.39 615	49	9.41 005	52	10.58 995	9.98 610	3	35	1 0.8 0.8 0.8
26	9.39 664	49	9.41 057	52	10.58 943	9.98 607	3	34	2 1.7 1.7 1.6
27	9.39 713	49	9.41 109	52	10.58 891	9.98 604	3	33	3 2.6 2.5 2.4
28	9.39 762	49	9.41 161	53	10.58 839	9.98 601	4	32	4 3.4 3.3 3.3
29	9.39 811	49	9.41 214	52	10.58 786	9.98 597	3	31	
30	9.39 860	49	9.41 266	52	10.58 734	9.98 594	3	30	5 4.2 4.2 4.1
31	9.39 909	49	9.41 318	52	10.58 682	9.98 591	3	29	6 5.1 5.0 4.9
32	9.39 958	48	9.41 370	52	10.58 630	9.98 588	4	28	7 6.0 5.8 5.7
33	9.40 006	49	9.41 422	52	10.58 578	9.98 584	3	27	8 6.8 6.7 6.5
34	9.40 055	48	9.41 474	52	10.58 526	9.98 581	3	26	9 7.6 7.5 7.4
35	9.40 103	49	9.41 526	52	10.58 474	9.98 578	4	25	10 8.5 8.3 8.2
36	9.40 152	48	9.41 578	51	10.58 422	9.98 574	3	24	20 17.0 16.7 16.3
37	9.40 200	49	9.41 629	52	10.58 371	9.98 571	3	23	30 25.5 25.0 24.5
38	9.40 249	48	9.41 681	52	10.58 319	9.98 568	3	22	40 34.0 33.3 32.7
39	9.40 297	49	9.41 733	51	10.58 267	9.98 565	4	21	50 42.5 41.7 40.8
40	9.40 346	48	9.41 784	52	10.58 216	9.98 561	3	20	//
41	9.40 394	48	9.41 836	51	10.58 164	9.98 558	3	19	1 0.8 0.8 0.1 0.0
42	9.40 442	48	9.41 887	52	10.58 113	9.98 555	4	18	2 1.6 1.6 0.1 0.1
43	9.40 490	48	9.41 939	51	10.58 061	9.98 551	3	17	3 2.4 2.4 0.2 0.2
44	9.40 538	48	9.41 990	51	10.58 010	9.98 548	3	16	4 3.2 3.1 0.3 0.2
45	9.40 586	48	9.42 041	52	10.57 959	9.98 545	4	15	5 4.0 3.9 0.3 0.2
46	9.40 634	48	9.42 093	51	10.57 907	9.98 541	3	14	6 4.8 4.7 0.4 0.3
47	9.40 682	48	9.42 144	51	10.57 856	9.98 538	3	13	7 5.6 5.5 0.5 0.4
48	9.40 730	48	9.42 195	51	10.57 805	9.98 535	4	12	8 6.4 6.3 0.5 0.4
49	9.40 778	47	9.42 246	51	10.57 754	9.98 531	3	11	9 7.2 7.0 0.6 0.4
50	9.40 825	48	9.42 297	51	10.57 703	9.98 528	3	10	10 8.0 7.8 0.7 0.5
51	9.40 873	48	9.42 348	51	10.57 652	9.98 525	4	9	20 16.0 15.7 1.3 1.0
52	9.40 921	47	9.42 399	51	10.57 601	9.98 521	3	8	30 24.0 23.5 2.0 1.5
53	9.40 968	48	9.42 450	51	10.57 550	9.98 518	3	7	40 32.0 31.3 2.7 2.0
54	9.41 016	47	9.42 501	51	10.57 499	9.98 515	4	6	50 40.0 39.2 3.3 2.5
55	9.41 063	48	9.42 552	51	10.57 448	9.98 511	3	5	
56	9.41 111	47	9.42 603	50	10.57 397	9.98 508	3	4	
57	9.41 158	47	9.42 653	51	10.57 347	9.98 505	4	3	
58	9.41 205	47	9.42 704	51	10.57 296	9.98 501	3	2	
59	9.41 252	48	9.42 755	50	10.57 245	9.98 498	4	1	
60	9.41 300	—	9.42 805	—	10.57 195	9.98 494	—	0	
/	L Cos	d	L Ctn	c d	L Tan	L Sin	d	/	Proportional parts

104° (284°)

(255°) 75°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

15° (195°)					(344°) 164°				
/	L Sin	d	L Tan	c d	L Ctn	L Cos	d	/	Proportional parts
0	9.41 300	47	9.42 805	51	10.57 195	9.98 494	3	60	
1	9.41 347	47	9.42 856	50	10.57 144	9.98 491	3	59	
2	9.41 394	47	9.42 906	51	10.57 094	9.98 488	4	58	
3	9.41 441	47	9.42 957	50	10.57 043	9.98 484	3	57	
4	9.41 488	47	9.43 007	50	10.56 993	9.98 481	4	56	
5	9.41 535	47	9.43 057	51	10.56 943	9.98 477	3	55	
6	9.41 582	46	9.43 108	50	10.56 892	9.98 474	3	54	
7	9.41 628	47	9.43 158	50	10.56 842	9.98 471	4	53	// 51 50 49
8	9.41 675	47	9.43 208	50	10.56 792	9.98 467	3	52	1 0.8 0.8 0.8
9	9.41 722	46	9.43 258	50	10.56 742	9.98 464	4	51	2 1.7 1.7 1.6
10	9.41 768	47	9.43 308	50	10.56 692	9.98 460	3	50	3 2.6 2.5 2.4
11	9.41 815	46	9.43 358	50	10.56 642	9.98 457	4	49	4 3.4 3.3 3.3
12	9.41 861	47	9.43 408	50	10.56 592	9.98 453	3	48	5 4.2 4.2 4.1
13	9.41 908	46	9.43 458	50	10.56 542	9.98 450	3	47	6 5.1 5.0 4.9
14	9.41 954	47	9.43 508	50	10.56 492	9.98 447	4	46	7 6.0 5.8 5.7
15	9.42 001	46	9.43 558	49	10.56 442	9.98 443	3	45	8 6.8 6.7 6.5
16	9.42 047	46	9.43 607	50	10.56 393	9.98 440	4	44	9 7.6 7.5 7.4
17	9.42 093	47	9.43 657	50	10.56 343	9.98 436	3	43	10 8.5 8.3 8.2
18	9.42 140	46	9.43 707	49	10.56 293	9.98 433	4	42	20 17.0 16.7 16.3
19	9.42 186	46	9.43 756	50	10.56 244	9.98 429	3	41	30 25.5 25.0 24.5
20	9.42 232	46	9.43 806	49	10.56 194	9.98 426	4	40	40 34.0 33.3 32.7
21	9.42 278	46	9.43 855	50	10.56 145	9.98 422	3	39	50 42.5 41.7 40.8
22	9.42 324	46	9.43 905	49	10.56 095	9.98 419	4	38	// 48 47 46
23	9.42 370	46	9.43 954	50	10.56 046	9.98 415	3	37	1 0.8 0.8 0.8
24	9.42 416	45	9.44 004	49	10.55 996	9.98 412	3	36	2 1.6 1.6 1.5
25	9.42 461	46	9.44 053	49	10.55 947	9.98 409	4	35	3 2.4 2.4 2.3
26	9.42 507	46	9.44 102	49	10.55 898	9.98 405	3	34	4 3.2 3.1 3.1
27	9.42 553	46	9.44 151	50	10.55 849	9.98 402	4	33	5 4.0 3.9 3.8
28	9.42 599	45	9.44 201	49	10.55 799	9.98 398	3	32	6 4.8 4.7 4.6
29	9.42 644	46	9.44 250	49	10.55 750	9.98 395	4	31	7 5.6 5.5 5.4
30	9.42 690	45	9.44 299	49	10.55 701	9.98 391	3	30	8 6.4 6.3 6.1
31	9.42 735	46	9.44 348	49	10.55 652	9.98 388	4	29	9 7.2 7.0 6.9
32	9.42 781	45	9.44 397	49	10.55 603	9.98 384	3	28	10 8.0 7.8 7.7
33	9.42 826	46	9.44 446	49	10.55 554	9.98 381	4	27	20 16.0 15.7 15.3
34	9.42 872	45	9.44 495	49	10.55 505	9.98 377	4	26	30 24.0 23.5 23.0
35	9.42 917	45	9.44 544	48	10.55 456	9.98 373	3	25	40 32.0 31.3 30.7
36	9.42 962	46	9.44 592	49	10.55 408	9.98 370	4	24	50 40.0 39.2 38.3
37	9.43 008	45	9.44 641	49	10.55 359	9.98 366	3	23	// 45 44 4 3
38	9.43 053	45	9.44 690	48	10.55 310	9.98 363	4	22	1 0.8 0.7 0.1 0.0
39	9.43 098	45	9.44 738	49	10.55 262	9.98 359	3	21	2 1.5 1.5 0.1 0.1
40	9.43 143	45	9.44 787	49	10.55 213	9.98 356	4	20	3 2.2 2.2 0.2 0.2
41	9.43 188	45	9.44 836	48	10.55 164	9.98 352	3	19	4 3.0 2.9 0.3 0.2
42	9.43 233	45	9.44 884	49	10.55 116	9.98 349	4	18	5 3.8 3.7 0.3 0.2
43	9.43 278	45	9.44 933	48	10.55 067	9.98 345	3	17	6 4.5 4.4 0.4 0.3
44	9.43 323	44	9.44 981	48	10.55 019	9.98 342	4	16	7 5.2 5.1 0.5 0.4
45	9.43 367	45	9.45 029	49	10.54 971	9.98 338	4	15	8 6.0 5.9 0.5 0.4
46	9.43 412	45	9.45 078	48	10.54 922	9.98 334	3	14	9 6.8 6.6 0.6 0.4
47	9.43 457	45	9.45 126	48	10.54 874	9.98 331	4	13	10 7.5 7.3 0.7 0.5
48	9.43 502	44	9.45 174	48	10.54 826	9.98 327	3	12	20 15.0 14.7 1.3 1.0
49	9.43 546	45	9.45 222	49	10.54 778	9.98 324	4	11	30 22.5 22.0 2.0 1.5
50	9.43 591	44	9.45 271	48	10.54 729	9.98 320	3	10	40 30.0 29.3 2.7 2.0
51	9.43 635	45	9.45 319	48	10.54 681	9.98 317	4	9	50 37.5 36.7 3.3 2.5
52	9.43 680	44	9.45 367	48	10.54 633	9.98 313	4	8	
53	9.43 724	45	9.45 415	48	10.54 585	9.98 309	3	7	
54	9.43 769	44	9.45 463	48	10.54 537	9.98 306	4	6	
55	9.43 813	44	9.45 511	48	10.54 489	9.98 302	3	5	
56	9.43 857	44	9.45 559	47	10.54 441	9.98 299	4	4	
57	9.43 901	45	9.45 606	48	10.54 394	9.98 295	4	3	
58	9.43 946	44	9.45 654	48	10.54 346	9.98 291	3	2	
59	9.43 990	44	9.45 702	48	10.54 298	9.98 288	4	1	
60	9.44 034	—	9.45 750	—	10.54 250	9.98 284	—	0	
/	L Cos	d	L Ctn	c d	L Tan	L Sin	d	/	Proportional parts

105° (285°)

(254°) 74°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

16° (196°)						(343°) 163°							
/	L Sin	d	L Tan	c d	L Ctn	L Cos	d	/	Proportional parts				
0	9.44 034	44	9.45 750	47	10.54 250	9.98 284	3	60					
1	9.44 078	44	9.45 797	48	10.54 203	9.98 281	4	59					
2	9.44 122	44	9.45 845	47	10.54 155	9.98 277	4	58					
3	9.44 166	44	9.45 892	48	10.54 108	9.98 273	3	57					
4	9.44 210	43	9.45 940	47	10.54 060	9.98 270	4	56					
5	9.44 253	44	9.45 987	48	10.54 013	9.98 266	4	55					
6	9.44 297	44	9.46 035	47	10.53 965	9.98 262	3	54					
7	9.44 341	44	9.46 082	48	10.53 918	9.98 259	4	53	//	48	47	46	
8	9.44 385	43	9.46 130	47	10.53 870	9.98 255	4	52	1	0.8	0.8	0.8	
9	9.44 428	44	9.46 177	47	10.53 823	9.98 251	3	51	2	1.6	1.6	1.5	
10	9.44 472	44	9.46 224	47	10.53 776	9.98 248	4	50	3	2.4	2.4	2.3	
11	9.44 516	43	9.46 271	48	10.53 729	9.98 244	4	49	4	3.2	3.1	3.1	
12	9.44 559	43	9.46 319	47	10.53 681	9.98 240	3	48	5	4.0	3.9	3.8	
13	9.44 602	44	9.46 366	47	10.53 634	9.98 237	4	47	6	4.8	4.7	4.6	
14	9.44 646	43	9.46 413	47	10.53 587	9.98 233	4	46	7	5.6	5.5	5.4	
15	9.44 689	44	9.46 460	47	10.53 540	9.98 229	3	45	8	6.4	6.3	6.1	
16	9.44 733	43	9.46 507	47	10.53 493	9.98 226	4	44	9	7.2	7.0	6.9	
17	9.44 776	43	9.46 554	47	10.53 446	9.98 222	4	43	10	8.0	7.8	7.7	
18	9.44 819	43	9.46 601	47	10.53 399	9.98 218	3	42	20	16.0	15.7	15.3	
19	9.44 862	43	9.46 648	46	10.53 352	9.98 215	4	41	30	24.0	23.5	23.0	
20	9.44 905	43	9.46 694	47	10.53 306	9.98 211	4	40	40	32.0	31.3	30.7	
21	9.44 948	44	9.46 741	47	10.53 259	9.98 207	3	39	50	40.0	39.2	38.3	
22	9.44 992	43	9.46 788	47	10.53 212	9.98 204	4	38	//	45	44	43	
23	9.45 035	42	9.46 835	46	10.53 165	9.98 200	4	37	1	0.8	0.7	0.7	
24	9.45 077	43	9.46 881	47	10.53 119	9.98 196	4	36	2	1.5	1.5	1.4	
25	9.45 120	43	9.46 928	47	10.53 072	9.98 192	3	35	3	2.2	2.2	2.2	
26	9.45 163	43	9.46 975	46	10.53 025	9.98 189	4	34	4	3.0	2.9	2.9	
27	9.45 206	43	9.47 021	47	10.52 979	9.98 185	4	33	5	3.8	3.7	3.6	
28	9.45 249	43	9.47 068	46	10.52 932	9.98 181	4	32	6	4.5	4.4	4.3	
29	9.45 292	42	9.47 114	46	10.52 886	9.98 177	3	31	7	5.2	5.1	5.0	
30	9.45 334	43	9.47 160	47	10.52 840	9.98 174	4	30	8	6.0	5.9	5.7	
31	9.45 377	42	9.47 207	46	10.52 793	9.98 170	4	29	9	6.8	6.6	6.4	
32	9.45 419	43	9.47 253	46	10.52 747	9.98 166	4	28	10	7.5	7.3	7.2	
33	9.45 462	42	9.47 299	47	10.52 701	9.98 162	3	27	20	15.0	14.7	14.3	
34	9.45 504	43	9.47 346	46	10.52 654	9.98 159	4	26	30	22.5	22.0	21.5	
35	9.45 547	42	9.47 392	46	10.52 608	9.98 155	4	25	40	30.0	29.3	28.7	
36	9.45 589	43	9.47 438	46	10.52 562	9.98 151	4	24	50	37.5	36.7	35.8	
37	9.45 632	42	9.47 484	46	10.52 516	9.98 147	3	23	//	42	41	4	3
38	9.45 674	42	9.47 530	46	10.52 470	9.98 144	4	22	1	0.7	0.7	0.1	0.1
39	9.45 716	42	9.47 576	46	10.52 424	9.98 140	4	21	2	1.4	1.4	0.1	0.1
40	9.45 758	43	9.47 622	46	10.52 378	9.98 136	4	20	3	2.1	2.0	0.2	0.2
41	9.45 801	42	9.47 668	46	10.52 332	9.98 132	3	19	4	2.8	2.7	0.3	0.2
42	9.45 843	42	9.47 714	46	10.52 286	9.98 129	4	18	5	3.5	3.4	0.3	0.2
43	9.45 885	42	9.47 760	46	10.52 240	9.98 125	4	17	6	4.2	4.1	0.4	0.3
44	9.45 927	42	9.47 806	46	10.52 194	9.98 121	4	16	7	4.9	4.8	0.5	0.4
45	9.45 969	42	9.47 852	45	10.52 148	9.98 117	4	15	8	5.6	5.5	0.5	0.4
46	9.46 011	42	9.47 897	46	10.52 103	9.98 113	3	14	9	6.3	6.2	0.6	0.4
47	9.46 053	42	9.47 943	46	10.52 057	9.98 110	4	13	10	7.0	6.8	0.7	0.5
48	9.46 095	41	9.47 989	46	10.52 011	9.98 106	4	12	20	14.0	13.7	1.3	1.0
49	9.46 136	42	9.48 035	45	10.51 965	9.98 102	4	11	30	21.0	20.5	2.0	1.5
50	9.46 178	42	9.48 080	46	10.51 920	9.98 098	4	10	40	28.0	27.3	2.7	2.0
51	9.46 220	42	9.48 126	45	10.51 874	9.98 094	4	9	50	35.0	34.2	3.3	2.5
52	9.46 262	41	9.48 171	46	10.51 829	9.98 090	3	8					
53	9.46 303	42	9.48 217	45	10.51 783	9.98 087	4	7					
54	9.46 345	41	9.48 262	45	10.51 738	9.98 083	4	6					
55	9.46 386	42	9.48 307	46	10.51 693	9.98 079	4	5					
56	9.46 428	41	9.48 353	45	10.51 647	9.98 075	4	4					
57	9.46 469	42	9.48 398	45	10.51 602	9.98 071	4	3					
58	9.46 511	41	9.48 443	46	10.51 557	9.98 067	4	2					
59	9.46 552	42	9.48 489	45	10.51 511	9.98 063	3	1					
60	9.46 594		9.48 534		10.51 466	9.98 060		0					
/	L Cos	d	L Ctn	c d	L Tan	L Sin	d	/	Proportional parts				

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

17° (197°)					(342°) 162°				
°	L Sin	d	L Tan	c d	L Ctn	L Cos	d	°	Proportional parts
0	9.46 594	41	9.48 534	45	10.51 466	9.98 060	4	60	
1	9.46 635	41	9.48 579	45	10.51 421	9.98 056	4	59	
2	9.46 676	41	9.48 624	45	10.51 376	9.98 052	4	58	
3	9.46 717	41	9.48 669	45	10.51 331	9.98 048	4	57	
4	9.46 758	42	9.48 714	45	10.51 286	9.98 044	4	56	
5	9.46 800	41	9.48 759	45	10.51 241	9.98 040	4	55	
6	9.46 841	41	9.48 804	45	10.51 196	9.98 036	4	54	
7	9.46 882	41	9.48 849	45	10.51 151	9.98 032	3	53	
8	9.46 923	41	9.48 894	45	10.51 106	9.98 029	4	52	
9	9.46 964	41	9.48 939	45	10.51 061	9.98 025	4	51	
10	9.47 005	40	9.48 984	45	10.51 016	9.98 021	4	50	
11	9.47 045	41	9.49 029	44	10.50 971	9.98 017	4	49	
12	9.47 086	41	9.49 073	45	10.50 927	9.98 013	4	48	
13	9.47 127	41	9.49 118	45	10.50 882	9.98 009	4	47	
14	9.47 168	41	9.49 163	44	10.50 837	9.98 005	4	46	
15	9.47 209	40	9.49 207	45	10.50 793	9.98 001	4	45	
16	9.47 249	41	9.49 252	44	10.50 748	9.97 997	4	44	
17	9.47 290	40	9.49 296	45	10.50 704	9.97 993	4	43	
18	9.47 330	41	9.49 341	44	10.50 659	9.97 989	3	42	
19	9.47 371	40	9.49 385	45	10.50 615	9.97 986	4	41	
20	9.47 411	41	9.49 430	44	10.50 570	9.97 982	4	40	
21	9.47 452	40	9.49 474	45	10.50 526	9.97 978	4	39	
22	9.47 492	41	9.49 519	44	10.50 481	9.97 974	4	38	
23	9.47 533	40	9.49 563	44	10.50 437	9.97 970	4	37	
24	9.47 573	40	9.49 607	45	10.50 393	9.97 966	4	36	
25	9.47 613	41	9.49 652	44	10.50 348	9.97 962	4	35	
26	9.47 654	40	9.49 696	44	10.50 304	9.97 958	4	34	
27	9.47 694	40	9.49 740	44	10.50 260	9.97 954	4	33	
28	9.47 734	40	9.49 784	44	10.50 216	9.97 950	4	32	
29	9.47 774	40	9.49 828	44	10.50 172	9.97 946	4	31	
30	9.47 814	40	9.49 872	44	10.50 128	9.97 942	4	30	
31	9.47 854	40	9.49 916	44	10.50 084	9.97 938	4	29	
32	9.47 894	40	9.49 960	44	10.50 040	9.97 934	4	28	
33	9.47 934	40	9.50 004	44	10.49 996	9.97 930	4	27	
34	9.47 974	40	9.50 048	44	10.49 952	9.97 926	4	26	
35	9.48 014	40	9.50 092	44	10.49 908	9.97 922	4	25	
36	9.48 054	40	9.50 136	44	10.49 864	9.97 918	4	24	
37	9.48 094	39	9.50 180	43	10.49 820	9.97 914	4	23	
38	9.48 133	40	9.50 223	44	10.49 777	9.97 910	4	22	
39	9.48 173	40	9.50 267	44	10.49 733	9.97 906	4	21	
40	9.48 213	39	9.50 311	44	10.49 689	9.97 902	4	20	
41	9.48 252	40	9.50 355	43	10.49 645	9.97 898	4	19	
42	9.48 292	40	9.50 398	44	10.49 602	9.97 894	4	18	
43	9.48 332	39	9.50 442	43	10.49 558	9.97 890	4	17	
44	9.48 371	40	9.50 485	44	10.49 515	9.97 886	4	16	
45	9.48 411	39	9.50 529	43	10.49 471	9.97 882	4	15	
46	9.48 450	40	9.50 572	44	10.49 428	9.97 878	4	14	
47	9.48 490	39	9.50 616	43	10.49 384	9.97 874	4	13	
48	9.48 529	39	9.50 659	44	10.49 341	9.97 870	4	12	
49	9.48 568	39	9.50 703	43	10.49 297	9.97 866	5	11	
50	9.48 607	40	9.50 746	43	10.49 254	9.97 861	4	10	
51	9.48 647	39	9.50 789	44	10.49 211	9.97 857	4	9	
52	9.48 686	39	9.50 833	43	10.49 167	9.97 853	4	8	
53	9.48 725	39	9.50 876	43	10.49 124	9.97 849	4	7	
54	9.48 764	39	9.50 919	43	10.49 081	9.97 845	4	6	
55	9.48 803	39	9.50 962	43	10.49 038	9.97 841	4	5	
56	9.48 842	39	9.51 005	43	10.48 995	9.97 837	4	4	
57	9.48 881	39	9.51 048	44	10.48 952	9.97 833	4	3	
58	9.48 920	39	9.51 092	43	10.48 908	9.97 829	4	2	
59	9.48 959	39	9.51 135	43	10.48 865	9.97 825	4	1	
60	9.48 998	—	9.51 178	—	10.48 822	9.97 821	—	0	
°	L Cos	d	L Ctn	c d	L Tan	L Sin	d	°	Proportional parts

107° (287°)

(252°) 72°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

18° (198°)						(341°) 161°																		
/	L Sin	d	L Tan	c d	L Ctn	L Cos	d	/	Proportional parts															
0	9.48 998	39	9.51 178	43	10.48 822	9.97 821	4	60		43	42	41												
1	9.49 037	39	9.51 221	43	10.48 779	9.97 817	6	59																
2	9.49 076	39	9.51 264	42	10.48 736	9.97 812	4	58																
3	9.49 115	38	9.51 306	43	10.48 694	9.97 808	4	57																
4	9.49 153	39	9.51 349	43	10.48 651	9.97 804	4	56																
5	9.49 192	39	9.51 392	43	10.48 608	9.97 800	4	55	11	1	0.7	0.7												
6	9.49 231	38	9.51 435	43	10.48 565	9.97 796	4	54					2	1.4	1.4	1.4								
7	9.49 269	39	9.51 478	42	10.48 522	9.97 792	4	53									3	2.2	2.1	2.0				
8	9.49 308	39	9.51 520	43	10.48 480	9.97 788	4	52													4	2.9	2.8	2.7
9	9.49 347	38	9.51 563	43	10.48 437	9.97 784	5	51																
10	9.49 385	39	9.51 606	42	10.48 394	9.97 779	4	50	6	4.3	4.2	4.1												
11	9.49 424	38	9.51 648	43	10.48 352	9.97 775	4	49					7	5.0	4.9	4.8								
12	9.49 462	38	9.51 691	43	10.48 309	9.97 771	4	48									8	5.7	5.6	5.5				
13	9.49 500	39	9.51 734	42	10.48 266	9.97 767	4	47													9	6.4	6.3	6.2
14	9.49 539	38	9.51 776	43	10.48 224	9.97 763	4	46																
15	9.49 577	38	9.51 819	42	10.48 181	9.97 759	5	45	20	14.3	14.0	13.7												
16	9.49 615	39	9.51 861	42	10.48 139	9.97 754	4	44					30	21.5	21.0	20.5								
17	9.49 654	38	9.51 903	43	10.48 097	9.97 750	4	43									40	28.7	28.0	27.3				
18	9.49 692	38	9.51 946	42	10.48 054	9.97 746	4	42													50	35.8	35.0	34.2
19	9.49 730	38	9.51 988	43	10.48 012	9.97 742	4	41																
20	9.49 768	38	9.52 031	42	10.47 969	9.97 738	4	40	2	1.3	1.3	1.2												
21	9.49 806	38	9.52 073	42	10.47 927	9.97 734	5	39					3	2.0	1.9	1.8								
22	9.49 844	38	9.52 115	42	10.47 885	9.97 729	4	38									4	2.6	2.5	2.5				
23	9.49 882	38	9.52 157	43	10.47 843	9.97 725	4	37													5	3.2	3.2	3.1
24	9.49 920	38	9.52 200	42	10.47 800	9.97 721	4	36																
25	9.49 958	38	9.52 242	42	10.47 758	9.97 717	4	35	7	4.6	4.4	4.3												
26	9.49 996	38	9.52 284	42	10.47 716	9.97 713	5	34					8	5.2	5.1	4.9								
27	9.50 034	38	9.52 326	42	10.47 674	9.97 708	4	33									9	5.8	5.7	5.6				
28	9.50 072	38	9.52 368	42	10.47 632	9.97 704	4	32													10	6.5	6.3	6.2
29	9.50 110	38	9.52 410	42	10.47 590	9.97 700	4	31																
30	9.50 148	37	9.52 452	42	10.47 548	9.97 696	5	30	30	19.5	19.0	18.5												
31	9.50 185	33	9.52 494	42	10.47 506	9.97 691	4	29					40	26.0	25.3	24.7								
32	9.50 223	38	9.52 536	42	10.47 464	9.97 687	4	28									50	32.5	31.7	30.8				
33	9.50 261	37	9.52 578	42	10.47 422	9.97 683	4	27													1	0.6	0.1	0.1
34	9.50 298	38	9.52 620	41	10.47 380	9.97 679	5	26																
35	9.50 336	38	9.52 661	42	10.47 339	9.97 674	4	25	3	1.8	0.3	0.3												
36	9.50 374	37	9.52 703	42	10.47 297	9.97 670	4	24					4	2.4	0.3	0.3								
37	9.50 411	38	9.52 745	42	10.47 255	9.97 666	4	23									5	3.0	0.4	0.3				
38	9.50 449	37	9.52 787	42	10.47 213	9.97 662	5	22													6	3.6	0.5	0.4
39	9.50 486	37	9.52 829	41	10.47 171	9.97 657	4	21																
40	9.50 523	38	9.52 870	42	10.47 130	9.97 653	4	20	8	4.8	0.7	0.5												
41	9.50 561	37	9.52 912	41	10.47 088	9.97 649	4	19					9	5.4	0.8	0.6								
42	9.50 598	37	9.52 953	42	10.47 047	9.97 645	5	18									10	6.0	0.8	0.7				
43	9.50 635	38	9.52 995	42	10.47 005	9.97 640	4	17													20	12.0	1.7	1.3
44	9.50 673	37	9.53 037	41	10.46 963	9.97 636	4	16																
45	9.50 710	37	9.53 078	42	10.46 922	9.97 632	4	15	40	24.0	3.3	2.7												
46	9.50 747	37	9.53 120	41	10.46 880	9.97 628	5	14					50	30.0	4.2	3.3								
47	9.50 784	37	9.53 161	41	10.46 839	9.97 623	4	13									1	0.6	0.1	0.1				
48	9.50 821	37	9.53 202	42	10.46 798	9.97 619	4	12													2	1.2	0.2	0.2
49	9.50 858	38	9.53 244	41	10.46 756	9.97 615	5	11																
50	9.50 896	37	9.53 285	42	10.46 715	9.97 610	4	10	4	2.4	0.3	0.3												
51	9.50 933	37	9.53 327	41	10.46 673	9.97 606	4	9					5	3.0	0.4	0.3								
52	9.50 970	37	9.53 368	41	10.46 632	9.97 602	5	8									6	3.6	0.5	0.4				
53	9.51 007	36	9.53 409	41	10.46 591	9.97 597	4	7													7	4.2	0.6	0.5
54	9.51 043	37	9.53 450	42	10.46 550	9.97 593	4	6																
55	9.51 080	37	9.53 492	41	10.46 508	9.97 589	5	5	9	5.4	0.8	0.6												
56	9.51 117	37	9.53 533	41	10.46 467	9.97 584	4	4					10	6.0	0.8	0.7								
57	9.51 154	37	9.53 574	41	10.46 426	9.97 580	4	3									20	12.0	1.7	1.3				
58	9.51 191	36	9.53 615	41	10.46 385	9.97 576	5	2													30	18.0	2.5	2.0
59	9.51 227	37	9.53 656	41	10.46 344	9.97 571	4	1																
60	9.51 264	—	9.53 697	—	10.46 303	9.97 567	—	0	50	30.0	4.2	3.3												
/	L Cos	d	L Ctn	c d	L Tan	L Sin	d	/					Proportional parts											
108° (288°)						(251°) 71°																		

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

19° (199°)					(340°) 160°				
/	L Sin	d	L Tan	c d	L Ctn	L Cos	d	/	Proportional parts
0	9.51 264	37	9.53 697	41	10.46 303	9.97 567	4	60	
1	9.51 301	37	9.53 738	41	10.46 262	9.97 563	5	59	
2	9.51 338	36	9.53 779	41	10.46 221	9.97 558	4	58	
3	9.51 374	37	9.53 820	41	10.46 180	9.97 554	4	57	
4	9.51 411	36	9.53 861	41	10.46 139	9.97 550	5	56	
5	9.51 447	37	9.53 902	41	10.46 098	9.97 545	4	55	
6	9.51 484	36	9.53 943	41	10.46 057	9.97 541	5	54	
7	9.51 520	37	9.53 984	41	10.46 016	9.97 536	4	53	
8	9.51 557	36	9.54 025	40	10.45 975	9.97 532	4	52	
9	9.51 593	36	9.54 065	41	10.45 935	9.97 528	5	51	
10	9.51 629	37	9.54 106	41	10.45 894	9.97 523	4	50	
11	9.51 666	36	9.54 147	40	10.45 853	9.97 519	4	49	
12	9.51 702	36	9.54 187	41	10.45 813	9.97 515	5	48	
13	9.51 738	36	9.54 228	41	10.45 772	9.97 510	4	47	
14	9.51 774	37	9.54 269	40	10.45 731	9.97 506	5	46	
15	9.51 811	36	9.54 309	41	10.45 691	9.97 501	4	45	
16	9.51 847	36	9.54 350	40	10.45 650	9.97 497	5	44	
17	9.51 883	36	9.54 390	41	10.45 610	9.97 492	4	43	
18	9.51 919	36	9.54 431	40	10.45 569	9.97 488	4	42	
19	9.51 955	36	9.54 471	41	10.45 529	9.97 484	5	41	
20	9.51 991	36	9.54 512	40	10.45 488	9.97 479	4	40	
21	9.52 027	36	9.54 552	41	10.45 448	9.97 475	5	39	
22	9.52 063	36	9.54 593	40	10.45 407	9.97 470	4	38	
23	9.52 099	36	9.54 633	40	10.45 367	9.97 466	5	37	
24	9.52 135	36	9.54 673	41	10.45 327	9.97 461	4	36	
25	9.52 171	36	9.54 714	40	10.45 286	9.97 457	4	35	
26	9.52 207	35	9.54 754	40	10.45 246	9.97 453	5	34	
27	9.52 242	36	9.54 794	41	10.45 206	9.97 448	4	33	
28	9.52 278	36	9.54 835	40	10.45 165	9.97 444	5	32	
29	9.52 314	36	9.54 875	40	10.45 125	9.97 439	4	31	
30	9.52 350	35	9.54 915	40	10.45 085	9.97 435	5	30	
31	9.52 385	36	9.54 955	40	10.45 045	9.97 430	4	29	
32	9.52 421	35	9.54 995	40	10.45 005	9.97 426	5	28	
33	9.52 456	36	9.55 035	40	10.44 965	9.97 421	4	27	
34	9.52 492	35	9.55 075	40	10.44 925	9.97 417	5	26	
35	9.52 527	36	9.55 115	40	10.44 885	9.97 412	4	25	
36	9.52 563	35	9.55 155	40	10.44 845	9.97 408	5	24	
37	9.52 598	36	9.55 195	40	10.44 805	9.97 403	4	23	
38	9.52 634	35	9.55 235	40	10.44 765	9.97 399	5	22	
39	9.52 669	36	9.55 275	40	10.44 725	9.97 394	4	21	
40	9.52 705	35	9.55 315	40	10.44 685	9.97 390	5	20	
41	9.52 740	35	9.55 355	40	10.44 645	9.97 385	4	19	
42	9.52 775	36	9.55 395	39	10.44 605	9.97 381	5	18	
43	9.52 811	35	9.55 434	40	10.44 566	9.97 376	4	17	
44	9.52 846	35	9.55 474	40	10.44 526	9.97 372	5	16	
45	9.52 881	35	9.55 514	40	10.44 486	9.97 367	4	15	
46	9.52 916	35	9.55 554	39	10.44 446	9.97 363	5	14	
47	9.52 951	35	9.55 593	40	10.44 407	9.97 358	4	13	
48	9.52 986	35	9.55 633	40	10.44 367	9.97 353	4	12	
49	9.53 021	35	9.55 673	39	10.44 327	9.97 349	5	11	
50	9.53 056	36	9.55 712	40	10.44 288	9.97 344	4	10	
51	9.53 092	34	9.55 752	39	10.44 248	9.97 340	5	9	
52	9.53 126	35	9.55 791	40	10.44 209	9.97 335	4	8	
53	9.53 161	35	9.55 831	39	10.44 169	9.97 331	5	7	
54	9.53 196	35	9.55 870	40	10.44 130	9.97 326	4	6	
55	9.53 231	35	9.55 910	39	10.44 090	9.97 322	5	5	
56	9.53 266	35	9.55 949	40	10.44 051	9.97 317	5	4	
57	9.53 301	35	9.55 989	39	10.44 011	9.97 312	4	3	
58	9.53 336	34	9.56 028	39	10.43 972	9.97 308	5	2	
59	9.53 370	35	9.56 067	40	10.43 933	9.97 303	4	1	
60	9.53 405		9.56 107		10.43 893	9.97 299		0	
/	L Cos	d	L Ctn	c d	L Tan	L Sin	d	/	Proportional parts

109° (289°)

(250°) 70°

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

(249°) 69°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

21° (201°)

(338°) 158°

/	L Sin d		L Tan c d		L Ctn		L Cos d		/	Proportional parts			
0	9.55	433	33	9.58	418	37	10.41	582	9.97	015	5	60	
1	9.55	466	33	9.58	455	38	10.41	545	9.97	010	5	59	
2	9.55	499	33	9.58	493	38	10.41	507	9.97	005	4	58	
3	9.55	532	32	9.58	531	38	10.41	469	9.97	001	5	57	
4	9.55	564	33	9.58	569	37	10.41	431	9.96	996	5	56	
5	9.55	597	33	9.58	606	38	10.41	394	9.96	991	5	55	
6	9.55	630	33	9.58	644	37	10.41	356	9.96	986	5	54	
7	9.55	663	32	9.58	681	38	10.41	319	9.96	981	5	53	
8	9.55	695	33	9.58	719	38	10.41	281	9.96	976	5	52	
9	9.55	728	33	9.58	757	37	10.41	243	9.96	971	5	51	
10	9.55	761	32	9.58	794	38	10.41	206	9.96	966	4	50	
11	9.55	793	33	9.58	832	37	10.41	168	9.96	962	5	49	
12	9.55	826	32	9.58	869	38	10.41	131	9.96	957	5	48	
13	9.55	858	33	9.58	907	37	10.41	093	9.96	952	5	47	
14	9.55	891	32	9.58	944	37	10.41	056	9.96	947	5	46	
15	9.55	923	33	9.58	981	38	10.41	019	9.96	942	5	45	
16	9.55	956	32	9.59	019	37	10.40	981	9.96	937	5	44	
17	9.55	988	33	9.59	056	38	10.40	944	9.96	932	5	43	
18	9.56	021	32	9.59	094	37	10.40	906	9.96	927	5	42	
19	9.56	053	32	9.59	131	37	10.40	869	9.96	922	5	41	
20	9.56	085	33	9.59	168	37	10.40	832	9.96	917	5	40	
21	9.56	118	32	9.59	205	38	10.40	795	9.96	912	5	39	
22	9.56	150	32	9.59	243	37	10.40	757	9.96	907	4	38	
23	9.56	182	33	9.59	280	37	10.40	720	9.96	903	5	37	
24	9.56	215	32	9.59	317	37	10.40	683	9.96	898	5	36	
25	9.56	247	32	9.59	354	37	10.40	646	9.96	893	5	35	
26	9.56	279	32	9.59	391	38	10.40	609	9.96	888	5	34	
27	9.56	311	32	9.59	429	37	10.40	571	9.96	883	5	33	
28	9.56	343	32	9.59	466	37	10.40	534	9.96	878	5	32	
29	9.56	375	33	9.59	503	37	10.40	497	9.96	873	5	31	
30	9.56	408	32	9.59	540	37	10.40	460	9.96	868	5	30	
31	9.56	440	32	9.59	577	37	10.40	423	9.96	863	5	29	
32	9.56	472	32	9.59	614	37	10.40	386	9.96	858	5	28	
33	9.56	504	32	9.59	651	37	10.40	349	9.96	853	5	27	
34	9.56	536	32	9.59	688	37	10.40	312	9.96	848	5	26	
35	9.56	568	31	9.59	725	37	10.40	275	9.96	843	5	25	
36	9.56	599	32	9.59	762	37	10.40	238	9.96	838	5	24	
37	9.56	631	32	9.59	799	36	10.40	201	9.96	833	5	23	
38	9.56	663	32	9.59	835	37	10.40	165	9.96	828	5	22	
39	9.56	695	32	9.59	872	37	10.40	128	9.96	823	5	21	
40	9.56	727	32	9.59	909	37	10.40	091	9.96	818	5	20	
41	9.56	759	31	9.59	946	37	10.40	054	9.96	813	5	19	
42	9.56	790	32	9.59	983	36	10.40	017	9.96	808	5	18	
43	9.56	822	32	9.60	019	37	10.39	981	9.96	803	5	17	
44	9.56	854	32	9.60	056	37	10.39	944	9.96	798	5	16	
45	9.56	886	31	9.60	093	37	10.39	907	9.96	793	5	15	
46	9.56	917	32	9.60	130	36	10.39	870	9.96	788	5	14	
47	9.56	949	31	9.60	166	37	10.39	834	9.96	783	5	13	
48	9.56	980	32	9.60	203	37	10.39	797	9.96	778	5	12	
49	9.57	012	32	9.60	240	36	10.39	760	9.96	772	5	11	
50	9.57	044	31	9.60	276	37	10.39	724	9.96	767	5	10	
51	9.57	075	32	9.60	313	36	10.39	687	9.96	762	5	9	
52	9.57	107	31	9.60	349	37	10.39	651	9.96	757	5	8	
53	9.57	138	31	9.60	386	36	10.39	614	9.96	752	5	7	
54	9.57	169	32	9.60	422	37	10.39	578	9.96	747	5	6	
55	9.57	201	31	9.60	459	36	10.39	541	9.96	742	5	5	
56	9.57	232	32	9.60	495	37	10.39	505	9.96	737	5	4	
57	9.57	264	31	9.60	532	36	10.39	468	9.96	732	5	3	
58	9.57	295	31	9.60	568	37	10.39	432	9.96	727	5	2	
59	9.57	326	32	9.60	605	36	10.39	395	9.96	722	5	1	
60	9.57	358		9.60	641		10.39	359	9.96	717		0	
/	L Cos d		L Ctn c d		L Tan		L Sin d		/	Proportional parts			
1										38	37	36	
2										0.6	0.6	0.6	
3										1.3	1.2	1.2	
4										1.9	1.8	1.8	
5										2.5	2.5	2.4	
6										3.2	3.1	3.0	
7										3.8	3.7	3.6	
8										4.4	4.3	4.2	
9										5.1	4.9	4.8	
10										5.7	5.6	5.4	
11										6.3	6.2	6.0	
12										12.7	12.3	12.0	
13										19.0	18.5	18.0	
14										25.3	24.7	24.0	
15										31.7	30.8	30.0	
16										33	32	31	
17										0.6	0.5	0.5	
18										1.1	1.1	1.0	
19										1.6	1.6	1.6	
20										2.2	2.1	2.1	
21										2.8	2.7	2.6	
22										3.3	3.2	3.1	
23										3.8	3.7	3.6	
24										4.4	4.3	4.1	
25										5.0	4.8	4.6	
26										5.5	5.3	5.2	
27										11.0	10.7	10.3	
28										16.5	16.0	15.5	
29										22.0	21.3	20.7	
30										27.5	26.7	25.8	
31										6	5	4	
32										0.1	0.1	0.1	
33										0.2	0.2	0.1	
34										0.3	0.2	0.2	
35										0.4	0.3	0.3	
36										0.5	0.4	0.3	
37										0.6	0.5	0.4	
38										0.7	0.6	0.5	
39										0.8	0.7	0.5	
40										0.9	0.8	0.6	
41										1.0	0.8	0.7	
42										2.0	1.7	1.3	
43										3.0	2.5	2.0	
44										4.0	3.3	2.7	
45										5.0	4.2	3.3	

111 (291°)

(248°) 68°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

22° (202°)						(337°) 157°						
/	L Sin	d	L Tan	c d	L Ctn	L Cos	d	/	Proportional parts			
0	9.57 358	31	9.60 641	36	10.39 359	9.96 717	6	60				
1	9.57 389	31	9.60 677	37	10.39 323	9.96 711	5	59				
2	9.57 420	31	9.60 714	36	10.39 286	9.96 706	5	58				
3	9.57 451	31	9.60 750	36	10.39 250	9.96 701	5	57				
4	9.57 482	32	9.60 786	37	10.39 214	9.96 696	5	56				
5	9.57 514	31	9.60 823	36	10.39 177	9.96 691	5	55				
6	9.57 545	31	9.60 859	36	10.39 141	9.96 686	5	54				
7	9.57 576	31	9.60 895	36	10.39 105	9.96 681	5	53	//	37	36	35
8	9.57 607	31	9.60 931	36	10.39 069	9.96 676	6	52	1	0.6	0.6	0.6
9	9.57 638	31	9.60 967	37	10.39 033	9.96 670	5	51	2	1.2	1.2	1.2
10	9.57 669	31	9.61 004	36	10.38 996	9.96 665	5	50	3	1.8	1.8	1.8
11	9.57 700	31	9.61 040	36	10.38 960	9.96 660	5	49	4	2.5	2.4	2.3
12	9.57 731	31	9.61 076	36	10.38 924	9.96 655	5	48	5	3.1	3.0	2.9
13	9.57 762	31	9.61 112	36	10.38 888	9.96 650	5	47	6	3.7	3.6	3.5
14	9.57 793	31	9.61 148	36	10.38 852	9.96 645	5	46	7	4.3	4.2	4.1
15	9.57 824	31	9.61 184	36	10.38 816	9.96 640	6	45	8	4.9	4.8	4.7
16	9.57 855	30	9.61 220	36	10.38 780	9.96 634	5	44	9	5.6	5.4	5.2
17	9.57 885	31	9.61 256	36	10.38 744	9.96 629	5	43	10	6.2	6.0	5.8
18	9.57 916	31	9.61 292	36	10.38 708	9.96 624	5	42	20	12.3	12.0	11.7
19	9.57 947	31	9.61 328	36	10.38 672	9.96 619	5	41	30	18.5	18.0	17.5
20	9.57 978	30	9.61 364	36	10.38 636	9.96 614	6	40	40	24.7	24.0	23.3
21	9.58 008	31	9.61 400	36	10.38 600	9.96 608	5	39	50	30.8	30.0	29.2
22	9.58 039	31	9.61 436	36	10.38 564	9.96 603	5	38				
23	9.58 070	31	9.61 472	36	10.38 528	9.96 598	5	37	//	32	31	30
24	9.58 101	30	9.61 508	36	10.38 492	9.96 593	5	36	1	0.5	0.5	0.5
25	9.58 131	31	9.61 544	35	10.38 456	9.96 588	6	35	2	1.1	1.0	1.0
26	9.58 162	30	9.61 579	36	10.38 421	9.96 582	5	34	3	1.6	1.6	1.5
27	9.58 192	31	9.61 615	36	10.38 385	9.96 577	5	33	4	2.1	2.1	2.0
28	9.58 223	30	9.61 651	36	10.38 349	9.96 572	5	32	5	2.7	2.6	2.5
29	9.58 253	31	9.61 687	35	10.38 313	9.96 567	5	31	6	3.2	3.1	3.0
30	9.58 284	30	9.61 722	36	10.38 278	9.96 562	6	30	7	3.7	3.6	3.5
31	9.58 314	31	9.61 758	36	10.38 242	9.96 556	5	29	8	4.3	4.1	4.0
32	9.58 345	30	9.61 794	36	10.38 206	9.96 551	5	28	9	4.8	4.6	4.5
33	9.58 375	31	9.61 830	35	10.38 170	9.96 546	5	27	10	5.3	5.2	5.0
34	9.58 406	30	9.61 865	36	10.38 135	9.96 541	6	26	20	10.7	10.3	10.0
35	9.58 436	31	9.61 901	35	10.38 099	9.96 535	5	25	30	16.0	15.5	15.0
36	9.58 467	30	9.61 936	36	10.38 064	9.96 530	5	24	40	21.3	20.7	20.0
37	9.58 497	30	9.61 972	36	10.38 028	9.96 525	5	23	50	26.7	25.8	25.0
38	9.58 527	30	9.62 008	35	10.37 992	9.96 520	6	22				
39	9.58 557	31	9.62 043	36	10.37 957	9.96 514	5	21	//	29	6	5
40	9.58 588	30	9.62 079	35	10.37 921	9.96 509	5	20	1	0.5	0.1	0.1
41	9.58 618	30	9.62 114	36	10.37 886	9.96 504	6	19	2	1.0	0.2	0.2
42	9.58 648	30	9.62 150	35	10.37 850	9.96 498	5	18	3	1.4	0.3	0.2
43	9.58 678	31	9.62 185	36	10.37 815	9.96 493	5	17	4	1.9	0.4	0.3
44	9.58 709	30	9.62 221	35	10.37 779	9.96 488	5	16	5	2.4	0.5	0.4
45	9.58 739	30	9.62 256	36	10.37 744	9.96 483	6	15	6	2.9	0.6	0.5
46	9.58 769	30	9.62 292	35	10.37 708	9.96 477	5	14	7	3.4	0.7	0.6
47	9.58 799	30	9.62 327	35	10.37 673	9.96 472	5	13	8	3.9	0.8	0.7
48	9.58 829	30	9.62 362	36	10.37 638	9.96 467	6	12	9	4.4	0.9	0.8
49	9.58 859	30	9.62 398	35	10.37 602	9.96 461	5	11	10	4.8	1.0	0.8
50	9.58 889	30	9.62 433	35	10.37 567	9.96 456	5	10	20	9.7	2.0	1.7
51	9.58 919	30	9.62 468	36	10.37 532	9.96 451	6	9	30	14.5	3.0	2.5
52	9.58 949	30	9.62 504	35	10.37 496	9.96 445	5	8	40	19.3	4.0	3.3
53	9.58 979	30	9.62 539	35	10.37 461	9.96 440	5	7	50	24.2	5.0	4.2
54	9.59 009	30	9.62 574	35	10.37 426	9.96 435	6	6				
55	9.59 039	30	9.62 609	36	10.37 391	9.96 429	5	5				
56	9.59 069	29	9.62 645	35	10.37 355	9.96 424	5	4				
57	9.59 098	30	9.62 680	35	10.37 320	9.96 419	6	3				
58	9.59 128	30	9.62 715	35	10.37 285	9.96 413	5	2				
59	9.59 158	30	9.62 750	35	10.37 250	9.96 408	5	1				
60	9.59 188	—	9.62 785	—	10.37 215	9.96 403	—	0				
/	L Cos	d	L Ctn	c d	L Tan	L Sin	d	/	Proportional parts			

112° (292°)

(247°) 67°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

23° (203°)

(336°) 156°

°	L Sin d			L Tan c d			L Ctn			L Cos d			°	Proportional parts			
0	9.59 188	30		9.62 785	35		10.37 215			9.96 403	6		60				
1	9.59 218	29		9.62 820	35		10.37 180			9.96 397	5		59				
2	9.59 247	30		9.62 855	35		10.37 145			9.96 392	5		58				
3	9.59 277	30		9.62 890	36		10.37 110			9.96 387	6		57				
4	9.59 307	29		9.62 926	35		10.37 074			9.96 381	6		56				
5	9.59 336	30		9.62 961	35		10.37 039			9.96 376	6		55				
6	9.59 366	30		9.62 996	35		10.37 004			9.96 370	5		54				
7	9.59 396	29		9.63 031	35		10.36 969			9.96 365	5		53				
8	9.59 425	30		9.63 066	35		10.36 934			9.96 360	6		52				
9	9.59 455	29		9.63 101	34		10.36 899			9.96 354	5		51				
10	9.59 484	30		9.63 135	35		10.36 865			9.96 349	6		50				
11	9.59 514	29		9.63 170	35		10.36 830			9.96 343	5		49				
12	9.59 543	30		9.63 205	35		10.36 795			9.96 338	5		48				
13	9.59 573	29		9.63 240	35		10.36 760			9.96 333	6		47				
14	9.59 602	30		9.63 275	35		10.36 725			9.96 327	5		46				
15	9.59 632	29		9.63 310	35		10.36 690			9.96 322	6		45				
16	9.59 661	29		9.63 345	34		10.36 655			9.96 316	5		44				
17	9.59 690	30		9.63 379	35		10.36 621			9.96 311	6		43				
18	9.59 720	29		9.63 414	35		10.36 586			9.96 305	5		42				
19	9.59 749	29		9.63 449	35		10.36 551			9.96 300	6		41				
20	9.59 778	30		9.63 484	35		10.36 516			9.96 294	5		40				
21	9.59 808	29		9.63 519	34		10.36 481			9.96 289	5		39				
22	9.59 837	29		9.63 553	35		10.36 447			9.96 284	6		38				
23	9.59 866	29		9.63 588	35		10.36 412			9.96 278	5		37				
24	9.59 895	29		9.63 623	34		10.36 377			9.96 273	6		36				
25	9.59 924	30		9.63 657	35		10.36 343			9.96 267	5		35				
26	9.59 954	29		9.63 692	34		10.36 308			9.96 262	6		34				
27	9.59 983	29		9.63 726	35		10.36 274			9.96 256	5		33				
28	9.60 012	29		9.63 761	35		10.36 239			9.96 251	6		32				
29	9.60 041	29		9.63 796	34		10.36 204			9.96 245	5		31				
30	9.60 070	29		9.63 830	35		10.36 170			9.96 240	6		30				
31	9.60 099	29		9.63 865	34		10.36 135			9.96 234	5		29				
32	9.60 128	29		9.63 899	35		10.36 101			9.96 229	6		28				
33	9.60 157	29		9.63 934	34		10.36 066			9.96 223	5		27				
34	9.60 186	29		9.63 968	35		10.36 032			9.96 218	6		26				
35	9.60 215	29		9.64 003	34		10.35 997			9.96 212	5		25				
36	9.60 244	29		9.64 037	35		10.35 963			9.96 207	6		24				
37	9.60 273	29		9.64 072	34		10.35 928			9.96 201	5		23				
38	9.60 302	29		9.64 106	34		10.35 894			9.96 196	6		22				
39	9.60 331	28		9.64 140	35		10.35 860			9.96 190	5		21				
40	9.60 359	29		9.64 175	34		10.35 825			9.96 185	6		20				
41	9.60 388	29		9.64 209	34		10.35 791			9.96 179	5		19				
42	9.60 417	29		9.64 243	35		10.35 757			9.96 174	6		18				
43	9.60 446	28		9.64 278	34		10.35 722			9.96 168	5		17				
44	9.60 474	29		9.64 312	34		10.35 688			9.96 162	5		16				
45	9.60 503	29		9.64 346	35		10.35 654			9.96 157	6		15				
46	9.60 532	29		9.64 381	34		10.35 619			9.96 151	5		14				
47	9.60 561	28		9.64 415	34		10.35 585			9.96 146	6		13				
48	9.60 589	29		9.64 449	34		10.35 551			9.96 140	5		12				
49	9.60 618	28		9.64 483	34		10.35 517			9.96 135	6		11				
50	9.60 646	29		9.64 517	35		10.35 483			9.96 129	5		10				
51	9.60 675	29		9.64 552	34		10.35 448			9.96 123	6		9				
52	9.60 704	28		9.64 586	34		10.35 414			9.96 118	5		8				
53	9.60 732	29		9.64 620	34		10.35 380			9.96 112	6		7				
54	9.60 761	28		9.64 654	34		10.35 346			9.96 107	5		6				
55	9.60 789	29		9.64 688	34		10.35 312			9.96 101	6		5				
56	9.60 818	28		9.64 722	34		10.35 278			9.96 095	5		4				
57	9.60 846	29		9.64 756	34		10.35 244			9.96 090	6		3				
58	9.60 875	28		9.64 790	34		10.35 210			9.96 084	5		2				
59	9.60 903	28		9.64 824	34		10.35 176			9.96 079	6		1				
60	9.60 931			9.64 858			10.35 142			9.96 073			0				
°	L Cos d			L Ctn c d			L Tan			L Sin d			°	Proportional parts			
113	9.29 297																

113° (297°)

(246°) 66°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

24° (204°)						(335°) 155°					
/	L Sin	d	L Tan	c d	L Ctn	L Cos	d	/	Proportional parts		
0	9.60 931	29	9.64 858	34	10.35 142	9.96 073	6	60			
1	9.60 960	28	9.64 892	34	10.35 108	9.96 067	5	59			
2	9.60 988	28	9.64 926	34	10.35 074	9.96 062	6	58			
3	9.61 016	29	9.64 960	34	10.35 040	9.96 056	6	57			
4	9.61 045	28	9.64 994	34	10.35 006	9.96 050	5	56			
5	9.61 073	28	9.65 028	34	10.34 972	9.96 045	6	55			
6	9.61 101	28	9.65 062	34	10.34 938	9.96 039	5	54			
7	9.61 129	29	9.65 096	34	10.34 904	9.96 034	6	53			
8	9.61 158	28	9.65 130	34	10.34 870	9.96 028	6	52			
9	9.61 186	28	9.65 164	33	10.34 836	9.96 022	5	51			
10	9.61 214	28	9.65 197	34	10.34 803	9.96 017	6	50			
11	9.61 242	28	9.65 231	34	10.34 769	9.96 011	6	49			
12	9.61 270	28	9.65 265	34	10.34 735	9.96 005	5	48			
13	9.61 298	28	9.65 299	34	10.34 701	9.96 000	6	47			
14	9.61 326	28	9.65 333	33	10.34 667	9.95 994	6	46			
15	9.61 354	28	9.65 366	34	10.34 634	9.95 988	6	45			
16	9.61 382	29	9.65 400	34	10.34 600	9.95 982	5	44			
17	9.61 411	27	9.65 434	33	10.34 566	9.95 977	6	43			
18	9.61 438	28	9.65 467	34	10.34 533	9.95 971	6	42			
19	9.61 466	28	9.65 501	34	10.34 499	9.95 965	5	41			
20	9.61 494	28	9.65 535	33	10.34 465	9.95 960	6	40			
21	9.61 522	28	9.65 568	34	10.34 432	9.95 954	6	39			
22	9.61 550	28	9.65 602	34	10.34 398	9.95 948	6	38			
23	9.61 578	28	9.65 636	33	10.34 364	9.95 942	5	37			
24	9.61 606	28	9.65 669	34	10.34 331	9.95 937	6	36			
25	9.61 634	28	9.65 703	33	10.34 297	9.95 931	6	35			
26	9.61 662	27	9.65 736	34	10.34 264	9.95 925	5	34			
27	9.61 689	28	9.65 770	33	10.34 230	9.95 920	6	33			
28	9.61 717	28	9.65 803	34	10.34 197	9.95 914	6	32			
29	9.61 745	28	9.65 837	33	10.34 163	9.95 908	6	31			
30	9.61 773	27	9.65 870	34	10.34 130	9.95 902	5	30			
31	9.61 800	28	9.65 904	33	10.34 096	9.95 897	6	29			
32	9.61 828	28	9.65 937	34	10.34 063	9.95 891	6	28			
33	9.61 856	27	9.65 971	33	10.34 029	9.95 885	6	27			
34	9.61 883	28	9.66 004	34	10.33 996	9.95 879	6	26			
35	9.61 911	28	9.66 038	33	10.33 962	9.95 873	5	25			
36	9.61 939	27	9.66 071	33	10.33 929	9.95 868	6	24			
37	9.61 966	28	9.66 104	34	10.33 896	9.95 862	6	23			
38	9.61 994	27	9.66 138	33	10.33 862	9.95 856	6	22			
39	9.62 021	28	9.66 171	33	10.33 829	9.95 850	6	21			
40	9.62 049	27	9.66 204	34	10.33 796	9.95 844	5	20			
41	9.62 076	28	9.66 238	33	10.33 762	9.95 839	6	19			
42	9.62 104	27	9.66 271	33	10.33 729	9.95 833	6	18			
43	9.62 131	28	9.66 304	33	10.33 696	9.95 827	6	17			
44	9.62 159	27	9.66 337	34	10.33 663	9.95 821	6	16			
45	9.62 186	28	9.66 371	33	10.33 629	9.95 815	5	15			
46	9.62 214	27	9.66 404	33	10.33 596	9.95 810	6	14			
47	9.62 241	27	9.66 437	33	10.33 563	9.95 804	6	13			
48	9.62 268	28	9.66 470	33	10.33 530	9.95 798	6	12			
49	9.62 296	27	9.66 503	34	10.33 497	9.95 792	6	11			
50	9.62 323	27	9.66 537	33	10.33 463	9.95 786	6	10			
51	9.62 350	27	9.66 570	33	10.33 430	9.95 780	5	9			
52	9.62 377	28	9.66 603	33	10.33 397	9.95 775	6	8			
53	9.62 405	27	9.66 636	33	10.33 364	9.95 769	6	7			
54	9.62 432	27	9.66 669	33	10.33 331	9.95 763	6	6			
55	9.62 459	27	9.66 702	33	10.33 298	9.95 757	6	5			
56	9.62 486	27	9.66 735	33	10.33 265	9.95 751	6	4			
57	9.62 513	28	9.66 768	33	10.33 232	9.95 745	6	3			
58	9.62 541	27	9.66 801	33	10.33 199	9.95 739	6	2			
59	9.62 568	27	9.66 834	33	10.33 166	9.95 733	5	1			
60	9.62 595		9.66 867		10.33 133	9.95 728		0			
/	L Cos	d	L Ctn	c d	L Tan	L Sin	d	/	Proportional parts		

114° (294°)

(245°) 65°

Table 6.15 (Cont'd)
COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

25 (205)					(334) 154				
<i>i</i>	L Sin	<i>d</i>	L Tan	<i>c d</i>	L Ctn	L Ciss	<i>d</i>	<i>i</i>	Proportional parts
0	9 62 595	27	9 66 861	33	10 33 133	9 95 728	6	60	
1	9 62 622	27	9 66 900	33	10 33 100	9 95 722	6	59	
2	9 62 649	27	9 66 933	33	10 33 067	9 95 716	6	58	
3	9 62 676	27	9 66 966	33	10 33 034	9 95 710	6	57	
4	9 62 703	27	9 66 999	33	10 33 001	9 95 704	6	56	
5	9 62 730	27	9 67 032	33	10 32 968	9 95 698	6	55	
6	9 62 757	27	9 67 065	33	10 32 935	9 95 692	6	54	
7	9 62 784	27	9 67 098	33	10 32 902	9 95 686	6	53	
8	9 62 811	27	9 67 131	32	10 32 869	9 95 680	6	52	
9	9 62 838	27	9 67 163	33	10 32 837	9 95 674	6	51	
10	9 62 865	27	9 67 196	33	10 32 804	9 95 668	6	50	
11	9 62 892	26	9 67 229	33	10 32 771	9 95 663	6	49	
12	9 62 918	27	9 67 262	33	10 32 738	9 95 657	6	48	
13	9 62 945	27	9 67 295	32	10 32 705	9 95 651	6	47	
14	9 62 972	27	9 67 327	33	10 32 673	9 95 645	6	46	
15	9 62 999	27	9 67 360	33	10 32 640	9 95 639	6	45	
16	9 63 026	26	9 67 393	33	10 32 607	9 95 633	6	44	
17	9 63 052	27	9 67 426	32	10 32 574	9 95 627	6	43	
18	9 63 079	27	9 67 458	33	10 32 542	9 95 621	6	42	
19	9 63 106	27	9 67 491	33	10 32 509	9 95 615	6	41	
20	9 63 133	26	9 67 524	32	10 32 476	9 95 609	6	40	
21	9 63 159	27	9 67 556	33	10 32 444	9 95 603	6	39	
22	9 63 186	27	9 67 589	33	10 32 411	9 95 597	6	38	
23	9 63 213	26	9 67 622	32	10 32 378	9 95 591	6	37	
24	9 63 239	27	9 67 654	33	10 32 346	9 95 585	6	36	
25	9 63 266	26	9 67 687	32	10 32 313	9 95 579	6	35	
26	9 63 292	27	9 67 719	33	10 32 281	9 95 573	6	34	
27	9 63 319	26	9 67 752	33	10 32 248	9 95 567	6	33	
28	9 63 345	27	9 67 785	32	10 32 215	9 95 561	6	32	
29	9 63 372	26	9 67 817	33	10 32 183	9 95 555	6	31	
30	9 63 398	27	9 67 850	32	10 32 150	9 95 549	6	30	
31	9 63 425	26	9 67 882	33	10 32 118	9 95 543	6	29	
32	9 63 451	27	9 67 915	32	10 32 085	9 95 537	6	28	
33	9 63 478	26	9 67 947	33	10 32 053	9 95 531	6	27	
34	9 63 504	27	9 67 980	32	10 32 020	9 95 525	6	26	
35	9 63 531	26	9 68 012	32	10 31 988	9 95 519	6	25	
36	9 63 557	27	9 68 044	33	10 31 956	9 95 513	6	24	
37	9 63 583	27	9 68 077	32	10 31 923	9 95 507	7	23	
38	9 63 610	26	9 68 109	33	10 31 891	9 95 500	6	22	
39	9 63 636	26	9 68 142	32	10 31 858	9 95 494	6	21	
40	9 63 662	27	9 68 174	32	10 31 826	9 95 488	6	20	
41	9 63 689	26	9 68 206	33	10 31 794	9 95 482	6	19	
42	9 63 715	26	9 68 239	32	10 31 761	9 95 476	6	18	
43	9 63 741	26	9 68 271	32	10 31 729	9 95 470	6	17	
44	9 63 767	27	9 68 303	33	10 31 697	9 95 464	6	16	
45	9 63 794	26	9 68 336	32	10 31 664	9 95 458	6	15	
46	9 63 820	26	9 68 368	32	10 31 632	9 95 452	6	14	
47	9 63 846	26	9 68 400	32	10 31 600	9 95 446	6	13	
48	9 63 872	26	9 68 432	33	10 31 568	9 95 440	6	12	
49	9 63 898	26	9 68 465	32	10 31 535	9 95 434	7	11	
50	9 63 924	26	9 68 497	32	10 31 503	9 95 427	6	10	
51	9 63 950	26	9 68 529	32	10 31 471	9 95 421	6	9	
52	9 63 976	26	9 68 561	32	10 31 439	9 95 415	6	8	
53	9 64 002	26	9 68 593	33	10 31 407	9 95 409	6	7	
54	9 64 028	26	9 68 626	32	10 31 374	9 95 403	6	6	
55	9 64 054	26	9 68 658	32	10 31 342	9 95 397	6	5	
56	9 64 080	26	9 68 690	32	10 31 310	9 95 391	7	4	
57	9 64 106	26	9 68 722	32	10 31 278	9 95 384	6	3	
58	9 64 132	26	9 68 754	32	10 31 246	9 95 378	6	2	
59	9 64 158	26	9 68 786	32	10 31 214	9 95 372	6	1	
60	9 64 184		9 68 818		10 31 182	9 95 366		0	
<i>i</i>	L Ciss	<i>d</i>	L Ctn	<i>c d</i>	L Tan	L Sin	<i>d</i>	<i>i</i>	Proportional parts
1									33 32
2									0.6 0.5
3									1.1 1.1
4									1.6 1.6
5									2.2 2.1
6									
7									2.8 2.7
8									3.3 3.2
9									3.8 3.7
10									4.4 4.3
11									5.0 4.8
12									
13									5.5 5.3
14									11.0 10.7
15									16.5 16.0
16									22.0 21.3
17									27.5 26.7
18									
19									27 26
20									0.4 0.4
21									0.9 0.9
22									1.4 1.3
23									1.8 1.7
24									
25									2.2 2.2
26									2.7 2.6
27									3.2 3.0
28									3.6 3.5
29									4.0 3.9
30									
31									4.5 4.3
32									9.0 8.7
33									13.5 13.0
34									18.0 17.3
35									22.5 21.7
36									
37									7 6 5
38									0.1 0.1
39									0.2 0.2
40									0.4 0.3
41									0.5 0.4
42									
43									0.6 0.5
44									0.7 0.6
45									0.8 0.7
46									0.9 0.8
47									1.0 0.9
48									
49									1.2 1.0
50									2.3 2.0
51									3.5 3.0
52									4.7 4.0
53									5.8 5.0
54									
55									
56									
57									
58									
59									
60									

Table 6.15 (Cont'd)

116° (296°)

(243°) 63°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

27° (207°)						(332°) 152°					
/	L Sin	d	L Tan	c d	L Ctn	L Cos	d	/	Proportional parts		
0	9.65 705	24	9.70 717	31	10.29 283	9.94 988	6	60			
1	9.65 729	25	9.70 748	31	10.29 252	9.94 982	7	59			
2	9.65 754	25	9.70 779	31	10.29 221	9.94 975	6	58			
3	9.65 779	25	9.70 810	31	10.29 190	9.94 969	7	57			
4	9.65 804	24	9.70 841	32	10.29 159	9.94 962	6	56			
5	9.65 828	25	9.70 873	31	10.29 127	9.94 956	7	55			
6	9.65 853	25	9.70 904	31	10.29 096	9.94 949	6	54			
7	9.65 878	24	9.70 935	31	10.29 065	9.94 943	7	53			
8	9.65 902	25	9.70 966	31	10.29 034	9.94 936	6	52			
9	9.65 927	25	9.70 997	31	10.29 003	9.94 930	7	51			
10	9.65 952	24	9.71 028	31	10.28 972	9.94 923	6	50			
11	9.65 976	25	9.71 059	31	10.28 941	9.94 917	6	49			
12	9.66 001	24	9.71 090	31	10.28 910	9.94 911	7	48			
13	9.66 025	25	9.71 121	32	10.28 879	9.94 904	6	47			
14	9.66 050	25	9.71 153	31	10.28 847	9.94 898	7	46			
15	9.66 075	24	9.71 184	31	10.28 816	9.94 891	6	45			
16	9.66 099	25	9.71 215	31	10.28 785	9.94 885	7	44			
17	9.66 124	24	9.71 246	31	10.28 754	9.94 878	7	43			
18	9.66 148	25	9.71 277	31	10.28 723	9.94 871	6	42			
19	9.66 173	24	9.71 308	31	10.28 692	9.94 865	7	41			
20	9.66 197	24	9.71 339	31	10.28 661	9.94 858	6	40			
21	9.66 221	25	9.71 370	31	10.28 630	9.94 852	7	39			
22	9.66 246	24	9.71 401	30	10.28 599	9.94 845	6	38			
23	9.66 270	25	9.71 431	31	10.28 569	9.94 839	7	37			
24	9.66 295	24	9.71 462	31	10.28 538	9.94 832	6	36			
25	9.66 319	24	9.71 493	31	10.28 507	9.94 826	7	35			
26	9.66 343	25	9.71 524	31	10.28 476	9.94 819	6	34			
27	9.66 368	24	9.71 555	31	10.28 445	9.94 813	7	33			
28	9.66 392	24	9.71 586	31	10.28 414	9.94 806	7	32			
29	9.66 416	25	9.71 617	31	10.28 383	9.94 799	6	31			
30	9.66 441	24	9.71 648	31	10.28 352	9.94 793	7	30			
31	9.66 465	24	9.71 679	30	10.28 321	9.94 786	6	29			
32	9.66 489	24	9.71 709	31	10.28 291	9.94 780	7	28			
33	9.66 513	24	9.71 740	31	10.28 260	9.94 773	6	27			
34	9.66 537	25	9.71 771	31	10.28 229	9.94 767	7	26			
35	9.66 562	24	9.71 802	31	10.28 198	9.94 760	7	25			
36	9.66 586	24	9.71 833	30	10.28 167	9.94 753	6	24			
37	9.66 610	24	9.71 863	31	10.28 137	9.94 747	7	23			
38	9.66 634	24	9.71 894	31	10.28 106	9.94 740	6	22			
39	9.66 658	24	9.71 925	30	10.28 075	9.94 734	7	21			
40	9.66 682	24	9.71 955	31	10.28 045	9.94 727	7	20			
41	9.66 706	25	9.71 986	31	10.28 014	9.94 720	6	19			
42	9.66 731	24	9.72 017	31	10.27 983	9.94 714	7	18			
43	9.66 755	24	9.72 048	30	10.27 952	9.94 707	7	17			
44	9.66 779	24	9.72 078	31	10.27 922	9.94 700	6	16			
45	9.66 803	24	9.72 109	31	10.27 891	9.94 694	7	15			
46	9.66 827	24	9.72 140	30	10.27 860	9.94 687	7	14			
47	9.66 851	24	9.72 170	31	10.27 830	9.94 680	6	13			
48	9.66 875	24	9.72 201	30	10.27 799	9.94 674	7	12			
49	9.66 899	23	9.72 231	31	10.27 769	9.94 667	7	11			
50	9.66 922	24	9.72 262	31	10.27 738	9.94 660	6	10			
51	9.66 946	24	9.72 293	30	10.27 707	9.94 654	7	9			
52	9.66 970	24	9.72 323	31	10.27 677	9.94 647	7	8			
53	9.66 994	24	9.72 354	30	10.27 646	9.94 640	6	7			
54	9.67 018	24	9.72 384	31	10.27 616	9.94 634	7	6			
55	9.67 042	24	9.72 415	30	10.27 585	9.94 627	7	5			
56	9.67 066	24	9.72 445	31	10.27 555	9.94 620	6	4			
57	9.67 090	23	9.72 476	30	10.27 524	9.94 614	7	3			
58	9.67 113	24	9.72 506	31	10.27 494	9.94 607	7	2			
59	9.67 137	24	9.72 537	30	10.27 463	9.94 600	7	1			
60	9.67 161		9.72 567		10.27 433	9.94 593		0			
/	L Cos	d	L Ctn	c d	L Tan	L Sin	d	/	Proportional parts		

117° (297°)

(242°) 62°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

28° (208°)						(331°) 151°										
/	L Sin	d	L Tan	c d	L Ctn	L Cos	d	/	Proportional parts							
0	9.67 161	24	9.72 567	31	10.27 433	9.94 593	6	60		31	30	29				
1	9.67 185	23	9.72 598	30	10.27 402	9.94 587	7	59								
2	9.67 208	24	9.72 628	31	10.27 372	9.94 580	7	58								
3	9.67 232	24	9.72 659	30	10.27 341	9.94 573	6	57								
4	9.67 256	24	9.72 689	31	10.27 311	9.94 567	7	56	1	0.6	0.5	0.6				
5	9.67 280	23	9.72 720	30	10.27 280	9.94 560	7	55								
6	9.67 303	24	9.72 750	30	10.27 250	9.94 553	7	54								
7	9.67 327	23	9.72 780	31	10.27 220	9.94 546	6	53								
8	9.67 350	24	9.72 811	30	10.27 189	9.94 540	7	52	2	1.0	1.0	1.0				
9	9.67 374	24	9.72 841	31	10.27 159	9.94 533	7	51								
10	9.67 398	23	9.72 872	30	10.27 128	9.94 526	7	50								
11	9.67 421	24	9.72 902	30	10.27 098	9.94 519	6	49								
12	9.67 445	23	9.72 932	31	10.27 068	9.94 513	7	48	3	1.6	1.5	1.4				
13	9.67 468	24	9.72 963	30	10.27 037	9.94 506	7	47								
14	9.67 492	23	9.72 993	30	10.27 007	9.94 499	7	46								
15	9.67 515	24	9.73 023	31	10.26 977	9.94 492	7	45								
16	9.67 539	23	9.73 054	30	10.26 946	9.94 485	6	44	4	2.1	2.0	1.9				
17	9.67 562	24	9.73 084	30	10.26 916	9.94 479	7	43								
18	9.67 586	23	9.73 114	30	10.26 886	9.94 472	7	42								
19	9.67 609	24	9.73 144	31	10.26 856	9.94 465	7	41								
20	9.67 633	23	9.73 175	30	10.26 825	9.94 458	7	40	5	2.6	2.5	2.4				
21	9.67 656	24	9.73 205	30	10.26 795	9.94 451	6	39								
22	9.67 680	23	9.73 235	30	10.26 765	9.94 445	7	38								
23	9.67 703	23	9.73 265	30	10.26 735	9.94 438	7	37								
24	9.67 726	24	9.73 295	31	10.26 705	9.94 431	7	36	6	3.1	3.0	2.9				
25	9.67 750	23	9.73 326	30	10.26 674	9.94 424	7	35								
26	9.67 773	23	9.73 356	30	10.26 644	9.94 417	7	34								
27	9.67 796	24	9.73 386	30	10.26 614	9.94 410	6	33								
28	9.67 820	23	9.73 416	30	10.26 584	9.94 404	7	32	7	3.6	3.5	3.4				
29	9.67 843	23	9.73 446	30	10.26 554	9.94 397	7	31								
30	9.67 866	24	9.73 476	31	10.26 524	9.94 390	7	30								
31	9.67 890	23	9.73 507	30	10.26 493	9.94 383	7	29								
32	9.67 913	23	9.73 537	30	10.26 463	9.94 376	7	28	8	4.1	4.0	3.9				
33	9.67 936	23	9.73 567	30	10.26 433	9.94 369	7	27								
34	9.67 959	23	9.73 597	30	10.26 403	9.94 362	7	26								
35	9.67 982	24	9.73 627	30	10.26 373	9.94 355	6	25								
36	9.68 006	23	9.73 657	30	10.26 343	9.94 349	7	24	9	4.6	4.5	4.4				
37	9.68 029	23	9.73 687	30	10.26 313	9.94 342	7	23								
38	9.68 052	23	9.73 717	30	10.26 283	9.94 335	7	22								
39	9.68 075	23	9.73 747	30	10.26 253	9.94 328	7	21								
40	9.68 098	23	9.73 777	30	10.26 223	9.94 321	7	20	10	5.2	5.0	4.8				
41	9.68 121	23	9.73 807	30	10.26 193	9.94 314	7	19								
42	9.68 144	23	9.73 837	30	10.26 163	9.94 307	7	18								
43	9.68 167	23	9.73 867	30	10.26 133	9.94 300	7	17								
44	9.68 190	23	9.73 897	30	10.26 103	9.94 293	7	16	11	5.7	5.6	5.5				
45	9.68 213	24	9.73 927	30	10.26 073	9.94 286	7	15								
46	9.68 237	23	9.73 957	30	10.26 043	9.94 279	6	14								
47	9.68 260	23	9.73 987	30	10.26 013	9.94 273	7	13								
48	9.68 283	22	9.74 017	30	10.25 983	9.94 266	7	12	12	6.2	6.0	5.8				
49	9.68 305	23	9.74 047	30	10.25 953	9.94 259	7	11								
50	9.68 328	23	9.74 077	30	10.25 923	9.94 252	7	10								
51	9.68 351	23	9.74 107	30	10.25 893	9.94 245	7	9								
52	9.68 374	23	9.74 137	29	10.25 863	9.94 238	7	8	13	6.7	6.5	6.3				
53	9.68 397	23	9.74 166	30	10.25 834	9.94 231	7	7								
54	9.68 420	23	9.74 196	30	10.25 804	9.94 224	7	6								
55	9.68 443	23	9.74 226	30	10.25 774	9.94 217	7	5								
56	9.68 466	23	9.74 256	30	10.25 744	9.94 210	7	4	14	7.2	7.0	6.8				
57	9.68 489	23	9.74 286	30	10.25 714	9.94 203	7	3								
58	9.68 512	22	9.74 316	29	10.25 684	9.94 196	7	2								
59	9.68 534	23	9.74 345	30	10.25 655	9.94 189	7	1								
60	9.68 557	—	9.74 375	—	10.25 625	9.94 182	—	0	15	7.7	7.5	7.3				
/	L Cos	d	L Ctn	c d	L Tan	L Sin	d	/								
													Proportional parts			

118° (298°)

(241°) 61°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

29° (209°)						(330°) 150°								
/	L Sin	d		L Tan	c d	L Ctn	L Cos	d	/	Proportional parts				
0	9.68 557	23		9.74 375	30	10.25 625	9.94 182	7	60		30	29	23	
1	9.68 580	23		9.74 405	30	10.25 595	9.94 175	7	59					
2	9.68 603	22		9.74 435	30	10.25 565	9.94 168	7	58					
3	9.68 625	23		9.74 465	29	10.25 535	9.94 161	7	57					
4	9.68 648	23		9.74 494	30	10.25 506	9.94 154	7	56					
5	9.68 671	23		9.74 524	30	10.25 476	9.94 147	7	55					
6	9.68 694	22		9.74 554	29	10.25 446	9.94 140	7	54					
7	9.68 716	23		9.74 583	30	10.25 417	9.94 133	7	53					
8	9.68 739	23		9.74 613	30	10.25 387	9.94 126	7	52					
9	9.68 762	22		9.74 643	30	10.25 357	9.94 119	7	51					
10	9.68 784	23		9.74 673	29	10.25 327	9.94 112	7	50					
11	9.68 807	22		9.74 702	30	10.25 298	9.94 105	7	49					
12	9.68 829	23		9.74 732	30	10.25 268	9.94 098	8	48					
13	9.68 852	23		9.74 762	29	10.25 238	9.94 090	7	47					
14	9.68 875	22		9.74 791	30	10.25 209	9.94 083	7	46					
15	9.68 897	23		9.74 821	30	10.25 179	9.94 076	7	45	''	1	0.5	0.5	0.4
16	9.68 920	22		9.74 851	29	10.25 149	9.94 069	7	44					
17	9.68 942	23		9.74 880	30	10.25 120	9.94 062	7	43					
18	9.68 965	22		9.74 910	29	10.25 090	9.94 055	7	42					
19	9.68 987	23		9.74 939	30	10.25 061	9.94 048	7	41	2	1.0	1.0	0.8	
20	9.69 010	22		9.74 969	29	10.25 031	9.94 041	7	40	3	1.5	1.4	1.2	
21	9.69 032	23		9.74 998	30	10.25 002	9.94 034	7	39	4	2.0	1.9	1.5	
22	9.69 055	22		9.75 028	30	10.24 972	9.94 027	7	38	5	2.5	2.4	1.9	
23	9.69 077	23		9.75 058	29	10.24 942	9.94 020	8	37	6	3.0	2.9	2.3	
24	9.69 100	22		9.75 087	30	10.24 913	9.94 012	7	36	7	3.5	3.4	2.7	
25	9.69 122	22		9.75 117	29	10.24 883	9.94 005	7	35	8	4.0	3.9	3.1	
26	9.69 144	23		9.75 146	30	10.24 854	9.93 998	7	34	9	4.5	4.4	3.4	
27	9.69 167	22		9.75 176	29	10.24 824	9.93 991	7	33	10	5.0	4.8	3.8	
28	9.69 189	23		9.75 205	30	10.24 795	9.93 984	7	32	20	10.0	9.7	7.7	
29	9.69 212	22		9.75 235	29	10.24 765	9.93 977	7	31	30	15.0	14.5	11.5	
30	9.69 234	22		9.75 264	30	10.24 736	9.93 970	7	30	40	20.0	19.3	15.3	
31	9.69 256	23		9.75 294	29	10.24 706	9.93 963	8	29	50	25.0	24.2	19.2	
32	9.69 279	22		9.75 323	30	10.24 677	9.93 955	7	28	''	22	8	7	
33	9.69 301	22		9.75 353	29	10.24 647	9.93 948	7	27					
34	9.69 323	22		9.75 382	29	10.24 618	9.93 941	7	26					
35	9.69 345	23		9.75 411	30	10.24 589	9.93 934	7	25					
36	9.69 368	22		9.75 441	29	10.24 559	9.93 927	7	24	1	0.4	0.1	0.1	
37	9.69 390	22		9.75 470	30	10.24 530	9.93 920	8	23	2	0.7	0.3	0.2	
38	9.69 412	22		9.75 500	29	10.24 500	9.93 912	7	22	3	1.1	0.4	0.4	
39	9.69 434	22		9.75 529	29	10.24 471	9.93 905	7	21	4	1.5	0.5	0.5	
40	9.69 456	23		9.75 558	30	10.24 442	9.93 898	7	20	5	1.8	0.7	0.6	
41	9.69 479	22		9.75 588	29	10.24 412	9.93 891	7	19	6	2.2	0.8	0.7	
42	9.69 501	22		9.75 617	30	10.24 383	9.93 884	8	18	7	2.6	0.9	0.8	
43	9.69 523	22		9.75 647	29	10.24 353	9.93 876	7	17	8	2.9	1.1	0.9	
44	9.69 545	22		9.75 676	29	10.24 324	9.93 869	7	16	9	3.3	1.2	1.0	
45	9.69 567	22		9.75 705	30	10.24 295	9.93 862	7	15	10	3.7	1.3	1.2	
46	9.69 589	22		9.75 735	29	10.24 265	9.93 855	8	14	20	7.3	2.7	2.3	
47	9.69 611	22		9.75 764	29	10.24 236	9.93 847	7	13	30	11.0	4.0	3.5	
48	9.69 633	22		9.75 793	29	10.24 207	9.93 840	7	12	40	14.7	5.3	4.7	
49	9.69 655	22		9.75 822	30	10.24 178	9.93 833	7	11	50	18.3	6.7	5.8	
50	9.69 677	22		9.75 852	29	10.24 148	9.93 826	7	10					
51	9.69 699	22		9.75 881	29	10.24 119	9.93 819	8	9					
52	9.69 721	22		9.75 910	29	10.24 090	9.93 811	7	8					
53	9.69 743	22		9.75 939	30	10.24 061	9.93 804	7	7					
54	9.69 765	22		9.75 969	29	10.24 031	9.93 797	8	6					
55	9.69 787	22		9.75 998	29	10.24 002	9.93 789	7	5					
56	9.69 809	22		9.76 027	29	10.23 973	9.93 782	7	4					
57	9.69 831	22		9.76 056	30	10.23 944	9.93 775	7	3					
58	9.69 853	22		9.76 086	29	10.23 914	9.93 768	8	2					
59	9.69 875	22		9.76 115	29	10.23 885	9.93 760	7	1					
60	9.69 897			9.76 144		10.23 856	9.93 753		0					
/	L Cos	d		L Ctn	c d	L Tan	L Sin	d	/	Proportional parts				

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

30° (210)						(329°) 149°									
/	L Sin	d	L Tan	c d	L Ctn	L Cos	d	/	Proportional parts						
0	9.69 897	22	9.76 144	29	10.23 856	9.93 753	7	60	//	30	29	28			
1	9.69 919	22	9.76 173	29	10.23 827	9.93 746	8	59							
2	9.69 941	22	9.76 202	29	10.23 798	9.93 738	7	58							
3	9.69 963	21	9.76 231	30	10.23 769	9.93 731	7	57							
4	9.69 984	22	9.76 261	29	10.23 739	9.93 724	7	56							
5	9.70 006	22	9.76 290	29	10.23 710	9.93 717	8	55	1	0.5	0.6	0.5			
6	9.70 028	22	9.76 319	29	10.23 681	9.93 709	7	54							
7	9.70 050	22	9.76 348	29	10.23 652	9.93 702	7	53							
8	9.70 072	21	9.76 377	29	10.23 623	9.93 695	8	52							
9	9.70 093	22	9.76 406	29	10.23 594	9.93 687	7	51							
10	9.70 115	22	9.76 435	29	10.23 565	9.93 680	7	50	5	2.5	2.4	2.3			
11	9.70 137	22	9.76 464	29	10.23 536	9.93 673	8	49							
12	9.70 159	21	9.76 493	29	10.23 507	9.93 665	7	48							
13	9.70 180	22	9.76 522	29	10.23 478	9.93 658	8	47							
14	9.70 202	22	9.76 551	29	10.23 449	9.93 650	7	46							
15	9.70 224	21	9.76 580	29	10.23 420	9.93 643	7	45	10	5.0	4.8	4.7			
16	9.70 245	22	9.76 609	30	10.23 391	9.93 636	8	44							
17	9.70 267	21	9.76 639	29	10.23 361	9.93 628	7	43							
18	9.70 288	22	9.76 668	29	10.23 332	9.93 621	7	42							
19	9.70 310	22	9.76 697	28	10.23 303	9.93 614	8	41							
20	9.70 332	21	9.76 725	29	10.23 275	9.93 606	7	40	20	10.0	9.7	9.3			
21	9.70 353	22	9.76 754	29	10.23 246	9.93 599	8	39							
22	9.70 375	21	9.76 783	29	10.23 217	9.93 591	7	38							
23	9.70 396	22	9.76 812	29	10.23 188	9.93 584	7	37							
24	9.70 418	21	9.76 841	29	10.23 159	9.93 577	8	36							
25	9.70 439	22	9.76 870	29	10.23 130	9.93 569	7	35	//	22	21				
26	9.70 461	21	9.76 899	29	10.23 101	9.93 562	8	34							
27	9.70 482	22	9.76 928	29	10.23 072	9.93 554	7	33							
28	9.70 504	21	9.76 957	29	10.23 043	9.93 547	8	32							
29	9.70 525	22	9.76 986	29	10.23 014	9.93 539	7	31							
30	9.70 547	21	9.77 015	29	10.22 985	9.93 532	7	30	6	1.8	1.8	1.7			
31	9.70 568	22	9.77 044	29	10.22 956	9.93 525	8	29							
32	9.70 590	21	9.77 073	28	10.22 927	9.93 517	7	28							
33	9.70 611	22	9.77 101	29	10.22 899	9.93 510	8	27							
34	9.70 633	21	9.77 130	29	10.22 870	9.93 502	7	26							
35	9.70 654	21	9.77 159	29	10.22 841	9.93 495	8	25	30	11.0	10.5	10.0			
36	9.70 675	22	9.77 188	29	10.22 812	9.93 487	7	24							
37	9.70 697	21	9.77 217	29	10.22 783	9.93 480	8	23							
38	9.70 718	21	9.77 246	28	10.22 754	9.93 472	7	22							
39	9.70 739	22	9.77 274	29	10.22 726	9.93 465	8	21							
40	9.70 761	21	9.77 303	29	10.22 697	9.93 457	7	20	//	8	7				
41	9.70 782	21	9.77 332	29	10.22 668	9.93 450	8	19							
42	9.70 803	21	9.77 361	29	10.22 639	9.93 442	7	18							
43	9.70 824	22	9.77 390	28	10.22 610	9.93 435	8	17							
44	9.70 846	21	9.77 418	29	10.22 582	9.93 427	7	16							
45	9.70 867	21	9.77 447	29	10.22 553	9.93 420	8	15	5	0.7	0.6				
46	9.70 888	21	9.77 476	29	10.22 524	9.93 412	7	14							
47	9.70 909	22	9.77 505	28	10.22 495	9.93 405	8	13							
48	9.70 931	21	9.77 533	29	10.22 467	9.93 397	7	12							
49	9.70 952	21	9.77 562	29	10.22 438	9.93 390	8	11							
50	9.70 973	21	9.77 591	28	10.22 409	9.93 382	7	10	10	1.3	1.2				
51	9.70 994	21	9.77 619	29	10.22 381	9.93 375	8	9							
52	9.71 015	21	9.77 648	29	10.22 352	9.93 367	7	8							
53	9.71 036	22	9.77 677	29	10.22 323	9.93 360	8	7							
54	9.71 058	21	9.77 706	28	10.22 294	9.93 352	8	6							
55	9.71 079	21	9.77 734	29	10.22 266	9.93 344	7	5	20	2.7	2.3				
56	9.71 100	21	9.77 763	28	10.22 237	9.93 337	8	4							
57	9.71 121	21	9.77 791	29	10.22 209	9.93 329	7	3							
58	9.71 142	21	9.77 820	29	10.22 180	9.93 322	8	2							
59	9.71 163	21	9.77 849	28	10.22 151	9.93 314	7	1							
60	9.71 184		9.77 877		10.22 123	9.93 307		0	//	22	21				
/	L Cos	d	L Ctn	c d	L Tan	L Sin	d	/							
													Proportional parts		

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

31° (211°)						(328°) 148°					
/	L Sin	d	L Tan	c d	L Ctn	L Cos	d	/	Proportional parts		
0	9.71 184	21	9.77 877	29	10.22 123	9.93 307	8	60			
1	9.71 205	21	9.77 906	29	10.22 094	9.93 299	8	59			
2	9.71 226	21	9.77 935	28	10.22 065	9.93 291	7	58			
3	9.71 247	21	9.77 963	29	10.22 037	9.93 284	8	57			
4	9.71 268	21	9.77 992	28	10.22 008	9.93 276	7	56			
5	9.71 289	21	9.78 020	29	10.21 980	9.93 269	8	55			
6	9.71 310	21	9.78 049	28	10.21 951	9.93 261	8	54			
7	9.71 331	21	9.78 077	29	10.21 923	9.93 253	7	53	//	29	28
8	9.71 352	21	9.78 106	29	10.21 894	9.93 246	8	52	1	0.5	0.5
9	9.71 373	20	9.78 135	28	10.21 865	9.93 238	8	51	2	1.0	0.9
10	9.71 393	21	9.78 163	29	10.21 837	9.93 230	7	50	3	1.4	1.4
11	9.71 414	21	9.78 192	28	10.21 808	9.93 223	8	49	4	1.9	1.9
12	9.71 435	21	9.78 220	29	10.21 780	9.93 215	8	48	5	2.4	2.3
13	9.71 456	21	9.78 249	28	10.21 751	9.93 207	7	47	6	2.9	2.8
14	9.71 477	21	9.78 277	29	10.21 723	9.93 200	8	46	7	3.4	3.3
15	9.71 498	21	9.78 306	28	10.21 694	9.93 192	8	45	8	3.9	3.7
16	9.71 519	20	9.78 334	29	10.21 666	9.93 184	7	44	9	4.4	4.2
17	9.71 539	21	9.78 363	28	10.21 637	9.93 177	8	43	10	4.8	4.7
18	9.71 560	21	9.78 391	29	10.21 609	9.93 169	8	42	20	9.7	9.3
19	9.71 581	21	9.78 419	29	10.21 581	9.93 161	7	41	30	14.5	14.0
20	9.71 602	20	9.78 448	28	10.21 552	9.93 154	8	40	40	19.3	18.7
21	9.71 622	21	9.78 476	29	10.21 524	9.93 146	8	39	50	24.2	23.3
22	9.71 643	21	9.78 505	28	10.21 495	9.93 138	7	38			
23	9.71 664	21	9.78 533	29	10.21 467	9.93 131	8	37	//	21	20
24	9.71 685	20	9.78 562	28	10.21 438	9.93 123	8	36	1	0.4	0.3
25	9.71 705	21	9.78 590	28	10.21 410	9.93 115	7	35	2	0.7	0.7
26	9.71 726	21	9.78 618	29	10.21 382	9.93 108	8	34	3	1.0	1.0
27	9.71 747	20	9.78 647	28	10.21 353	9.93 100	8	33	4	1.4	1.3
28	9.71 767	21	9.78 675	29	10.21 325	9.93 092	8	32			
29	9.71 788	21	9.78 704	28	10.21 296	9.93 084	7	31	5	1.8	1.7
30	9.71 809	20	9.78 732	28	10.21 268	9.93 077	8	30	6	2.1	2.0
31	9.71 829	21	9.78 760	29	10.21 240	9.93 069	8	29	7	2.4	2.3
32	9.71 850	20	9.78 789	28	10.21 211	9.93 061	8	28	8	2.8	2.7
33	9.71 870	21	9.78 817	28	10.21 183	9.93 053	7	27	9	3.2	3.0
34	9.71 891	20	9.78 845	29	10.21 155	9.93 046	8	26	10	3.5	3.3
35	9.71 911	21	9.78 874	28	10.21 126	9.93 038	8	25	20	7.0	6.7
36	9.71 932	20	9.78 902	28	10.21 098	9.93 030	8	24	30	10.5	10.0
37	9.71 952	21	9.78 930	29	10.21 070	9.93 022	8	23	40	14.0	13.3
38	9.71 973	21	9.78 959	28	10.21 041	9.93 014	7	22	50	17.5	16.7
39	9.71 994	20	9.78 987	28	10.21 013	9.93 007	8	21			
40	9.72 014	20	9.79 015	28	10.20 985	9.92 999	8	20	//	8	7
41	9.72 034	21	9.79 043	29	10.20 957	9.92 991	8	19	1	0.1	0.1
42	9.72 055	20	9.79 072	28	10.20 928	9.92 983	7	18	2	0.3	0.2
43	9.72 075	21	9.79 100	28	10.20 900	9.92 976	8	17	3	0.4	0.4
44	9.72 096	20	9.79 128	28	10.20 872	9.92 968	8	16	4	0.5	0.5
45	9.72 116	21	9.79 156	29	10.20 844	9.92 960	8	15	5	0.7	0.6
46	9.72 137	20	9.79 185	28	10.20 815	9.92 952	8	14	6	0.8	0.7
47	9.72 157	20	9.79 213	28	10.20 787	9.92 944	8	13	7	0.9	0.8
48	9.72 177	21	9.79 241	28	10.20 759	9.92 936	7	12	8	1.1	0.9
49	9.72 198	20	9.79 269	28	10.20 731	9.92 929	8	11	9	1.2	1.0
50	9.72 218	20	9.79 297	29	10.20 703	9.92 921	8	10	10	1.3	1.2
51	9.72 238	21	9.79 326	28	10.20 674	9.92 913	8	9	20	2.7	2.3
52	9.72 259	20	9.79 354	28	10.20 646	9.92 905	8	8	30	4.0	3.5
53	9.72 279	20	9.79 382	28	10.20 618	9.92 897	8	7	40	5.3	4.7
54	9.72 299	21	9.79 410	28	10.20 590	9.92 889	8	6	50	6.7	6.8
55	9.72 320	20	9.79 438	28	10.20 562	9.92 881	7	5			
56	9.72 340	20	9.79 466	29	10.20 534	9.92 874	8	4			
57	9.72 360	21	9.79 495	28	10.20 505	9.92 866	8	3			
58	9.72 381	20	9.79 523	28	10.20 477	9.92 858	8	2			
59	9.72 401	20	9.79 551	28	10.20 449	9.92 850	8	1			
60	9.72 421	—	9.79 579	—	10.20 421	9.92 842	—	0			
/	L Cos	d	L Ctn	c d	L Tan	L Sin	d	/	Proportional parts		

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

32° (212°)					(327°) 147°				
/	L Sin	d	L Tan	c d	L Ctn	L Cos	d	/	Proportional parts
0	9.72 421	20	9.79 579	28	10.20 421	9.92 842	8	60	
1	9.72 441	20	9.79 607	28	10.20 393	9.92 834	8	59	
2	9.72 461	21	9.79 635	28	10.20 365	9.92 826	8	58	
3	9.72 482	20	9.79 663	28	10.20 337	9.92 818	8	57	
4	9.72 502	20	9.79 691	28	10.20 309	9.92 810	7	56	
5	9.72 522	20	9.79 719	28	10.20 281	9.92 803	8	55	
6	9.72 542	20	9.79 747	29	10.20 253	9.92 795	8	54	
7	9.72 562	20	9.79 776	28	10.20 224	9.92 787	8	53	// 29 28 27
8	9.72 582	20	9.79 804	28	10.20 196	9.92 779	8	52	1 0.5 0.5 0.4
9	9.72 602	20	9.79 832	28	10.20 168	9.92 771	8	51	2 1.0 0.9 0.9
10	9.72 622	21	9.79 860	28	10.20 140	9.92 763	8	50	3 1.4 1.4 1.4
11	9.72 643	20	9.79 888	28	10.20 112	9.92 755	8	49	4 1.9 1.9 1.8
12	9.72 663	20	9.79 916	28	10.20 084	9.92 747	8	48	5 2.4 2.3 2.2
13	9.72 683	20	9.79 944	28	10.20 056	9.92 739	8	47	6 2.9 2.8 2.7
14	9.72 703	20	9.79 972	28	10.20 028	9.92 731	8	46	7 3.4 3.3 3.2
15	9.72 723	20	9.80 000	28	10.20 000	9.92 723	8	45	8 3.9 3.7 3.6
16	9.72 743	20	9.80 028	28	10.19 972	9.92 715	8	44	9 4.4 4.2 4.0
17	9.72 763	20	9.80 056	28	10.19 944	9.92 707	8	43	10 4.8 4.7 4.5
18	9.72 783	20	9.80 084	28	10.19 916	9.92 699	8	42	20 9.7 9.3 9.0
19	9.72 803	20	9.80 112	28	10.19 888	9.92 691	8	41	30 14.5 14.0 13.5
20	9.72 823	20	9.80 140	28	10.19 860	9.92 683	8	40	40 19.3 18.7 18.0
21	9.72 843	20	9.80 168	27	10.19 832	9.92 675	8	39	50 24.2 23.3 22.5
22	9.72 863	20	9.80 195	28	10.19 805	9.92 667	8	38	
23	9.72 883	19	9.80 223	28	10.19 777	9.92 659	8	37	// 21 20 19
24	9.72 902	20	9.80 251	28	10.19 749	9.92 651	8	36	1 0.4 0.3 0.3
25	9.72 922	20	9.80 279	28	10.19 721	9.92 643	8	35	2 0.7 0.7 0.6
26	9.72 942	20	9.80 307	28	10.19 693	9.92 635	8	34	3 1.0 1.0 1.0
27	9.72 962	20	9.80 335	28	10.19 665	9.92 627	8	33	4 1.4 1.3 1.3
28	9.72 982	20	9.80 363	28	10.19 637	9.92 619	8	32	
29	9.73 002	20	9.80 391	28	10.19 609	9.92 611	8	31	5 1.8 1.7 1.6
30	9.73 022	19	9.80 419	28	10.19 581	9.92 603	8	30	6 2.1 2.0 1.9
31	9.73 041	20	9.80 447	27	10.19 553	9.92 595	8	29	7 2.4 2.3 2.2
32	9.73 061	20	9.80 474	28	10.19 526	9.92 587	8	28	8 2.8 2.7 2.5
33	9.73 081	20	9.80 502	28	10.19 498	9.92 579	8	27	9 3.2 3.0 2.8
34	9.73 101	20	9.80 530	28	10.19 470	9.92 571	8	26	10 3.5 3.3 3.2
35	9.73 121	19	9.80 558	28	10.19 442	9.92 563	8	25	20 7.0 6.7 6.3
36	9.73 140	20	9.80 586	28	10.19 414	9.92 555	9	24	30 10.5 10.0 9.5
37	9.73 160	20	9.80 614	28	10.19 386	9.92 546	8	23	40 14.0 13.3 12.7
38	9.73 180	20	9.80 642	27	10.19 358	9.92 538	8	22	50 17.5 16.7 15.8
39	9.73 200	19	9.80 669	28	10.19 331	9.92 530	8	21	// 9 8 7
40	9.73 219	20	9.80 697	28	10.19 303	9.92 522	8	20	1 0.2 0.1 0.1
41	9.73 239	20	9.80 725	28	10.19 275	9.92 514	8	19	2 0.3 0.3 0.2
42	9.73 259	19	9.80 753	28	10.19 247	9.92 506	8	18	3 0.4 0.4 0.4
43	9.73 278	20	9.80 781	27	10.19 219	9.92 498	8	17	4 0.6 0.5 0.5
44	9.73 298	20	9.80 808	28	10.19 192	9.92 490	8	16	
45	9.73 318	19	9.80 836	28	10.19 164	9.92 482	9	15	5 0.8 0.7 0.6
46	9.73 337	20	9.80 864	28	10.19 136	9.92 473	8	14	6 0.9 0.8 0.7
47	9.73 357	20	9.80 892	27	10.19 108	9.92 465	8	13	7 1.0 0.9 0.8
48	9.73 377	19	9.80 919	28	10.19 081	9.92 457	8	12	8 1.2 1.1 0.9
49	9.73 396	20	9.80 947	28	10.19 053	9.92 449	8	11	9 1.4 1.2 1.0
50	9.73 416	19	9.80 975	28	10.19 025	9.92 441	8	10	10 1.5 1.3 1.2
51	9.73 435	20	9.81 003	27	10.18 997	9.92 433	8	9	20 3.0 2.7 2.3
52	9.73 455	19	9.81 030	28	10.18 970	9.92 425	9	8	30 4.5 4.0 3.5
53	9.73 474	20	9.81 058	28	10.18 942	9.92 416	8	7	40 6.0 5.3 4.7
54	9.73 494	19	9.81 086	27	10.18 914	9.92 408	8	6	50 7.5 6.7 5.8
55	9.73 513	20	9.81 113	28	10.18 887	9.92 400	8	5	
56	9.73 533	19	9.81 141	28	10.18 859	9.92 392	8	4	
57	9.73 552	20	9.81 169	27	10.18 831	9.92 384	8	3	
58	9.73 572	19	9.81 196	28	10.18 804	9.92 376	9	2	
59	9.73 591	20	9.81 224	28	10.18 776	9.92 367	8	1	
60	9.73 611	—	9.81 252	—	10.18 748	9.92 359	—	0	
/	L Cos	d	L Ctn	c d	L Tan	L Sin	d	/	Proportional parts

122° (302°)

(237°) 57°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

33° (213°)

(326°) 146°

/	L Sin	d	L Tan	c d	L Ctn	L Cos	d	/	Proportional parts			
0	9.73 611	19	9.81 252	27	10.18 748	9.92 359	8	60				
1	9.73 630	20	9.81 279	28	10.18 721	9.92 351	8	59				
2	9.73 650	19	9.81 307	28	10.18 693	9.92 343	8	58				
3	9.73 669	20	9.81 335	27	10.18 665	9.92 335	9	57				
4	9.73 689	19	9.81 362	28	10.18 638	9.92 326	8	56				
5	9.73 708	19	9.81 390	28	10.18 610	9.92 318	8	55				
6	9.73 727	20	9.81 418	27	10.18 582	9.92 310	8	54				
7	9.73 747	19	9.81 445	28	10.18 555	9.92 302	9	53	//	28	27	
8	9.73 766	19	9.81 473	27	10.18 527	9.92 293	8	52	1	0.5	0.4	
9	9.73 785	20	9.81 500	28	10.18 500	9.92 285	8	51	2	0.9	0.9	
									3	1.4	1.4	
10	9.73 805	19	9.81 528	28	10.18 472	9.92 277	8	50	4	1.9	1.8	
11	9.73 824	19	9.81 556	27	10.18 444	9.92 269	9	49				
12	9.73 843	20	9.81 583	28	10.18 417	9.92 260	8	48	5	2.3	2.2	
13	9.73 863	19	9.81 611	27	10.18 389	9.92 252	8	47	6	2.8	2.7	
14	9.73 882	19	9.81 638	28	10.18 362	9.92 244	9	46	7	3.3	3.2	
									8	3.7	3.6	
15	9.73 901	20	9.81 666	27	10.18 334	9.92 235	8	45	9	4.2	4.0	
16	9.73 921	19	9.81 693	28	10.18 307	9.92 227	8	44				
17	9.73 940	19	9.81 721	27	10.18 279	9.92 219	8	43	10	4.7	4.5	
18	9.73 959	19	9.81 748	28	10.18 252	9.92 211	9	42	20	9.3	9.0	
19	9.73 978	19	9.81 776	27	10.18 224	9.92 202	8	41	30	14.0	13.5	
									40	18.7	18.0	
20	9.73 997	20	9.81 803	28	10.18 197	9.92 194	8	40	50	23.3	22.5	
21	9.74 017	19	9.81 831	27	10.18 169	9.92 186	9	39				
22	9.74 036	19	9.81 858	28	10.18 142	9.92 177	8	38				
23	9.74 055	19	9.81 886	27	10.18 114	9.92 169	8	37	//	20	19	18
24	9.74 074	19	9.81 913	28	10.18 087	9.92 161	9	36	1	0.3	0.3	0.3
									2	0.7	0.6	0.6
25	9.74 093	20	9.81 941	27	10.18 059	9.92 152	8	35	3	1.0	1.0	0.9
26	9.74 113	19	9.81 968	28	10.18 032	9.92 144	8	34	4	1.3	1.3	1.2
27	9.74 132	19	9.81 996	27	10.18 004	9.92 136	9	33				
28	9.74 151	19	9.82 023	28	10.17 977	9.92 127	8	32	5	1.7	1.6	1.5
29	9.74 170	19	9.82 051	27	10.17 949	9.92 119	8	31	6	2.0	1.9	1.8
									7	2.3	2.2	2.1
30	9.74 189	19	9.82 078	28	10.17 922	9.92 111	9	30	8	2.7	2.5	2.4
31	9.74 208	19	9.82 106	27	10.17 894	9.92 102	8	29	9	3.0	2.8	2.7
32	9.74 227	19	9.82 133	28	10.17 867	9.92 094	8	28				
33	9.74 246	19	9.82 161	27	10.17 839	9.92 086	9	27	10	3.3	3.2	3.0
34	9.74 265	19	9.82 188	27	10.17 812	9.92 077	8	26	20	6.7	6.3	6.0
									30	10.0	9.5	9.0
35	9.74 284	19	9.82 215	28	10.17 785	9.92 069	9	25	40	13.3	12.7	12.0
36	9.74 303	19	9.82 243	27	10.17 757	9.92 060	8	24	50	16.7	15.8	15.0
37	9.74 322	19	9.82 270	28	10.17 730	9.92 052	8	23				
38	9.74 341	19	9.82 298	27	10.17 702	9.92 044	9	22				
39	9.74 360	19	9.82 325	27	10.17 675	9.92 035	8	21	//	9	8	
									1	0.2	0.1	
40	9.74 379	19	9.82 352	28	10.17 648	9.92 027	9	20	2	0.3	0.3	
41	9.74 398	19	9.82 380	27	10.17 620	9.92 018	8	19	3	0.4	0.4	
42	9.74 417	19	9.82 407	28	10.17 593	9.92 010	8	18	4	0.6	0.5	
43	9.74 436	19	9.82 435	27	10.17 565	9.92 002	9	17				
44	9.74 455	19	9.82 462	27	10.17 538	9.91 993	8	16	5	0.8	0.7	
									6	0.9	0.8	
45	9.74 474	19	9.82 489	28	10.17 511	9.91 985	9	15	7	1.0	0.9	
46	9.74 493	19	9.82 517	27	10.17 483	9.91 976	8	14	8	1.2	1.1	
47	9.74 512	19	9.82 544	27	10.17 456	9.91 968	9	13	9	1.4	1.2	
48	9.74 531	18	9.82 571	28	10.17 429	9.91 959	8	12				
49	9.74 549	19	9.82 599	27	10.17 401	9.91 951	9	11	10	1.5	1.3	
									20	3.0	2.7	
50	9.74 568	19	9.82 626	27	10.17 374	9.91 942	8	10	30	4.5	4.0	
51	9.74 587	19	9.82 653	28	10.17 347	9.91 934	9	9	40	6.0	5.3	
52	9.74 606	19	9.82 681	27	10.17 319	9.91 925	8	8	50	7.5	6.7	
53	9.74 625	19	9.82 708	27	10.17 292	9.91 917	9	7				
54	9.74 644	18	9.82 735	27	10.17 265	9.91 908	8	6				
55	9.74 662	19	9.82 762	28	10.17 238	9.91 900	9	5				
56	9.74 681	19	9.82 790	27	10.17 210	9.91 891	8	4				
57	9.74 700	19	9.82 817	27	10.17 183	9.91 883	9	3				
58	9.74 719	18	9.82 844	27	10.17 156	9.91 874	8	2				
59	9.74 737	19	9.82 871	28	10.17 129	9.91 866	9	1				
60	9.74 756	—	9.82 899	—	10.17 101	9.91 857	—	0				
/	L Cos	d	L Ctn	c d	L Tan	L Sin	d	/	Proportional parts			

123° (303°)

(236°) 56°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

34° (214°)					(325°) 145°				
/	L Sin	d	L Tan	c d	L Ctn	L Cos	d	/	Proportional parts
0	9.74 756	19	9.82 899	27	10.17 101	9.91 857	8	60	
1	9.74 775	19	9.82 926	27	10.17 074	9.91 849	9	59	
2	9.74 794	18	9.82 953	27	10.17 047	9.91 840	8	58	
3	9.74 812	19	9.82 980	28	10.17 020	9.91 832	9	57	
4	9.74 831	19	9.83 008	27	10.16 992	9.91 823	8	56	
5	9.74 850	18	9.83 035	27	10.16 965	9.91 815	9	55	
6	9.74 868	19	9.83 062	27	10.16 938	9.91 806	8	54	
7	9.74 887	19	9.83 089	28	10.16 911	9.91 798	9	53	
8	9.74 906	18	9.83 117	27	10.16 883	9.91 789	8	52	
9	9.74 924	19	9.83 144	27	10.16 856	9.91 781	9	51	
10	9.74 943	18	9.83 171	27	10.16 829	9.91 772	9	50	
11	9.74 961	19	9.83 198	27	10.16 802	9.91 763	8	49	
12	9.74 980	19	9.83 225	27	10.16 775	9.91 755	9	48	
13	9.74 999	18	9.83 252	28	10.16 748	9.91 746	8	47	
14	9.75 017	19	9.83 280	27	10.16 720	9.91 738	9	46	
15	9.75 036	18	9.83 307	27	10.16 693	9.91 729	9	45	
16	9.75 054	19	9.83 334	27	10.16 666	9.91 720	8	44	
17	9.75 073	18	9.83 361	27	10.16 639	9.91 712	9	43	
18	9.75 091	19	9.83 388	27	10.16 612	9.91 703	8	42	
19	9.75 110	18	9.83 415	27	10.16 585	9.91 695	9	41	
20	9.75 128	19	9.83 442	28	10.16 558	9.91 686	9	40	
21	9.75 147	18	9.83 470	27	10.16 530	9.91 677	8	39	
22	9.75 165	19	9.83 497	27	10.16 503	9.91 669	9	38	
23	9.75 184	18	9.83 524	27	10.16 476	9.91 660	9	37	
24	9.75 202	19	9.83 551	27	10.16 449	9.91 651	8	36	
25	9.75 221	18	9.83 578	27	10.16 422	9.91 643	9	35	
26	9.75 239	19	9.83 605	27	10.16 395	9.91 634	9	34	
27	9.75 258	18	9.83 632	27	10.16 368	9.91 625	8	33	
28	9.75 276	18	9.83 659	27	10.16 341	9.91 617	9	32	
29	9.75 294	19	9.83 686	27	10.16 314	9.91 608	9	31	
30	9.75 313	18	9.83 713	27	10.16 287	9.91 599	8	30	
31	9.75 331	19	9.83 740	28	10.16 260	9.91 591	9	29	
32	9.75 350	18	9.83 768	27	10.16 232	9.91 582	8	28	
33	9.75 368	18	9.83 795	27	10.16 205	9.91 573	9	27	
34	9.75 386	19	9.83 822	27	10.16 178	9.91 565	9	26	
35	9.75 405	18	9.83 849	27	10.16 151	9.91 556	9	25	
36	9.75 423	18	9.83 876	27	10.16 124	9.91 547	9	24	
37	9.75 441	18	9.83 903	27	10.16 097	9.91 538	8	23	
38	9.75 459	19	9.83 930	27	10.16 070	9.91 530	9	22	
39	9.75 478	18	9.83 957	27	10.16 043	9.91 521	9	21	
40	9.75 496	18	9.83 984	27	10.16 016	9.91 512	8	20	
41	9.75 514	19	9.84 011	27	10.15 989	9.91 504	9	19	
42	9.75 533	18	9.84 038	27	10.15 962	9.91 495	9	18	
43	9.75 551	18	9.84 065	27	10.15 935	9.91 486	9	17	
44	9.75 569	18	9.84 092	27	10.15 908	9.91 477	8	16	
45	9.75 587	18	9.84 119	27	10.15 881	9.91 469	9	15	
46	9.75 605	19	9.84 146	27	10.15 854	9.91 460	9	14	
47	9.75 624	18	9.84 173	27	10.15 827	9.91 451	9	13	
48	9.75 642	18	9.84 200	27	10.15 800	9.91 442	9	12	
49	9.75 660	18	9.84 227	27	10.15 773	9.91 433	8	11	
50	9.75 678	18	9.84 254	26	10.15 746	9.91 425	9	10	
51	9.75 696	18	9.84 280	27	10.15 720	9.91 416	9	9	
52	9.75 714	19	9.84 307	27	10.15 693	9.91 407	9	8	
53	9.75 733	18	9.84 334	27	10.15 666	9.91 398	9	7	
54	9.75 751	18	9.84 361	27	10.15 639	9.91 389	8	6	
55	9.75 769	18	9.84 388	27	10.15 612	9.91 381	9	5	
56	9.75 787	18	9.84 415	27	10.15 585	9.91 372	9	4	
57	9.75 805	18	9.84 442	27	10.15 558	9.91 363	9	3	
58	9.75 823	18	9.84 469	27	10.15 531	9.91 354	9	2	
59	9.75 841	18	9.84 496	27	10.15 504	9.91 345	9	1	
60	9.75 859	—	9.84 523	—	10.15 477	9.91 336	—	0	
/	L Cos	d	L Ctn	c d	L Tan	L Sin	d	/	Proportional parts

124° (304°)

(235°) 55°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

35° (215°)					(324°) 144				
/	L Sin	d	L Tan	cd	L Ctn	L Cos	d	/	Proportional parts
0	9.75 859	18	9.84 523	27	10.15 477	9.91 336	8	60	
1	9.75 877	18	9.84 550	26	10.15 450	9.91 328	9	59	
2	9.75 895	18	9.84 576	27	10.15 424	9.91 319	9	58	
3	9.75 913	18	9.84 603	27	10.15 397	9.91 310	9	57	
4	9.75 931	18	9.84 630	27	10.15 370	9.91 301	9	56	
5	9.75 949	18	9.84 657	27	10.15 343	9.91 292	9	55	
6	9.75 967	18	9.84 684	27	10.15 316	9.91 283	9	54	
7	9.75 985	18	9.84 711	27	10.15 289	9.91 274	8	53	
8	9.76 003	18	9.84 738	26	10.15 262	9.91 266	9	52	
9	9.76 021	18	9.84 764	27	10.15 236	9.91 257	9	51	
10	9.76 039	18	9.84 791	27	10.15 209	9.91 248	9	50	
11	9.76 057	18	9.84 818	27	10.15 182	9.91 239	9	49	
12	9.76 075	18	9.84 845	27	10.15 155	9.91 230	9	48	
13	9.76 093	18	9.84 872	27	10.15 128	9.91 221	9	47	
14	9.76 111	18	9.84 899	26	10.15 101	9.91 212	9	46	
15	9.76 129	17	9.84 925	27	10.15 075	9.91 203	9	45	
16	9.76 146	18	9.84 952	27	10.15 048	9.91 194	9	44	
17	9.76 164	18	9.84 979	27	10.15 021	9.91 185	9	43	
18	9.76 182	18	9.85 006	27	10.14 994	9.91 176	9	42	
19	9.76 200	18	9.85 033	26	10.14 967	9.91 167	9	41	
20	9.76 218	18	9.85 059	27	10.14 941	9.91 158	9	40	
21	9.76 236	17	9.85 086	27	10.14 914	9.91 149	8	39	
22	9.76 253	18	9.85 113	27	10.14 887	9.91 141	9	38	
23	9.76 271	18	9.85 140	26	10.14 860	9.91 132	9	37	
24	9.76 289	18	9.85 166	27	10.14 834	9.91 123	9	36	
25	9.76 307	17	9.85 193	27	10.14 807	9.91 114	9	35	
26	9.76 324	18	9.85 220	27	10.14 780	9.91 105	9	34	
27	9.76 342	18	9.85 247	26	10.14 753	9.91 096	9	33	
28	9.76 360	18	9.85 273	27	10.14 727	9.91 087	9	32	
29	9.76 378	17	9.85 300	27	10.14 700	9.91 078	9	31	
30	9.76 395	18	9.85 327	27	10.14 673	9.91 069	9	30	
31	9.76 413	18	9.85 354	26	10.14 646	9.91 060	9	29	
32	9.76 431	17	9.85 380	27	10.14 620	9.91 051	9	28	
33	9.76 448	18	9.85 407	27	10.14 593	9.91 042	9	27	
34	9.76 466	18	9.85 434	26	10.14 566	9.91 033	10	26	
35	9.76 484	17	9.85 460	27	10.14 540	9.91 023	9	25	
36	9.76 501	18	9.85 487	27	10.14 513	9.91 014	9	24	
37	9.76 519	18	9.85 514	26	10.14 486	9.91 005	9	23	
38	9.76 537	17	9.85 540	27	10.14 460	9.90 996	9	22	
39	9.76 554	18	9.85 567	27	10.14 433	9.90 987	9	21	
40	9.76 572	18	9.85 594	26	10.14 406	9.90 978	9	20	
41	9.76 590	17	9.85 620	27	10.14 380	9.90 969	9	19	
42	9.76 607	18	9.85 647	27	10.14 353	9.90 960	9	18	
43	9.76 625	17	9.85 674	26	10.14 326	9.90 951	9	17	
44	9.76 642	18	9.85 700	27	10.14 300	9.90 942	9	16	
45	9.76 660	17	9.85 727	27	10.14 273	9.90 933	9	15	
46	9.76 677	18	9.85 754	26	10.14 246	9.90 924	9	14	
47	9.76 695	17	9.85 780	27	10.14 220	9.90 915	9	13	
48	9.76 712	18	9.85 807	27	10.14 193	9.90 906	10	12	
49	9.76 730	17	9.85 834	26	10.14 166	9.90 896	9	11	
50	9.76 747	18	9.85 860	27	10.14 140	9.90 887	9	10	
51	9.76 765	17	9.85 887	26	10.14 113	9.90 878	9	9	
52	9.76 782	18	9.85 913	27	10.14 087	9.90 869	9	8	
53	9.76 800	17	9.85 940	27	10.14 060	9.90 860	9	7	
54	9.76 817	18	9.85 967	26	10.14 033	9.90 851	9	6	
55	9.76 835	17	9.85 993	27	10.14 007	9.90 842	10	5	
56	9.76 852	18	9.86 020	26	10.13 980	9.90 832	9	4	
57	9.76 870	17	9.86 046	27	10.13 954	9.90 823	9	3	
58	9.76 887	17	9.86 073	27	10.13 927	9.90 814	9	2	
59	9.76 904	18	9.86 100	26	10.13 900	9.90 805	9	1	
60	9.76 922		9.86 126		10.13 874	9.90 796		0	
/	L Cos	d	L Ctn	cd	L Tan	L Sin	d	/	Proportional parts

125° (305°)

(234°) 54°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

36° (216)					(323°) 143°				
°	L Sin	d	L Tan	c d	L Ctn	L Cos	d	°	Proportional parts
0	9.76 922	17	9.86 126	27	10.13 874	9.90 796	9	60	
1	9.76 939	18	9.86 153	26	10.13 847	9.90 787	10	59	
2	9.76 957	17	9.86 179	27	10.13 821	9.90 777	9	58	
3	9.76 974	17	9.86 206	26	10.13 794	9.90 768	9	57	
4	9.76 991	18	9.86 232	27	10.13 768	9.90 759	9	56	
5	9.77 009	17	9.86 259	26	10.13 741	9.90 750	9	55	
6	9.77 026	17	9.86 285	27	10.13 715	9.90 741	10	54	
7	9.77 043	18	9.86 312	26	10.13 688	9.90 731	9	53	
8	9.77 061	17	9.86 338	27	10.13 662	9.90 722	9	52	
9	9.77 078	17	9.86 365	27	10.13 635	9.90 713	9	51	
10	9.77 095	17	9.86 392	26	10.13 608	9.90 704	10	50	
11	9.77 112	18	9.86 418	27	10.13 582	9.90 694	9	49	
12	9.77 130	17	9.86 445	26	10.13 555	9.90 685	9	48	
13	9.77 147	17	9.86 471	27	10.13 529	9.90 676	9	47	
14	9.77 164	17	9.86 498	26	10.13 502	9.90 667	10	46	
15	9.77 181	18	9.86 524	27	10.13 476	9.90 657	9	45	
16	9.77 199	17	9.86 551	26	10.13 449	9.90 648	9	44	
17	9.77 216	17	9.86 577	26	10.13 423	9.90 639	9	43	
18	9.77 233	17	9.86 603	27	10.13 397	9.90 630	10	42	
19	9.77 250	18	9.86 630	26	10.13 370	9.90 620	9	41	
20	9.77 268	17	9.86 656	27	10.13 344	9.90 611	9	40	
21	9.77 285	17	9.86 683	26	10.13 317	9.90 602	10	39	
22	9.77 302	17	9.86 709	27	10.13 291	9.90 592	9	38	
23	9.77 319	17	9.86 736	26	10.13 264	9.90 583	9	37	
24	9.77 336	17	9.86 762	27	10.13 238	9.90 574	9	36	
25	9.77 353	17	9.86 789	26	10.13 211	9.90 565	10	35	
26	9.77 370	17	9.86 815	27	10.13 185	9.90 555	9	34	
27	9.77 387	18	9.86 842	26	10.13 158	9.90 546	9	33	
28	9.77 405	17	9.86 868	26	10.13 132	9.90 537	10	32	
29	9.77 422	17	9.86 894	27	10.13 106	9.90 527	9	31	
30	9.77 439	17	9.86 921	26	10.13 079	9.90 518	9	30	
31	9.77 456	17	9.86 947	27	10.13 053	9.90 509	10	29	
32	9.77 473	17	9.86 974	26	10.13 026	9.90 499	9	28	
33	9.77 490	17	9.87 000	27	10.13 000	9.90 490	10	27	
34	9.77 507	17	9.87 027	26	10.12 973	9.90 480	9	26	
35	9.77 524	17	9.87 053	26	10.12 947	9.90 471	9	25	
36	9.77 541	17	9.87 079	27	10.12 921	9.90 462	10	24	
37	9.77 558	17	9.87 106	26	10.12 894	9.90 452	9	23	
38	9.77 575	17	9.87 132	26	10.12 868	9.90 443	9	22	
39	9.77 592	17	9.87 158	27	10.12 842	9.90 434	10	21	
40	9.77 609	17	9.87 185	26	10.12 815	9.90 424	9	20	
41	9.77 626	17	9.87 211	27	10.12 789	9.90 415	10	19	
42	9.77 643	17	9.87 238	26	10.12 762	9.90 405	9	18	
43	9.77 660	17	9.87 264	26	10.12 736	9.90 396	10	17	
44	9.77 677	17	9.87 290	27	10.12 710	9.90 386	9	16	
45	9.77 694	17	9.87 317	26	10.12 683	9.90 377	9	15	
46	9.77 711	17	9.87 343	26	10.12 657	9.90 368	10	14	
47	9.77 728	16	9.87 369	27	10.12 631	9.90 358	9	13	
48	9.77 744	17	9.87 396	26	10.12 604	9.90 349	10	12	
49	9.77 761	17	9.87 422	26	10.12 578	9.90 339	9	11	
50	9.77 778	17	9.87 448	27	10.12 552	9.90 330	10	10	
51	9.77 795	17	9.87 475	26	10.12 525	9.90 320	9	9	
52	9.77 812	17	9.87 501	26	10.12 499	9.90 311	10	8	
53	9.77 829	17	9.87 527	27	10.12 473	9.90 301	9	7	
54	9.77 846	16	9.87 554	26	10.12 446	9.90 292	10	6	
55	9.77 862	17	9.87 580	26	10.12 420	9.90 282	9	5	
56	9.77 879	17	9.87 606	27	10.12 394	9.90 273	10	4	
57	9.77 896	17	9.87 633	26	10.12 367	9.90 263	9	3	
58	9.77 913	17	9.87 659	26	10.12 341	9.90 254	10	2	
59	9.77 930	16	9.87 685	26	10.12 315	9.90 244	9	1	
60	9.77 946	—	9.87 711	—	10.12 289	9.90 235	—	0	
°	L Cos	d	L Ctn	c d	L Tan	L Sin	d	°	Proportional parts

126° (306°)

(233°) 53°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

37° (217°)						(322°) 142°					
<i>i</i>	L Sin	<i>d</i>	L Tan	<i>c d</i>	L Ctn	L Cos	<i>d</i>	<i>i</i>	Proportional parts		
0	9.77 946	17	9.87 711	27	10.12 289	9.90 235	10	60			
1	9.77 963	17	9.87 738	26	10.12 262	9.90 225	9	59			
2	9.77 980	17	9.87 764	26	10.12 236	9.90 216	10	58			
3	9.77 997	16	9.87 790	27	10.12 210	9.90 206	9	57			
4	9.78 013	17	9.87 817	26	10.12 183	9.90 197	10	56			
5	9.78 030	17	9.87 843	26	10.12 157	9.90 187	9	55			
6	9.78 047	16	9.87 869	26	10.12 131	9.90 178	10	54			
7	9.78 063	17	9.87 895	27	10.12 105	9.90 168	9	53			
8	9.78 080	17	9.87 922	26	10.12 078	9.90 159	10	52			
9	9.78 097	16	9.87 948	26	10.12 052	9.90 149	10	51			
10	9.78 113	17	9.87 974	26	10.12 026	9.90 139	9	50			
11	9.78 130	17	9.88 000	27	10.12 000	9.90 130	10	49			
12	9.78 147	16	9.88 027	26	10.11 973	9.90 120	9	48			
13	9.78 163	17	9.88 053	26	10.11 947	9.90 111	10	47			
14	9.78 180	17	9.88 079	26	10.11 921	9.90 101	10	46			
15	9.78 197	16	9.88 105	26	10.11 895	9.90 091	9	45			
16	9.78 213	17	9.88 131	27	10.11 869	9.90 082	10	44			
17	9.78 230	16	9.88 158	26	10.11 842	9.90 072	9	43			
18	9.78 246	17	9.88 184	26	10.11 816	9.90 063	10	42			
19	9.78 263	17	9.88 210	26	10.11 790	9.90 053	10	41			
20	9.78 280	16	9.88 236	26	10.11 764	9.90 043	9	40			
21	9.78 296	17	9.88 262	27	10.11 738	9.90 034	10	39			
22	9.78 313	16	9.88 289	26	10.11 711	9.90 024	10	38			
23	9.78 329	17	9.88 315	26	10.11 685	9.90 014	9	37			
24	9.78 346	16	9.88 341	26	10.11 659	9.90 005	10	36			
25	9.78 362	17	9.88 367	26	10.11 633	9.89 995	10	35			
26	9.78 379	16	9.88 393	27	10.11 607	9.89 985	9	34			
27	9.78 395	17	9.88 420	26	10.11 580	9.89 976	10	33			
28	9.78 412	16	9.88 446	26	10.11 554	9.89 966	10	32			
29	9.78 428	17	9.88 472	26	10.11 528	9.89 956	9	31			
30	9.78 445	16	9.88 498	26	10.11 502	9.89 947	10	30			
31	9.78 461	17	9.88 524	26	10.11 476	9.89 937	10	29			
32	9.78 478	16	9.88 550	27	10.11 450	9.89 927	9	28			
33	9.78 494	16	9.88 577	26	10.11 423	9.89 918	10	27			
34	9.78 510	17	9.88 603	26	10.11 397	9.89 908	10	26			
35	9.78 527	16	9.88 629	26	10.11 371	9.89 898	10	25			
36	9.78 543	17	9.88 655	26	10.11 345	9.89 888	9	24			
37	9.78 560	16	9.88 681	26	10.11 319	9.89 879	10	23			
38	9.78 576	16	9.88 707	26	10.11 293	9.89 869	10	22			
39	9.78 592	17	9.88 733	26	10.11 267	9.89 859	10	21			
40	9.78 609	16	9.88 759	27	10.11 241	9.89 849	9	20			
41	9.78 625	17	9.88 786	26	10.11 214	9.89 840	10	19			
42	9.78 642	16	9.88 812	26	10.11 188	9.89 830	10	18			
43	9.78 658	16	9.88 838	26	10.11 162	9.89 820	10	17			
44	9.78 674	17	9.88 864	26	10.11 136	9.89 810	9	16			
45	9.78 691	16	9.88 890	26	10.11 110	9.89 801	10	15			
46	9.78 707	16	9.88 916	26	10.11 084	9.89 791	10	14			
47	9.78 723	16	9.88 942	26	10.11 058	9.89 781	10	13			
48	9.78 739	17	9.88 968	26	10.11 032	9.89 771	10	12			
49	9.78 756	16	9.88 994	26	10.11 006	9.89 761	9	11			
50	9.78 772	16	9.89 020	26	10.10 980	9.89 752	10	10			
51	9.78 788	17	9.89 046	27	10.10 954	9.89 742	10	9			
52	9.78 805	16	9.89 073	26	10.10 927	9.89 732	10	8			
53	9.78 821	16	9.89 099	26	10.10 901	9.89 722	10	7			
54	9.78 837	16	9.89 125	26	10.10 875	9.89 712	10	6			
55	9.78 853	16	9.89 151	26	10.10 849	9.89 702	9	5			
56	9.78 869	17	9.89 177	26	10.10 823	9.89 693	10	4			
57	9.78 886	16	9.89 203	26	10.10 797	9.89 683	10	3			
58	9.78 902	16	9.89 229	26	10.10 771	9.89 673	10	2			
59	9.78 918	16	9.89 255	26	10.10 745	9.89 663	10	1			
60	9.78 934	—	9.89 281	—	10.10 719	9.89 653	—	0			
<i>i</i>	L Cos	<i>d</i>	L Ctn	<i>c d</i>	L Tan	L Sin	<i>d</i>	<i>i</i>	Proportional parts		

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

38° (218°)						(321°) 141°					
/	L Sin	d	L Tan	c d	L Ctn	L Cos	d	/	Proportional parts		
0	9.78 934	16	9.89 281	26	10.10 719	9.89 653	10	60			
1	9.78 950	17	9.89 307	26	10.10 693	9.89 643	10	59			
2	9.78 967	16	9.89 333	26	10.10 667	9.89 633	9	58			
3	9.78 983	16	9.89 359	26	10.10 641	9.89 624	10	57			
4	9.78 999	16	9.89 385	26	10.10 615	9.89 614	10	56			
5	9.79 015	16	9.89 411	26	10.10 589	9.89 604	10	55			
6	9.79 031	16	9.89 437	26	10.10 563	9.89 594	10	54			
7	9.79 047	16	9.89 463	26	10.10 537	9.89 584	10	53			
8	9.79 063	16	9.89 489	26	10.10 511	9.89 574	10	52			
9	9.79 079	16	9.89 515	26	10.10 485	9.89 564	10	51			
10	9.79 095	16	9.89 541	26	10.10 459	9.89 554	10	50			
11	9.79 111	17	9.89 567	26	10.10 433	9.89 544	10	49			
12	9.79 128	16	9.89 593	26	10.10 407	9.89 534	10	48			
13	9.79 144	16	9.89 619	26	10.10 381	9.89 524	10	47			
14	9.79 160	16	9.89 645	26	10.10 355	9.89 514	10	46			
15	9.79 176	16	9.89 671	26	10.10 329	9.89 504	9	45			
16	9.79 192	16	9.89 697	26	10.10 303	9.89 495	10	44			
17	9.79 208	16	9.89 723	26	10.10 277	9.89 485	10	43			
18	9.79 224	16	9.89 749	26	10.10 251	9.89 475	10	42			
19	9.79 240	16	9.89 775	26	10.10 225	9.89 465	10	41			
20	9.79 256	16	9.89 801	26	10.10 199	9.89 455	10	40			
21	9.79 272	16	9.89 827	26	10.10 173	9.89 445	10	39			
22	9.79 288	16	9.89 853	26	10.10 147	9.89 435	10	38			
23	9.79 304	15	9.89 879	26	10.10 121	9.89 425	10	37			
24	9.79 319	16	9.89 905	26	10.10 095	9.89 415	10	36			
25	9.79 335	16	9.89 931	26	10.10 069	9.89 405	10	35			
26	9.79 351	16	9.89 957	26	10.10 043	9.89 395	10	34			
27	9.79 367	16	9.89 983	26	10.10 017	9.89 385	10	33			
28	9.79 383	16	9.90 009	26	10.09 991	9.89 375	11	32			
29	9.79 399	16	9.90 035	26	10.09 965	9.89 364	10	31			
30	9.79 415	16	9.90 061	25	10.09 939	9.89 354	10	30			
31	9.79 431	16	9.90 086	26	10.09 914	9.89 344	10	29			
32	9.79 447	16	9.90 112	26	10.09 888	9.89 334	10	28			
33	9.79 463	15	9.90 138	26	10.09 862	9.89 324	10	27			
34	9.79 478	16	9.90 164	26	10.09 836	9.89 314	10	26			
35	9.79 494	16	9.90 190	26	10.09 810	9.89 304	10	25			
36	9.79 510	16	9.90 216	26	10.09 784	9.89 294	10	24			
37	9.79 526	16	9.90 242	26	10.09 758	9.89 284	10	23			
38	9.79 542	16	9.90 268	26	10.09 732	9.89 274	10	22			
39	9.79 558	15	9.90 294	26	10.09 706	9.89 264	10	21			
40	9.79 573	16	9.90 320	26	10.09 680	9.89 254	10	20			
41	9.79 589	16	9.90 346	25	10.09 654	9.89 244	11	19			
42	9.79 605	16	9.90 371	26	10.09 629	9.89 233	10	18			
43	9.79 621	15	9.90 397	26	10.09 603	9.89 223	10	17			
44	9.79 636	16	9.90 423	26	10.09 577	9.89 213	10	16			
45	9.79 652	16	9.90 449	26	10.09 551	9.89 203	10	15			
46	9.79 668	16	9.90 475	26	10.09 525	9.89 193	10	14			
47	9.79 684	15	9.90 501	26	10.09 499	9.89 183	10	13			
48	9.79 699	16	9.90 527	26	10.09 473	9.89 173	11	12			
49	9.79 715	16	9.90 553	25	10.09 447	9.89 162	10	11			
50	9.79 731	15	9.90 578	26	10.09 422	9.89 152	10	10			
51	9.79 746	16	9.90 604	26	10.09 396	9.89 142	10	9			
52	9.79 762	16	9.90 630	26	10.09 370	9.89 132	10	8			
53	9.79 778	15	9.90 656	26	10.09 344	9.89 122	10	7			
54	9.79 793	16	9.90 682	26	10.09 318	9.89 112	11	6			
55	9.79 809	16	9.90 708	26	10.09 292	9.89 101	10	5			
56	9.79 825	15	9.90 734	25	10.09 266	9.89 091	10	4			
57	9.79 840	16	9.90 759	26	10.09 241	9.89 081	10	3			
58	9.79 856	16	9.90 785	26	10.09 215	9.89 071	11	2			
59	9.79 872	15	9.90 811	26	10.09 189	9.89 060	10	1			
60	9.79 887		9.90 837		10.09 163	9.89 050		0	Proportional parts		
/	L Cos	d	L Ctn	c d	L Tan	L Sin	d	/			

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

39° (219°)						(320°) 140°					
/	L Sin	d	L Tan	c d	L Ctn	L Cos	d	/	Proportional parts		
0	9.79 887	16	9.90 837	26	10.09 163	9.89 050	10	60			
1	9.79 903	15	9.90 863	26	10.09 137	9.89 040	10	59			
2	9.79 918	16	9.90 889	25	10.09 111	9.89 030	10	58			
3	9.79 934	16	9.90 914	26	10.09 086	9.89 020	11	57			
4	9.79 950	15	9.90 940	26	10.09 060	9.89 009	10	56			
5	9.79 965	16	9.90 966	26	10.09 034	9.88 999	10	55			
6	9.79 981	15	9.90 992	26	10.09 008	9.88 989	11	54			
7	9.79 996	16	9.91 018	25	10.08 982	9.88 978	10	53	//	26	25
8	9.80 012	15	9.91 043	26	10.08 957	9.88 968	10	52	1	0.4	0.4
9	9.80 027	16	9.91 069	26	10.08 931	9.88 958	10	51	2	0.9	0.8
10	9.80 043	15	9.91 095	26	10.08 905	9.88 948	11	50	3	1.3	1.2
11	9.80 058	16	9.91 121	26	10.08 879	9.88 937	10	49	4	1.7	1.7
12	9.80 074	15	9.91 147	25	10.08 853	9.88 927	10	48	5	2.2	2.1
13	9.80 089	16	9.91 172	26	10.08 828	9.88 917	11	47	6	2.6	2.5
14	9.80 105	15	9.91 198	26	10.08 802	9.88 906	10	46	7	3.0	2.9
15	9.80 120	16	9.91 224	26	10.08 776	9.88 896	10	45	8	3.5	3.3
16	9.80 136	15	9.91 250	26	10.08 750	9.88 886	11	44	9	3.9	3.8
17	9.80 151	15	9.91 276	25	10.08 724	9.88 875	10	43			
18	9.80 166	16	9.91 301	26	10.08 699	9.88 865	10	42	10	4.3	4.2
19	9.80 182	15	9.91 327	26	10.08 673	9.88 855	11	41	20	8.7	8.3
20	9.80 197	16	9.91 353	26	10.08 647	9.88 844	10	40	30	13.0	12.5
21	9.80 213	15	9.91 379	25	10.08 621	9.88 834	10	39	40	17.3	16.7
22	9.80 228	16	9.91 404	26	10.08 596	9.88 824	11	38	50	21.7	20.8
23	9.80 244	15	9.91 430	26	10.08 570	9.88 813	10	37			
24	9.80 259	15	9.91 456	26	10.08 544	9.88 803	10	36	//	16	15
25	9.80 274	16	9.91 482	25	10.08 518	9.88 793	11	35	1	0.3	0.2
26	9.80 290	15	9.91 507	26	10.08 493	9.88 782	10	34	2	0.5	0.5
27	9.80 305	15	9.91 533	26	10.08 467	9.88 772	11	33	3	0.8	0.8
28	9.80 320	16	9.91 559	26	10.08 441	9.88 761	10	32	4	1.1	1.0
29	9.80 336	15	9.91 585	25	10.08 415	9.88 751	10	31	5	1.3	1.2
30	9.80 351	15	9.91 610	26	10.08 390	9.88 741	11	30	6	1.6	1.5
31	9.80 366	16	9.91 636	26	10.08 364	9.88 730	10	29	7	1.9	1.8
32	9.80 382	15	9.91 662	26	10.08 338	9.88 720	11	28	8	2.1	2.0
33	9.80 397	15	9.91 688	25	10.08 312	9.88 709	10	27	9	2.4	2.2
34	9.80 412	16	9.91 713	26	10.08 287	9.88 699	11	26	10	2.7	2.5
35	9.80 428	15	9.91 739	26	10.08 261	9.88 688	10	25	20	5.3	5.0
36	9.80 443	15	9.91 765	26	10.08 235	9.88 678	10	24	30	8.0	7.5
37	9.80 458	15	9.91 791	25	10.08 209	9.88 668	11	23	40	10.7	10.0
38	9.80 473	16	9.91 816	26	10.08 184	9.88 657	10	22	50	13.3	12.5
39	9.80 489	15	9.91 842	26	10.08 158	9.88 647	11	21			
40	9.80 504	15	9.91 868	25	10.08 132	9.88 636	10	20	//	11	10
41	9.80 519	15	9.91 893	26	10.08 107	9.88 626	11	19	1	0.2	0.2
42	9.80 534	16	9.91 919	26	10.08 081	9.88 615	10	18	2	0.4	0.3
43	9.80 550	15	9.91 945	26	10.08 055	9.88 605	11	17	3	0.6	0.5
44	9.80 565	15	9.91 971	25	10.08 029	9.88 594	10	16	4	0.7	0.7
45	9.80 580	15	9.91 996	26	10.08 004	9.88 584	11	15	5	0.9	0.8
46	9.80 595	15	9.92 022	26	10.07 978	9.88 573	10	14	6	1.1	1.0
47	9.80 610	15	9.92 048	25	10.07 952	9.88 563	11	13	7	1.3	1.2
48	9.80 625	16	9.92 073	26	10.07 927	9.88 552	10	12	8	1.5	1.3
49	9.80 641	15	9.92 099	26	10.07 901	9.88 542	11	11	9	1.6	1.5
50	9.80 656	15	9.92 125	25	10.07 875	9.88 531	10	10	10	1.8	1.7
51	9.80 671	15	9.92 150	26	10.07 850	9.88 521	11	9	20	3.7	3.3
52	9.80 686	15	9.92 176	26	10.07 824	9.88 510	11	8	30	5.5	5.0
53	9.80 701	15	9.92 202	25	10.07 798	9.88 499	10	7	40	7.3	6.7
54	9.80 716	15	9.92 227	26	10.07 773	9.88 489	11	6	50	9.2	8.3
55	9.80 731	15	9.92 253	26	10.07 747	9.88 478	10	5			
56	9.80 746	16	9.92 279	25	10.07 721	9.88 468	11	4			
57	9.80 762	15	9.92 304	26	10.07 696	9.88 457	10	3			
58	9.80 777	15	9.92 330	26	10.07 670	9.88 447	11	2			
59	9.80 792	15	9.92 356	25	10.07 644	9.88 436	11	1			
60	9.80 807		9.92 381		10.07 619	9.88 425		0			
/	L Cos	d	L Ctn	c d	L Tan	L Sin	d	/	Proportional parts		

129° (309°)

(230°) 50°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

40° (220°)						(319°) 139°						Proportional parts		
/	L Sin	d	L Tan	c d	L Ctn	L Cos	d	/						
0	9.80 807	15	9.92 381	26	10.07 619	9.88 425	10	60						
1	9.80 822	15	9.92 407	26	10.07 593	9.88 415	11	59						
2	9.80 837	15	9.92 433	25	10.07 567	9.88 404	10	58						
3	9.80 852	15	9.92 458	26	10.07 542	9.88 394	11	57						
4	9.80 867	15	9.92 484	26	10.07 516	9.88 383	11	56						
5	9.80 882	15	9.92 510	25	10.07 490	9.88 372	10	55						
6	9.80 897	15	9.92 535	26	10.07 465	9.88 362	11	54						
7	9.80 912	15	9.92 561	26	10.07 439	9.88 351	11	53						
8	9.80 927	15	9.92 587	25	10.07 413	9.88 340	10	52						
9	9.80 942	15	9.92 612	26	10.07 388	9.88 330	11	51						
10	9.80 957	15	9.92 638	25	10.07 362	9.88 319	11	50						
11	9.80 972	15	9.92 663	26	10.07 337	9.88 308	10	49						
12	9.80 987	15	9.92 689	26	10.07 311	9.88 298	11	48						
13	9.81 002	15	9.92 715	25	10.07 285	9.88 287	11	47						
14	9.81 017	15	9.92 740	26	10.07 260	9.88 276	10	46						
15	9.81 032	15	9.92 766	26	10.07 234	9.88 266	11	45						
16	9.81 047	14	9.92 792	25	10.07 208	9.88 255	11	44						
17	9.81 061	15	9.92 817	26	10.07 183	9.88 244	10	43						
18	9.81 076	15	9.92 843	25	10.07 157	9.88 234	11	42						
19	9.81 091	15	9.92 868	26	10.07 132	9.88 223	11	41						
20	9.81 106	15	9.92 894	26	10.07 106	9.88 212	11	40						
21	9.81 121	15	9.92 920	25	10.07 080	9.88 201	10	39						
22	9.81 136	15	9.92 945	26	10.07 055	9.88 191	11	38						
23	9.81 151	15	9.92 971	25	10.07 029	9.88 180	11	37						
24	9.81 166	14	9.92 996	26	10.07 004	9.88 169	11	36						
25	9.81 180	15	9.93 022	26	10.06 978	9.88 158	10	35						
26	9.81 195	15	9.93 048	25	10.06 952	9.88 148	11	34						
27	9.81 210	15	9.93 073	26	10.06 927	9.88 137	11	33						
28	9.81 225	15	9.93 099	25	10.06 901	9.88 126	11	32						
29	9.81 240	14	9.93 124	26	10.06 876	9.88 115	10	31						
30	9.81 254	15	9.93 150	25	10.06 850	9.88 105	11	30						
31	9.81 269	15	9.93 175	26	10.06 825	9.88 094	11	29						
32	9.81 284	15	9.93 201	26	10.06 799	9.88 083	11	28						
33	9.81 299	15	9.93 227	25	10.06 773	9.88 072	11	27						
34	9.81 314	14	9.93 252	26	10.06 748	9.88 061	10	26						
35	9.81 328	15	9.93 278	25	10.06 722	9.88 051	11	25						
36	9.81 343	15	9.93 303	26	10.06 697	9.88 040	11	24						
37	9.81 358	14	9.93 329	25	10.06 671	9.88 029	11	23						
38	9.81 372	15	9.93 354	26	10.06 646	9.88 018	11	22						
39	9.81 387	15	9.93 380	26	10.06 620	9.88 007	11	21						
40	9.81 402	15	9.93 406	25	10.06 594	9.87 996	11	20						
41	9.81 417	14	9.93 431	26	10.06 569	9.87 985	10	19						
42	9.81 431	15	9.93 457	25	10.06 543	9.87 975	11	18						
43	9.81 446	15	9.93 482	26	10.06 518	9.87 964	11	17						
44	9.81 461	14	9.93 508	25	10.06 492	9.87 953	11	16						
45	9.81 475	15	9.93 533	26	10.06 467	9.87 942	11	15						
46	9.81 490	15	9.93 559	25	10.06 441	9.87 931	11	14						
47	9.81 505	14	9.93 584	26	10.06 416	9.87 920	11	13						
48	9.81 519	15	9.93 610	26	10.06 390	9.87 909	11	12						
49	9.81 534	15	9.93 636	25	10.06 364	9.87 898	11	11						
50	9.81 549	14	9.93 661	26	10.06 339	9.87 887	10	10						
51	9.81 563	15	9.93 687	25	10.06 313	9.87 877	11	9						
52	9.81 578	14	9.93 712	26	10.06 288	9.87 866	11	8						
53	9.81 592	15	9.93 738	25	10.06 262	9.87 855	11	7						
54	9.81 607	15	9.93 763	26	10.06 237	9.87 844	11	6						
55	9.81 622	14	9.93 789	25	10.06 211	9.87 833	11	5						
56	9.81 636	15	9.93 814	26	10.06 186	9.87 822	11	4						
57	9.81 651	14	9.93 840	25	10.06 160	9.87 811	11	3						
58	9.81 665	15	9.93 865	26	10.06 135	9.87 800	11	2						
59	9.81 680	14	9.93 891	25	10.06 109	9.87 789	11	1						
60	9.81 694	—	9.93 916	—	10.06 084	9.87 778	—	0						
/	L Cos	d	L Ctn	c d	L Tan	L Sin	d	/	Proportional parts					

130° (310°)

(229°) 49°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

41° (221°)

(318°) 138°

/	L Sin	d	L Tan	c d	L Ctn	L Cos	d	/	Proportional parts	
0	9.81 694	15	9.93 916	26	10.06 084	9.87 778	11	60		
1	9.81 709	14	9.93 942	25	10.06 058	9.87 767	11	59		
2	9.81 723	15	9.93 967	26	10.06 033	9.87 756	11	58		
3	9.81 738	14	9.93 993	25	10.06 007	9.87 745	11	57		
4	9.81 752	15	9.94 018	26	10.05 982	9.87 734	11	56		
5	9.81 767	14	9.94 044	25	10.05 956	9.87 723	11	55		
6	9.81 781	15	9.94 069	26	10.05 931	9.87 712	11	54		
7	9.81 796	14	9.94 095	25	10.05 905	9.87 701	11	53	//	26 25
8	9.81 810	15	9.94 120	26	10.05 880	9.87 690	11	52	1	0.4 0.4
9	9.81 825	14	9.94 146	25	10.05 854	9.87 679	11	51	2	0.9 0.8
10	9.81 839	15	9.94 171	26	10.05 829	9.87 668	11	50	3	1.3 1.2
11	9.81 854	14	9.94 197	25	10.05 803	9.87 657	11	49	4	1.7 1.7
12	9.81 868	14	9.94 222	26	10.05 778	9.87 646	11	48	5	2.2 2.1
13	9.81 882	15	9.94 248	25	10.05 752	9.87 635	11	47	6	2.6 2.5
14	9.81 897	14	9.94 273	26	10.05 727	9.87 624	11	46	7	3.0 2.9
15	9.81 911	15	9.94 299	25	10.05 701	9.87 613	12	45	8	3.5 3.3
16	9.81 926	14	9.94 324	26	10.05 676	9.87 601	11	44	9	3.9 3.8
17	9.81 940	15	9.94 350	25	10.05 650	9.87 590	11	43	10	4.3 4.2
18	9.81 955	14	9.94 375	26	10.05 625	9.87 579	11	42	20	8.7 8.3
19	9.81 969	14	9.94 401	25	10.05 599	9.87 568	11	41	30	13.0 12.5
20	9.81 983	15	9.94 426	26	10.05 574	9.87 557	11	40	40	17.3 16.7
21	9.81 998	14	9.94 452	25	10.05 548	9.87 546	11	39	50	21.7 20.8
22	9.82 012	14	9.94 477	26	10.05 523	9.87 535	11	38	//	15 14
23	9.82 026	15	9.94 503	25	10.05 497	9.87 524	11	37	1	0.2 0.2
24	9.82 041	14	9.94 528	26	10.05 472	9.87 513	12	36	2	0.5 0.5
25	9.82 055	14	9.94 554	25	10.05 446	9.87 501	11	35	3	0.8 0.7
26	9.82 069	15	9.94 579	25	10.05 421	9.87 490	11	34	4	1.0 0.9
27	9.82 084	14	9.94 604	26	10.05 396	9.87 479	11	33	5	1.2 1.2
28	9.82 098	14	9.94 630	25	10.05 370	9.87 468	11	32	6	1.5 1.4
29	9.82 112	14	9.94 655	26	10.05 345	9.87 457	11	31	7	1.8 1.6
30	9.82 126	15	9.94 681	25	10.05 319	9.87 446	12	30	8	2.0 1.9
31	9.82 141	14	9.94 706	26	10.05 294	9.87 434	11	29	9	2.2 2.1
32	9.82 155	14	9.94 732	25	10.05 268	9.87 423	11	28	10	2.5 2.3
33	9.82 169	15	9.94 757	26	10.05 243	9.87 412	11	27	20	5.0 4.7
34	9.82 184	14	9.94 783	25	10.05 217	9.87 401	11	26	30	7.5 7.0
35	9.82 198	14	9.94 808	26	10.05 192	9.87 390	12	25	40	10.0 9.3
36	9.82 212	14	9.94 834	25	10.05 166	9.87 378	11	24	50	12.5 11.7
37	9.82 226	14	9.94 859	25	10.05 141	9.87 367	11	23	//	12 11
38	9.82 240	15	9.94 884	26	10.05 116	9.87 356	11	22	1	0.2 0.2
39	9.82 255	14	9.94 910	25	10.05 090	9.87 345	11	21	2	0.4 0.4
40	9.82 269	14	9.94 935	26	10.05 065	9.87 334	12	20	3	0.6 0.6
41	9.82 283	14	9.94 961	25	10.05 039	9.87 322	11	19	4	0.8 0.7
42	9.82 297	14	9.94 986	26	10.05 014	9.87 311	11	18	5	1.0 0.9
43	9.82 311	15	9.95 012	25	10.04 988	9.87 300	12	17	6	1.2 1.1
44	9.82 326	14	9.95 037	25	10.04 963	9.87 288	11	16	7	1.4 1.3
45	9.82 340	14	9.95 062	26	10.04 938	9.87 277	11	15	8	1.6 1.5
46	9.82 354	14	9.95 088	25	10.04 912	9.87 266	11	14	9	1.8 1.6
47	9.82 368	14	9.95 113	26	10.04 887	9.87 255	12	13	10	2.0 1.8
48	9.82 382	14	9.95 139	25	10.04 861	9.87 243	11	12	20	4.0 3.7
49	9.82 396	14	9.95 164	26	10.04 836	9.87 232	11	11	30	6.0 5.5
50	9.82 410	14	9.95 190	25	10.04 810	9.87 221	12	10	40	8.0 7.3
51	9.82 424	15	9.95 215	25	10.04 785	9.87 209	11	9	50	10.0 9.2
52	9.82 439	14	9.95 240	26	10.04 760	9.87 198	11	8		
53	9.82 453	14	9.95 266	25	10.04 734	9.87 187	12	7		
54	9.82 467	14	9.95 291	26	10.04 709	9.87 175	11	6		
55	9.82 481	14	9.95 317	25	10.04 683	9.87 164	11	5		
56	9.82 495	14	9.95 342	26	10.04 658	9.87 153	12	4		
57	9.82 509	14	9.95 368	25	10.04 632	9.87 141	11	3		
58	9.82 523	14	9.95 393	25	10.04 607	9.87 130	11	2		
59	9.82 537	14	9.95 418	26	10.04 582	9.87 119	12	1		
60	9.82 551	—	9.95 444	—	10.04 556	9.87 107	—	0		
/	L Cos	d	L Ctn	c d	L Tan	L Sin	d	/	Proportional parts	

131 (311°)

(228°) 48°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

42° (222°)						(317°) 137°					
/	L Sin	d	L Tan	c d	L Ctn	L Cos	d	/	Proportional parts		
0	9.82 551	14	9.95 444	25	10.04 556	9.87 107	11	60			
1	9.82 565	14	9.95 469	26	10.04 531	9.87 096	11	59			
2	9.82 579	14	9.95 495	25	10.04 505	9.87 085	12	58			
3	9.82 593	14	9.95 520	25	10.04 480	9.87 073	11	57			
4	9.82 607	14	9.95 545	26	10.04 455	9.87 062	12	56			
5	9.82 621	14	9.95 571	25	10.04 429	9.87 050	11	55			
6	9.82 635	14	9.95 596	26	10.04 404	9.87 039	11	54			
7	9.82 649	14	9.95 622	25	10.04 378	9.87 028	12	53	//	26 25	
8	9.82 663	14	9.95 647	25	10.04 353	9.87 016	11	52	1	0.4 0.4	
9	9.82 677	14	9.95 672	26	10.04 328	9.87 005	12	51	2	0.9 0.8	
10	9.82 691	14	9.95 698	25	10.04 302	9.86 993	11	50	3	1.3 1.2	
11	9.82 705	14	9.95 723	25	10.04 277	9.86 982	12	49	4	1.7 1.7	
12	9.82 719	14	9.95 748	26	10.04 252	9.86 970	11	48	5	2.2 2.1	
13	9.82 733	14	9.95 774	25	10.04 226	9.86 959	12	47	6	2.6 2.5	
14	9.82 747	14	9.95 799	26	10.04 201	9.86 947	11	46	7	3.0 2.9	
15	9.82 761	14	9.95 825	25	10.04 175	9.86 936	12	45	8	3.5 3.3	
16	9.82 775	13	9.95 850	25	10.04 150	9.86 924	11	44	9	3.9 3.8	
17	9.82 788	14	9.95 875	26	10.04 125	9.86 913	11	43			
18	9.82 802	14	9.95 901	25	10.04 099	9.86 902	12	42	10	4.3 4.2	
19	9.82 816	14	9.95 926	26	10.04 074	9.86 890	11	41	20	8.7 8.3	
20	9.82 830	14	9.95 952	25	10.04 048	9.86 879	12	40	30	13.0 12.5	
21	9.82 844	14	9.95 977	25	10.04 023	9.86 867	12	39	40	17.3 16.7	
22	9.82 858	14	9.96 002	26	10.03 998	9.86 855	11	38	50	21.7 20.8	
23	9.82 872	13	9.96 028	25	10.03 972	9.86 844	12	37			
24	9.82 885	14	9.96 053	25	10.03 947	9.86 832	11	36	//	14 13	
25	9.82 899	14	9.96 078	26	10.03 922	9.86 821	12	35	1	0.2 0.2	
26	9.82 913	14	9.96 104	25	10.03 896	9.86 809	11	34	2	0.5 0.4	
27	9.82 927	14	9.96 129	26	10.03 871	9.86 798	12	33	3	0.7 0.6	
28	9.82 941	14	9.96 155	25	10.03 845	9.86 786	11	32	4	0.9 0.9	
29	9.82 955	13	9.96 180	25	10.03 820	9.86 775	12	31	5	1.2 1.1	
30	9.82 968	14	9.96 205	26	10.03 795	9.86 763	11	30	6	1.4 1.3	
31	9.82 982	14	9.96 231	25	10.03 769	9.86 752	12	29	7	1.6 1.5	
32	9.82 996	14	9.96 256	25	10.03 744	9.86 740	12	28	8	1.9 1.7	
33	9.83 010	13	9.96 281	26	10.03 719	9.86 728	11	27	9	2.1 2.0	
34	9.83 023	14	9.96 307	25	10.03 693	9.86 717	12	26	10	2.3 2.2	
35	9.83 037	14	9.96 332	25	10.03 668	9.86 705	11	25	20	4.7 4.3	
36	9.83 051	14	9.96 357	26	10.03 643	9.86 694	12	24	30	7.0 6.5	
37	9.83 065	13	9.96 383	25	10.03 617	9.86 682	12	23	40	9.3 8.7	
38	9.83 078	14	9.96 408	25	10.03 592	9.86 670	11	22	50	11.7 10.8	
39	9.83 092	14	9.96 433	26	10.03 567	9.86 659	12	21			
40	9.83 106	14	9.96 459	25	10.03 541	9.86 647	12	20	//	12 11	
41	9.83 120	13	9.96 484	26	10.03 516	9.86 635	11	19	1	0.2 0.2	
42	9.83 133	14	9.96 510	25	10.03 490	9.86 624	12	18	2	0.4 0.4	
43	9.83 147	14	9.96 535	25	10.03 465	9.86 612	12	17	3	0.6 0.6	
44	9.83 161	13	9.96 560	26	10.03 440	9.86 600	11	16	4	0.8 0.7	
45	9.83 174	14	9.96 586	25	10.03 414	9.86 589	12	15	5	1.0 0.9	
46	9.83 188	14	9.96 611	25	10.03 389	9.86 577	12	14	6	1.2 1.1	
47	9.83 202	13	9.96 636	26	10.03 364	9.86 565	11	13	7	1.4 1.3	
48	9.83 215	14	9.96 662	25	10.03 338	9.86 554	12	12	8	1.6 1.5	
49	9.83 229	13	9.96 687	25	10.03 313	9.86 542	12	11	9	1.8 1.6	
50	9.83 242	14	9.96 712	26	10.03 288	9.86 530	12	10	10	2.0 1.8	
51	9.83 256	14	9.96 738	25	10.03 262	9.86 518	11	9	20	4.0 3.7	
52	9.83 270	13	9.96 763	25	10.03 237	9.86 507	12	8	30	6.0 5.5	
53	9.83 283	14	9.96 788	26	10.03 212	9.86 495	12	7	40	8.0 7.3	
54	9.83 297	13	9.96 814	25	10.03 186	9.86 483	11	6	50	10.0 9.2	
55	9.83 310	14	9.96 839	25	10.03 161	9.86 472	12	5			
56	9.83 324	14	9.96 864	26	10.03 136	9.86 460	12	4			
57	9.83 338	13	9.96 890	25	10.03 110	9.86 448	12	3			
58	9.83 351	14	9.96 915	25	10.03 085	9.86 436	11	2			
59	9.83 365	13	9.96 940	26	10.03 060	9.86 425	12	1			
60	9.83 378	—	9.96 966	—	10.03 034	9.86 413	—	0			
/	L Cos	d	L Ctn	c d	L Tan	L Sin	d	/	Proportional parts		

132° (312°)

(227°) 47°

Table 6.15 (Cont'd)

COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

43° (223°)					(316°) 136°				
/	L Sin	d	L Tan	c d	L Ctn	L Cos	d	/	Proportional parts
0	9.83 378	14	9.96 966	25	10.03 034	9.86 413	12	60	
1	9.83 392	13	9.96 991	25	10.03 009	9.86 401	12	59	
2	9.83 405	14	9.97 016	26	10.02 984	9.86 389	12	58	
3	9.83 419	13	9.97 042	25	10.02 958	9.86 377	11	57	
4	9.83 432	14	9.97 067	25	10.02 933	9.86 366	12	56	
5	9.83 446	13	9.97 092	26	10.02 908	9.86 354	12	55	
6	9.83 459	14	9.97 118	25	10.02 882	9.86 342	12	54	
7	9.83 473	13	9.97 143	25	10.02 857	9.86 330	12	53	// 26 25
8	9.83 486	14	9.97 168	25	10.02 832	9.86 318	12	52	1 0.4 0.4
9	9.83 500	13	9.97 193	26	10.02 807	9.86 306	11	51	2 0.9 0.8
10	9.83 513	14	9.97 219	25	10.02 781	9.86 295	12	50	3 1.3 1.2
11	9.83 527	13	9.97 244	25	10.02 756	9.86 283	12	49	4 1.7 1.7
12	9.83 540	14	9.97 269	26	10.02 731	9.86 271	12	48	5 2.2 2.1
13	9.83 554	13	9.97 295	25	10.02 705	9.86 259	12	47	6 2.6 2.5
14	9.83 567	14	9.97 320	25	10.02 680	9.86 247	12	46	7 3.0 2.9
15	9.83 581	13	9.97 345	26	10.02 655	9.86 235	12	45	8 3.5 3.3
16	9.83 594	14	9.97 371	25	10.02 629	9.86 223	12	44	9 3.9 3.8
17	9.83 608	13	9.97 396	25	10.02 604	9.86 211	11	43	10 4.3 4.2
18	9.83 621	13	9.97 421	26	10.02 579	9.86 200	12	42	20 8.7 8.3
19	9.83 634	14	9.97 447	25	10.02 553	9.86 188	12	41	30 13.0 12.5
20	9.83 648	13	9.97 472	25	10.02 528	9.86 176	12	40	40 17.3 16.7
21	9.83 661	13	9.97 497	26	10.02 503	9.86 164	12	39	50 21.7 20.8
22	9.83 674	14	9.97 523	25	10.02 477	9.86 152	12	38	
23	9.83 688	13	9.97 548	25	10.02 452	9.86 140	12	37	// 14 13
24	9.83 701	14	9.97 573	25	10.02 427	9.86 128	12	36	1 0.2 0.2
25	9.83 715	13	9.97 598	26	10.02 402	9.86 116	12	35	2 0.5 0.4
26	9.83 728	13	9.97 624	25	10.02 376	9.86 104	12	34	3 0.7 0.6
27	9.83 741	14	9.97 649	25	10.02 351	9.86 092	12	33	4 0.9 0.9
28	9.83 755	13	9.97 674	26	10.02 326	9.86 080	12	32	
29	9.83 768	13	9.97 700	25	10.02 300	9.86 068	12	31	5 1.2 1.1
30	9.83 781	14	9.97 725	25	10.02 275	9.86 056	12	30	6 1.4 1.3
31	9.83 795	13	9.97 750	26	10.02 250	9.86 044	12	29	7 1.6 1.5
32	9.83 808	13	9.97 776	25	10.02 224	9.86 032	12	28	8 1.9 1.7
33	9.83 821	13	9.97 801	25	10.02 199	9.86 020	12	27	9 2.1 2.0
34	9.83 834	14	9.97 826	25	10.02 174	9.86 008	12	26	10 2.3 2.2
35	9.83 848	13	9.97 851	26	10.02 149	9.85 996	12	25	20 4.7 4.3
36	9.83 861	13	9.97 877	25	10.02 123	9.85 984	12	24	30 7.0 6.5
37	9.83 874	13	9.97 902	25	10.02 098	9.85 972	12	23	40 9.3 8.7
38	9.83 887	14	9.97 927	26	10.02 073	9.85 960	12	22	50 11.7 10.8
39	9.83 901	13	9.97 953	25	10.02 047	9.85 948	12	21	
40	9.83 914	13	9.97 978	25	10.02 022	9.85 936	12	20	// 12 11
41	9.83 927	13	9.98 003	26	10.01 997	9.85 924	12	19	1 0.2 0.2
42	9.83 940	14	9.98 029	25	10.01 971	9.85 912	12	18	2 0.4 0.4
43	9.83 954	13	9.98 054	25	10.01 946	9.85 900	12	17	3 0.6 0.6
44	9.83 967	13	9.98 079	25	10.01 921	9.85 888	12	16	4 0.8 0.7
45	9.83 980	13	9.98 104	26	10.01 896	9.85 876	12	15	5 1.0 0.9
46	9.83 993	13	9.98 130	25	10.01 870	9.85 864	13	14	6 1.2 1.1
47	9.84 006	14	9.98 155	25	10.01 845	9.85 851	12	13	7 1.4 1.3
48	9.84 020	13	9.98 180	26	10.01 820	9.85 839	12	12	8 1.6 1.5
49	9.84 033	13	9.98 206	25	10.01 794	9.85 827	12	11	9 1.8 1.6
50	9.84 046	13	9.98 231	25	10.01 769	9.85 815	12	10	10 2.0 1.8
51	9.84 059	13	9.98 256	25	10.01 744	9.85 803	12	9	20 4.0 3.7
52	9.84 072	13	9.98 281	26	10.01 719	9.85 791	12	8	30 6.0 5.5
53	9.84 085	13	9.98 307	25	10.01 693	9.85 779	13	7	40 8.0 7.3
54	9.84 098	14	9.98 332	25	10.01 668	9.85 766	12	6	50 10.0 9.2
55	9.84 112	13	9.98 357	26	10.01 643	9.85 754	12	5	
56	9.84 125	13	9.98 383	25	10.01 617	9.85 742	12	4	
57	9.84 138	13	9.98 408	25	10.01 592	9.85 730	12	3	
58	9.84 151	13	9.98 433	25	10.01 567	9.85 718	12	2	
59	9.84 164	13	9.98 458	26	10.01 542	9.85 706	13	1	
60	9.84 177	—	9.98 484	—	10.01 516	9.85 693	—	0	
/	L Ctn	d	L Ctn	c d	L Tan	L Sin	d	/	Proportional parts

6.16 DEFINITION

The **exponential function** e^x is the inverse of the logarithm function and is given by

$$e^x = \ln^{-1}(x).$$

6.17 DEFINITION

The **exponential function** a^x is defined by

$$a^x = \log_a^{-1}(x) \quad a > 0, a \neq 1.$$

6.18 PROPERTIES OF EXPONENTIAL FUNCTIONS

$$(1) \quad a^x = e^{x \ln a}.$$

$$(2) \quad a^{x_1} a^{x_2} = a^{x_1 + x_2}.$$

$$(3) \quad \frac{a^{x_1}}{a^{x_2}} = a^{x_1 - x_2}.$$

$$(4) \quad (a^{x_1})^{x_2} = a^{x_1 x_2}.$$

$$(5) \quad e^{\ln x} = \ln e^x = x.$$

6.19 DEFINITIONS OF HYPERBOLIC FUNCTIONS AND THEIR INVERSES

$$(1) \quad \sinh x = \frac{1}{2}(e^x - e^{-x}).$$

$$(2) \quad \cosh x = \frac{1}{2}(e^x + e^{-x}).$$

$$(3) \quad \tanh x = \frac{\sinh x}{\cosh x}.$$

$$(4) \quad \coth x = \frac{1}{\tanh x}.$$

$$(5) \quad \operatorname{sech} x = \frac{1}{\cosh x}.$$

$$(6) \quad \operatorname{csch} x = \frac{1}{\sinh x}.$$

$$(7) \quad \sinh^{-1} x = \ln(x + \sqrt{x^2 + 1}). \quad \text{Domain: } (-\infty, \infty).$$

$$(8) \quad \cosh^{-1} x = \ln (x + \sqrt{x^2 - 1}). \quad \text{Domain: } [1, \infty).$$

$$(9) \quad \tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right). \quad \text{Domain: } (-1, 1).$$

$$(10) \quad \coth^{-1} x = \frac{1}{2} \ln \left(\frac{x+1}{x-1} \right). \quad \text{Domain: } (-\infty, -1), (1, \infty).$$

$$(11) \quad \operatorname{sech}^{-1} x = \ln \left(\frac{1 + \sqrt{1-x^2}}{x} \right). \quad \text{Domain: } (0, 1].$$

$$(12) \quad \operatorname{csch}^{-1} x = \ln \left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|} \right). \quad \text{Domain: } (-\infty, 0), (0, \infty).$$

6.20 PROPERTIES OF HYPERBOLIC FUNCTIONS

$$(1) \quad \cosh^2 x - \sinh^2 x = 1.$$

$$(2) \quad \operatorname{sech}^2 x + \tanh^2 x = 1.$$

$$(3) \quad \operatorname{csch}^2 x - \coth^2 x = -1.$$

$$(4) \quad \sinh(-x) = -\sinh x.$$

$$(5) \quad \cosh(-x) = \cosh x.$$

$$(6) \quad \tanh(-x) = -\tanh x.$$

$$(7) \quad \sinh(x_1 \pm x_2) = \sinh x_1 \cosh x_2 \pm \cosh x_1 \sinh x_2.$$

$$(8) \quad \cosh(x_1 \pm x_2) = \cosh x_1 \cosh x_2 \pm \sinh x_1 \sinh x_2.$$

$$(9) \quad \sinh 2x = 2 \sinh x \cosh x.$$

$$(10) \quad \cosh 2x = \cosh^2 x + \sinh^2 x.$$

$$(11) \quad 2 \sinh^2 \frac{x}{2} = \cosh x - 1.$$

$$(12) \quad 2 \cosh^2 \frac{x}{2} = \cosh x + 1.$$

Table 6.21

VALUES OF EXPONENTIAL AND HYPERBOLIC FUNCTIONS

x	e^x	e^{-x}	$\sinh x$	$\cosh x$	$\tanh x$
0.00	1.0000	1.00000	0.0000	1.0000	.00000
0.01	1.0101	0.99005	0.0100	1.0001	.01000
0.02	1.0202	.98020	0.0200	1.0002	.02000
0.03	1.0305	.97045	0.0300	1.0005	.02999
0.04	1.0408	.96079	0.0400	1.0008	.03998
0.05	1.0513	.95123	0.0500	1.0013	.04996
0.06	1.0618	.94176	0.0600	1.0018	.05993
0.07	1.0725	.93239	0.0701	1.0025	.06989
0.08	1.0833	.92312	0.0801	1.0032	.07983
0.09	1.0942	.91393	0.0901	1.0041	.08976
0.10	1.1052	.90484	0.1002	1.0050	.09967
0.11	1.1163	.89583	0.1102	1.0061	.10956
0.12	1.1275	.88692	0.1203	1.0072	.11943
0.13	1.1388	.87809	0.1304	1.0085	.12927
0.14	1.1503	.86936	0.1405	1.0098	.13909
0.15	1.1618	.86071	0.1506	1.0113	.14889
0.16	1.1735	.85214	0.1607	1.0128	.15865
0.17	1.1853	.84366	0.1708	1.0145	.16838
0.18	1.1972	.83527	0.1810	1.0162	.17808
0.19	1.2092	.82696	0.1911	1.0181	.18775
0.20	1.2214	.81873	0.2013	1.0201	.19738
0.21	1.2337	.81058	0.2115	1.0221	.20697
0.22	1.2461	.80252	0.2218	1.0243	.21652
0.23	1.2586	.79453	0.2320	1.0266	.22603
0.24	1.2712	.78663	0.2423	1.0289	.23550
0.25	1.2840	.77880	0.2526	1.0314	.24492
0.26	1.2969	.77105	0.2629	1.0340	.25430
0.27	1.3100	.76338	0.2733	1.0367	.26362
0.28	1.3231	.75578	0.2837	1.0395	.27291
0.29	1.3364	.74826	0.2941	1.0423	.28213
0.30	1.3499	.74082	0.3045	1.0453	.29131
0.31	1.3634	.73345	0.3150	1.0484	.30044
0.32	1.3771	.72615	0.3255	1.0516	.30951
0.33	1.3910	.71892	0.3360	1.0549	.31852
0.34	1.4049	.71177	0.3466	1.0584	.32748
0.35	1.4191	.70469	0.3572	1.0619	.33638
0.36	1.4333	.69768	0.3678	1.0655	.34521
0.37	1.4477	.69073	0.3785	1.0692	.35399
0.38	1.4623	.68386	0.3892	1.0731	.36271
0.39	1.4770	.67706	0.4000	1.0770	.37136
0.40	1.4918	.67032	0.4108	1.0811	.37995
0.41	1.5068	.66365	0.4216	1.0852	.38847
0.42	1.5220	.65705	0.4325	1.0895	.39693
0.43	1.5373	.65051	0.4434	1.0939	.40532
0.44	1.5527	.64404	0.4543	1.0984	.41364
0.45	1.5683	.63763	0.4653	1.1030	.42190
0.46	1.5841	.63128	0.4764	1.1077	.43008
0.47	1.6000	.62500	0.4875	1.1125	.43820
0.48	1.6161	.61878	0.4986	1.1174	.44624
0.49	1.6323	.61263	0.5098	1.1225	.45422
0.50	1.6487	.60653	0.5211	1.1276	.46212

Table 6.21 (Cont'd)

VALUES OF EXPONENTIAL AND HYPERBOLIC FUNCTIONS

x	e^x	e^{-x}	$\sinh x$	$\cosh x$	$\tanh x$
0.50	1.6487	0.60653	0.5211	1.1276	.46212
0.51	1.6653	0.60050	0.5374	1.1329	.46995
0.52	1.6820	0.59457	0.5438	1.1383	.47770
0.53	1.6989	0.58860	0.5552	1.1438	.48538
0.54	1.7160	0.58275	0.5666	1.1494	.49299
0.55	1.7333	0.57695	0.5782	1.1551	.50052
0.56	1.7507	0.57121	0.5897	1.1609	.50798
0.57	1.7683	0.56553	0.6014	1.1669	.51536
0.58	1.7860	0.55990	0.6131	1.1730	.52267
0.59	1.8040	0.55433	0.6248	1.1792	.52990
0.60	1.8221	0.54881	0.6367	1.1855	.53705
0.61	1.8404	0.54335	0.6485	1.1919	.54413
0.62	1.8589	0.53794	0.6605	1.1984	.55113
0.63	1.8776	0.53259	0.6725	1.2051	.55805
0.64	1.8965	0.52729	0.6846	1.2119	.56490
0.65	1.9155	0.52205	0.6967	1.2188	.57167
0.66	1.9348	0.51685	0.7090	1.2258	.57836
0.67	1.9542	0.51171	0.7213	1.2330	.58498
0.68	1.9739	0.50662	0.7336	1.2402	.59152
0.69	1.9937	0.50158	0.7461	1.2476	.59798
0.70	2.0138	0.49659	0.7586	1.2552	.60437
0.71	2.0340	0.49164	0.7712	1.2628	.61068
0.72	2.0544	0.48675	0.7838	1.2706	.61691
0.73	2.0751	0.48191	0.7966	1.2785	.62307
0.74	2.0959	0.47711	0.8094	1.2865	.62915
0.75	2.1170	0.47237	0.8223	1.2947	.63515
0.76	2.1383	0.46767	0.8353	1.3030	.64108
0.77	2.1598	0.46301	0.8484	1.3114	.64693
0.78	2.1815	0.45841	0.8615	1.3199	.65271
0.79	2.2034	0.45384	0.8748	1.3286	.65841
0.80	2.2255	0.44933	0.8881	1.3374	.66404
0.81	2.2479	0.44486	0.9015	1.3464	.66959
0.82	2.2705	0.44043	0.9150	1.3555	.67507
0.83	2.2933	0.43605	0.9286	1.3647	.68048
0.84	2.3164	0.43171	0.9423	1.3740	.68581
0.85	2.3396	0.42741	0.9561	1.3835	.69107
0.86	2.3632	0.42316	0.9700	1.3932	.69626
0.87	2.3869	0.41895	0.9840	1.4029	.70137
0.88	2.4109	0.41478	0.9981	1.4128	.70642
0.89	2.4351	0.41066	1.0122	1.4229	.71139
0.90	2.4596	0.40657	1.0265	1.4331	.71630
0.91	2.4843	0.40252	1.0409	1.4434	.72113
0.92	2.5093	0.39852	1.0554	1.4539	.72590
0.93	2.5345	0.39455	1.0700	1.4645	.73059
0.94	2.5600	0.39063	1.0847	1.4753	.73522
0.95	2.5857	0.38674	1.0995	1.4862	.73978
0.96	2.6117	0.38289	1.1144	1.4973	.74428
0.97	2.6379	0.37908	1.1294	1.5085	.74870
0.98	2.6645	0.37531	1.1446	1.5199	.75307
0.99	2.6912	0.37158	1.1598	1.5314	.75735
1.00	2.7183	0.36788	1.1752	1.5431	.76159

Table 6.21 (Cont'd)

VALUES OF EXPONENTIAL AND HYPERBOLIC FUNCTIONS

x	e^x	e^{-x}	$\sinh x$	$\cosh x$	$\tanh x$
1.00	2.7183	.36788	1.1752	1.5431	.76159
1.01	2.7456	.36422	1.1907	1.5549	.76576
1.02	2.7732	.36060	1.2063	1.5669	.76987
1.03	2.8011	.35701	1.2220	1.5790	.77391
1.04	2.8292	.35345	1.2379	1.5913	.77789
1.05	2.8577	.34994	1.2539	1.6038	.78181
1.06	2.8864	.34646	1.2700	1.6164	.78566
1.07	2.9154	.34301	1.2862	1.6292	.78946
1.08	2.9447	.33960	1.3025	1.6421	.79320
1.09	2.9743	.33622	1.3190	1.6552	.79688
1.10	3.0042	.33287	1.3356	1.6685	.80050
1.11	3.0344	.32956	1.3524	1.6820	.80406
1.12	3.0649	.32628	1.3693	1.6956	.80757
1.13	3.0957	.32303	1.3863	1.7093	.81102
1.14	3.1268	.31982	1.4035	1.7233	.81441
1.15	3.1582	.31664	1.4208	1.7374	.81775
1.16	3.1899	.31349	1.4382	1.7517	.82104
1.17	3.2220	.31037	1.4558	1.7662	.82427
1.18	3.2544	.30728	1.4735	1.7808	.82745
1.19	3.2871	.30422	1.4914	1.7957	.83058
1.20	3.3201	.30119	1.5095	1.8107	.83365
1.21	3.3535	.29820	1.5276	1.8258	.83668
1.22	3.3872	.29523	1.5460	1.8412	.83965
1.23	3.4212	.29229	1.5645	1.8568	.84258
1.24	3.4556	.28938	1.5831	1.8725	.84546
1.25	3.4903	.28650	1.6019	1.8884	.84828
1.26	3.5254	.28365	1.6209	1.9045	.85106
1.27	3.5609	.28083	1.6400	1.9208	.85380
1.28	3.5966	.27804	1.6593	1.9373	.85648
1.29	3.6328	.27527	1.6788	1.9540	.85913
1.30	3.6693	.27253	1.6984	1.9709	.86172
1.31	3.7062	.26982	1.7182	1.9880	.86428
1.32	3.7434	.26714	1.7381	2.0055	.86678
1.33	3.7810	.26448	1.7583	2.0228	.86925
1.34	3.8190	.26185	1.7786	2.0404	.87167
1.35	3.8574	.25924	1.7991	2.0583	.87405
1.36	3.8962	.25666	1.8198	2.0764	.87639
1.37	3.9354	.25411	1.8406	2.0947	.87869
1.38	3.9749	.25158	1.8617	2.1132	.88095
1.39	4.0149	.24908	1.8829	2.1320	.88317
1.40	4.0552	.24660	1.9043	2.1509	.88535
1.41	4.0960	.24414	1.9259	2.1700	.88749
1.42	4.1371	.24171	1.9477	2.1894	.88960
1.43	4.1787	.23931	1.9697	2.2090	.89167
1.44	4.2207	.23693	1.9919	2.2288	.89370
1.45	4.2631	.23457	2.0143	2.2488	.89569
1.46	4.3060	.23224	2.0369	2.2691	.89765
1.47	4.3492	.22993	2.0597	2.2896	.89958
1.48	4.3929	.22764	2.0827	2.3103	.90147
1.49	4.4371	.22537	2.1059	2.3312	.90332
1.50	4.4817	.22313	2.1293	2.3524	.90515

Table 6.21 (Cont'd)

VALUES OF EXPONENTIAL AND HYPERBOLIC FUNCTIONS

x	e^x	e^{-x}	$\sinh x$	$\cosh x$	$\tanh x$
1.50	4.4817	.22313	2.1293	2.3524	.90515
1.51	4.5267	.22091	2.1529	2.3738	.90694
1.52	4.5722	.21871	2.1768	2.3955	.90870
1.53	4.6182	.21654	2.2008	2.4174	.91042
1.54	4.6646	.21438	2.2251	2.4395	.91212
1.55	4.7115	.21225	2.2496	2.4619	.91379
1.56	4.7588	.21014	2.2743	2.4845	.91542
1.57	4.8066	.20805	2.2993	2.5073	.91703
1.58	4.8550	.20598	2.3245	2.5305	.91860
1.59	4.9037	.20393	2.3499	2.5538	.92015
1.60	4.9530	.20190	2.3756	2.5775	.92167
1.61	5.0028	.19989	2.4015	2.6013	.92316
1.62	5.0531	.19790	2.4276	2.6255	.92462
1.63	5.1039	.19593	2.4540	2.6498	.92606
1.64	5.1552	.19398	2.4806	2.6746	.92747
1.65	5.2070	.19205	2.5075	2.6995	.92886
1.66	5.2593	.19014	2.5346	2.7247	.93022
1.67	5.3122	.18825	2.5620	2.7502	.93155
1.68	5.3656	.18637	2.5896	2.7760	.93286
1.69	5.4195	.18452	2.6175	2.8020	.93415
1.70	5.4739	.18268	2.6456	2.8283	.93541
1.71	5.5290	.18087	2.6740	2.8549	.93665
1.72	5.5845	.17907	2.7027	2.8818	.93786
1.73	5.6407	.17728	2.7317	2.9090	.93906
1.74	5.6973	.17552	2.7609	2.9364	.94023
1.75	5.7546	.17377	2.7904	2.9642	.94138
1.76	5.8124	.17204	2.8202	2.9922	.94250
1.77	5.8709	.17033	2.8503	3.0206	.94361
1.78	5.9299	.16864	2.8806	3.0492	.94470
1.79	5.9895	.16696	2.9112	3.0782	.94576
1.80	6.0496	.16530	2.9422	3.1075	.94681
1.81	6.1104	.16365	2.9734	3.1371	.94783
1.82	6.1719	.16203	3.0049	3.1669	.94884
1.83	6.2339	.16041	3.0367	3.1972	.94983
1.84	6.2965	.15882	3.0689	3.2277	.95080
1.85	6.3598	.15724	3.1013	3.2585	.95175
1.86	6.4237	.15567	3.1340	3.2897	.95268
1.87	6.4883	.15412	3.1671	3.3212	.95359
1.88	6.5535	.15259	3.2005	3.3530	.95449
1.89	6.6194	.15107	3.2341	3.3852	.95537
1.90	6.6859	.14957	3.2682	3.4177	.95624
1.91	6.7531	.14808	3.3025	3.4506	.95709
1.92	6.8210	.14661	3.3372	3.4838	.95792
1.93	6.8895	.14515	3.3722	3.5173	.95873
1.94	6.9588	.14370	3.4075	3.5512	.95953
1.95	7.0287	.14227	3.4432	3.5855	.96032
1.96	7.0993	.14086	3.4792	3.6201	.96109
1.97	7.1707	.13946	3.5156	3.6551	.96185
1.98	7.2427	.13807	3.5523	3.6904	.96259
1.99	7.3155	.13670	3.5894	3.7261	.96331
2.00	7.3891	.13534	3.6269	3.7622	.96403

Table 6.21 (Cont'd)
VALUES OF EXPONENTIAL AND HYPERBOLIC FUNCTIONS

x	e^x	e^{-x}	$\sinh x$	$\cosh x$	$\tanh x$
2.00	7.3891	.13534	3.6269	3.7622	.96403
2.01	7.4633	.13399	3.6647	3.7987	.96473
2.02	7.5383	.13266	3.7028	3.8355	.96541
2.03	7.6141	.13134	3.7414	3.8727	.96609
2.04	7.6906	.13003	3.7803	3.9103	.96675
2.05	7.7679	.12873	3.8196	3.9483	.96740
2.06	7.8460	.12745	3.8593	3.9867	.96803
2.07	7.9248	.12619	3.8993	4.0255	.96865
2.08	8.0045	.12493	3.9398	4.0647	.96926
2.09	8.0849	.12369	3.9806	4.1043	.96986
2.10	8.1662	.12246	4.0219	4.1443	.97045
2.11	8.2482	.12124	4.0635	4.1847	.97103
2.12	8.3311	.12003	4.1056	4.2256	.97159
2.13	8.4149	.11884	4.1480	4.2669	.97215
2.14	8.4994	.11765	4.1909	4.3085	.97269
2.15	8.5849	.11648	4.2342	4.3507	.97323
2.16	8.6711	.11533	4.2779	4.3932	.97375
2.17	8.7583	.11418	4.3221	4.4362	.97426
2.18	8.8463	.11304	4.3666	4.4797	.97477
2.19	8.9352	.11192	4.4116	4.5236	.97526
2.20	9.0250	.11080	4.4571	4.5679	.97574
2.21	9.1157	.10970	4.5030	4.6127	.97622
2.22	9.2073	.10861	4.5494	4.6580	.97668
2.23	9.2999	.10753	4.5962	4.7037	.97714
2.24	9.3933	.10646	4.6434	4.7499	.97759
2.25	9.4877	.10540	4.6912	4.7966	.97803
2.26	9.5831	.10435	4.7394	4.8437	.97846
2.27	9.6794	.10331	4.7880	4.8914	.97888
2.28	9.7767	.10228	4.8372	4.9395	.97929
2.29	9.8749	.10127	4.8868	4.9881	.97970
2.30	9.9742	.10026	4.9370	5.0372	.98010
2.31	10.074	.09926	4.9876	5.0868	.98049
2.32	10.176	.09827	5.0387	5.1370	.98087
2.33	10.278	.09730	5.0903	5.1876	.98124
2.34	10.381	.09633	5.1425	5.2388	.98161
2.35	10.486	.09537	5.1951	5.2905	.98197
2.36	10.591	.09442	5.2483	5.3427	.98233
2.37	10.697	.09348	5.3020	5.3954	.98267
2.38	10.805	.09255	5.3562	5.4487	.98301
2.39	10.913	.09163	5.4109	5.5026	.98335
2.40	11.023	.09072	5.4662	5.5569	.98367
2.41	11.134	.08982	5.5221	5.6119	.98400
2.42	11.246	.08892	5.5785	5.6674	.98431
2.43	11.359	.08804	5.6354	5.7235	.98462
2.44	11.473	.08716	5.6929	5.7801	.98492
2.45	11.588	.08629	5.7510	5.8373	.98522
2.46	11.705	.08543	5.8097	5.8951	.98551
2.47	11.822	.08458	5.8689	5.9535	.98579
2.48	11.941	.08374	5.9288	6.0125	.98607
2.49	12.061	.08291	5.9892	6.0721	.98635
2.50	12.182	.08208	6.0502	6.1323	.98661

Table 6.21 (Cont'd)
VALUES OF EXPONENTIAL AND HYPERBOLIC FUNCTIONS

x	e^x	e^{-x}	$\sinh x$	$\cosh x$	$\tanh x$
2.50	12.182	.08208	6.0502	6.1323	.98661
2.51	12.305	.08127	6.1118	6.1931	.98688
2.52	12.429	.08046	6.1741	6.2545	.98714
2.53	12.554	.07966	6.2369	6.3166	.98739
2.54	12.680	.07887	6.3004	6.3793	.98764
2.55	12.807	.07808	6.3645	6.4426	.98788
2.56	12.936	.07730	6.4293	6.5066	.98812
2.57	13.066	.07654	6.4946	6.5712	.98835
2.58	13.197	.07577	6.5607	6.6365	.98858
2.59	13.330	.07502	6.6274	6.7024	.98881
2.60	13.464	.07427	6.6947	6.7690	.98903
2.61	13.599	.07353	6.7628	6.8363	.98924
2.62	13.736	.07280	6.8315	6.9043	.98946
2.63	13.874	.07208	6.9008	6.9729	.98966
2.64	14.013	.07136	6.9709	7.0423	.98987
2.65	14.154	.07065	7.0417	7.1123	.99007
2.66	14.296	.06995	7.1132	7.1831	.99026
2.67	14.440	.06925	7.1854	7.2546	.99045
2.68	14.585	.06856	7.2583	7.3268	.99064
2.69	14.732	.06788	7.3319	7.3998	.99083
2.70	14.880	.06721	7.4063	7.4735	.99101
2.71	15.029	.06654	7.4814	7.5479	.99118
2.72	15.180	.06587	7.5572	7.6231	.99136
2.73	15.333	.06522	7.6338	7.6991	.99153
2.74	15.487	.06457	7.7112	7.7758	.99170
2.75	15.643	.06393	7.7894	7.8533	.99186
2.76	15.800	.06329	7.8683	7.9316	.99202
2.77	15.959	.06266	7.9480	8.0106	.99218
2.78	16.119	.06204	8.0285	8.0905	.99233
2.79	16.281	.06142	8.1098	8.1712	.99248
2.80	16.445	.06081	8.1919	8.2527	.99263
2.81	16.610	.06020	8.2749	8.3351	.99278
2.82	16.777	.05961	8.3586	8.4182	.99292
2.83	16.945	.05901	8.4432	8.5022	.99306
2.84	17.116	.05843	8.5287	8.5871	.99320
2.85	17.288	.05784	8.6150	8.6728	.99333
2.86	17.462	.05727	8.7021	8.7594	.99346
2.87	17.637	.05670	8.7902	8.8469	.99359
2.88	17.814	.05613	8.8791	8.9352	.99372
2.89	17.993	.05558	8.9689	9.0244	.99384
2.90	18.174	.05502	9.0596	9.1146	.99396
2.91	18.357	.05448	9.1512	9.2056	.99408
2.92	18.541	.05393	9.2437	9.2976	.99420
2.93	18.728	.05340	9.3371	9.3905	.99431
2.94	18.916	.05287	9.4315	9.4844	.99443
2.95	19.106	.05234	9.5268	9.5791	.99454
2.96	19.298	.05182	9.6231	9.6749	.99464
2.97	19.492	.05130	9.7203	9.7716	.99475
2.98	19.688	.05079	9.8185	9.8693	.99485
2.99	19.886	.05029	9.9177	9.9680	.99496
3.00	20.086	.04979	10.018	10.068	.99505

Table 6.21 (Cont'd)

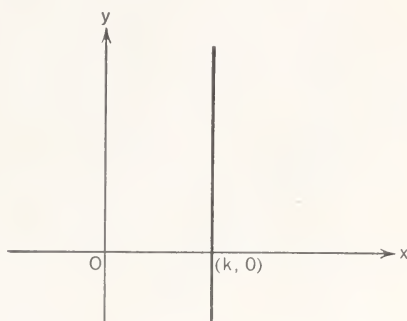
VALUES OF EXPONENTIAL AND HYPERBOLIC FUNCTIONS

x	e^x	e^{-x}	$\sinh x$	$\cosh x$	$\tanh x$
3.00	20.086	.04979	10.018	10.068	.99505
3.05	21.115	.04736	10.534	10.581	.99552
3.10	22.198	.04505	11.076	11.122	.99595
3.15	23.336	.04285	11.647	11.690	.99633
3.20	24.533	.04076	12.246	12.287	.99668
3.25	25.790	.03877	12.876	12.915	.99700
3.30	27.113	.03688	13.538	13.575	.99728
3.35	28.503	.03508	14.234	14.269	.99754
3.40	29.964	.03337	14.965	14.999	.99777
3.45	31.500	.03175	15.734	15.766	.99799
3.50	33.115	.03020	16.543	16.573	.99818
3.55	34.813	.02872	17.392	17.421	.99835
3.60	36.598	.02732	18.286	18.313	.99851
3.65	38.475	.02599	19.224	19.250	.99865
3.70	40.447	.02472	20.211	20.236	.99878
3.75	42.521	.02352	21.249	21.272	.99889
3.80	44.701	.02237	22.339	22.362	.99900
3.85	46.993	.02128	23.486	23.507	.99909
3.90	49.402	.02024	24.691	24.711	.99918
3.95	51.935	.01925	25.958	25.977	.99926
4.00	54.598	.01832	27.290	27.308	.99933
4.10	60.340	.01657	30.162	30.178	.99945
4.20	66.686	.01500	33.336	33.351	.99955
4.30	73.700	.01357	36.843	36.857	.99963
4.40	81.451	.01227	40.719	40.732	.99970
4.50	90.017	.01111	45.003	45.014	.99975
4.60	99.484	.01005	49.737	49.747	.99980
4.70	109.95	.00910	54.969	54.978	.99983
4.80	121.51	.00823	60.751	60.755	.99986
4.90	134.29	.00745	67.141	67.149	.99989
5.00	148.41	.00674	74.203	74.210	.99991
5.10	164.02	.00610	82.008	82.014	.99993
5.20	181.27	.00552	90.633	90.639	.99994
5.30	200.34	.00499	100.17	100.17	.99995
5.40	221.41	.00452	110.70	110.71	.99996
5.50	244.69	.00409	122.34	122.35	.99997
5.60	270.43	.00370	135.21	135.22	.99997
5.70	298.87	.00335	149.43	149.44	.99998
5.80	330.30	.00303	165.15	165.15	.99998
5.90	365.04	.00274	182.52	182.52	.99998
6.00	403.43	.00248	201.71	201.72	.99999
6.25	518.01	.00193	259.01	259.01	.99999
6.50	665.14	.00150	332.57	332.57	1.0000
6.75	854.06	.00117	427.03	427.03	1.0000
7.00	1096.6	.00091	548.32	548.32	1.0000
7.50	1808.0	.00055	904.02	904.02	1.0000
8.00	2981.0	.00034	1490.5	1490.5	1.0000
8.50	4914.8	.00020	2457.4	2457.4	1.0000
9.00	8103.1	.00012	4051.5	4051.5	1.0000
9.50	13360.	.00007	6679.9	6679.9	1.0000
10.00	22026.	.00005	11013.	11013.	1.0000

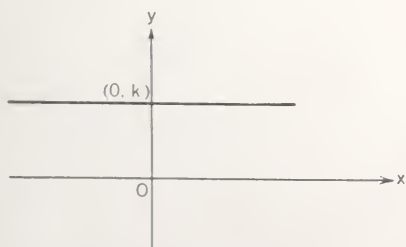
7

TWO- AND THREE-DIMENSIONAL GRAPHS

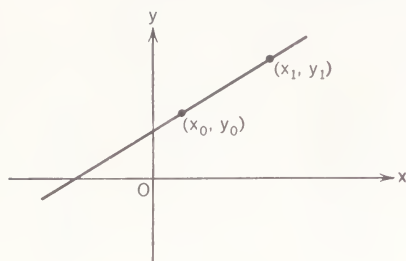
7.1 THE STRAIGHT LINE



(a) $x = k$

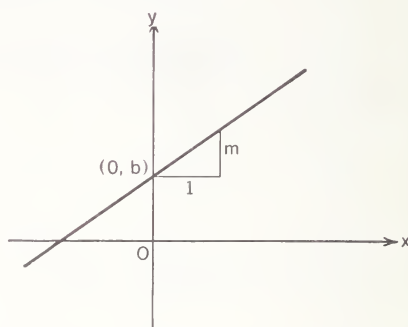


(b) $y = k$

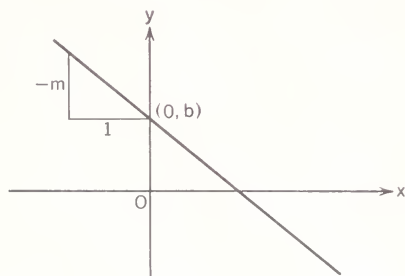


$$(c) \quad y - y_0 = \frac{y_1 - y_0}{x_1 - x_0} (x - x_0),$$

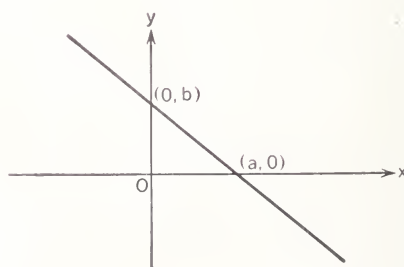
$$x_1 \neq x_0$$



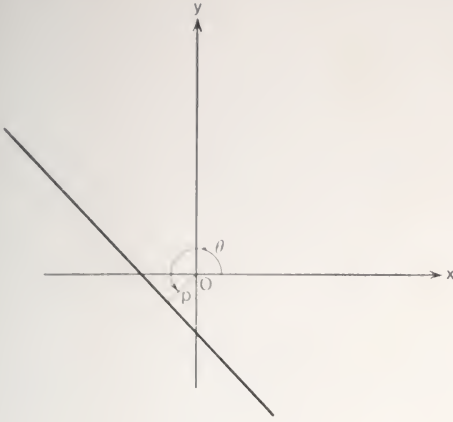
$$(d) \quad y = mx + b, \quad m > 0$$



$$(e) \quad y = mx + b, \quad m < 0$$

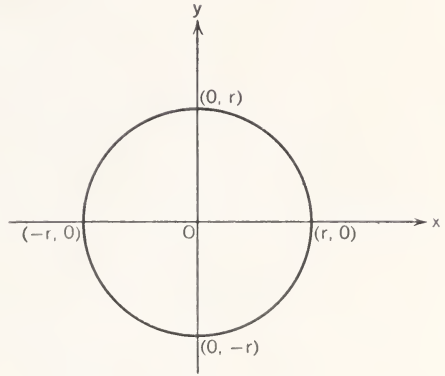


$$(f) \quad \frac{x}{a} + \frac{y}{b} = 1$$

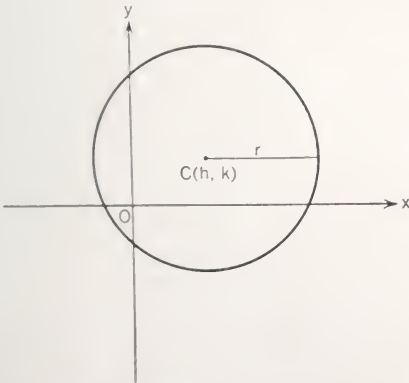


(g) $x \cos \theta + y \sin \theta - p = 0$

7.2 THE CIRCLE

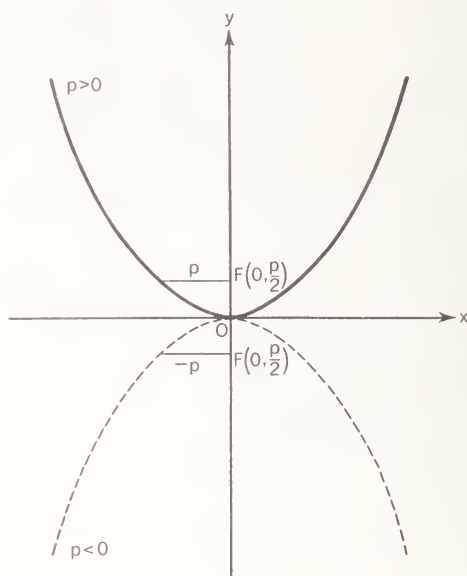


(a) $x^2 + y^2 = r^2$

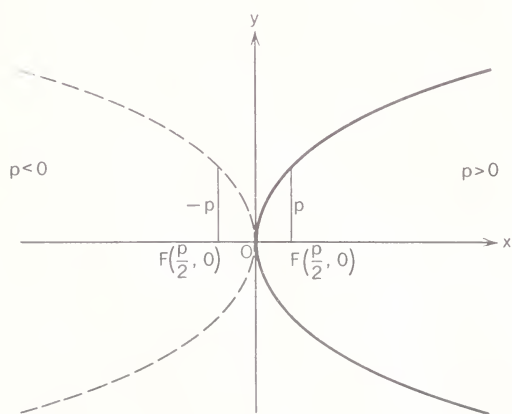


(b) $(x - h)^2 + (y - k)^2 = r^2$

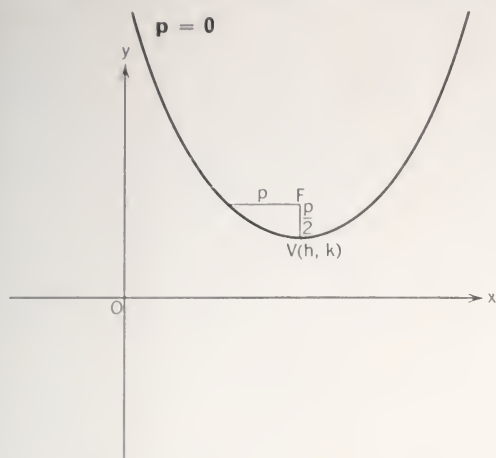
7.3 THE PARABOLA



$$(a) \ y = \frac{1}{2p} x^2$$

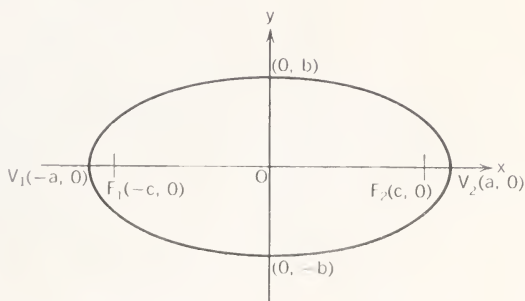


$$(b) \ y^2 = 2px$$



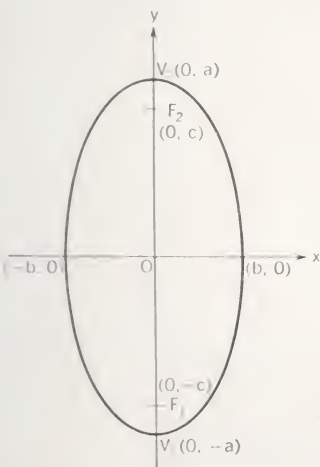
$$(c) (y - k) = \frac{1}{2p} (x - h)^2$$

7.4 THE ELLIPSE



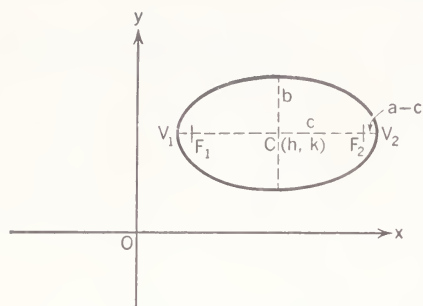
$$(a) \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a > b$$

$$c^2 = a^2 - b^2$$



$$(b) \frac{y^2}{a^2} + \frac{x^2}{b^2} = 1, \quad a > b$$

$$c^2 = a^2 - b^2$$



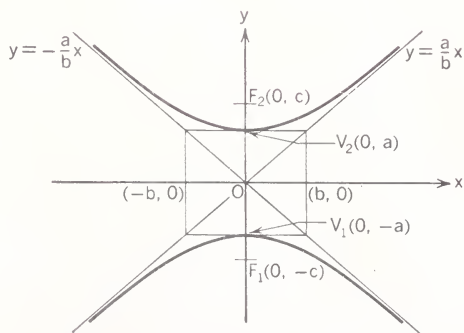
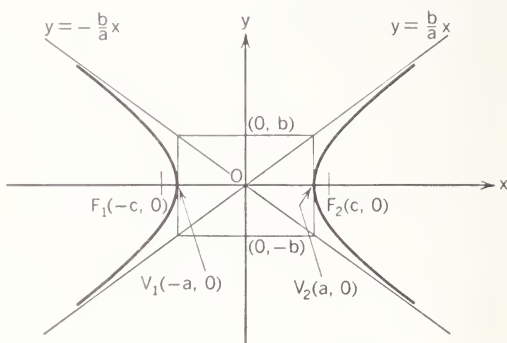
$$(c) \quad \frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1, \quad a > b$$

$$c^2 = a^2 - b^2$$

7.5 THE HYPERBOLA

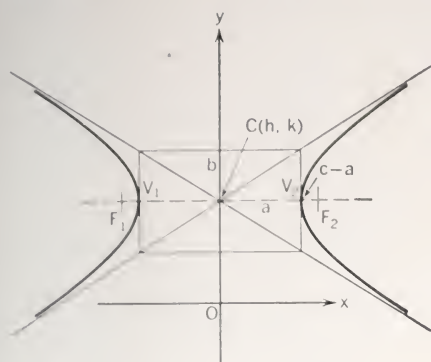
$$(a) \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$c^2 = a^2 + b^2$$



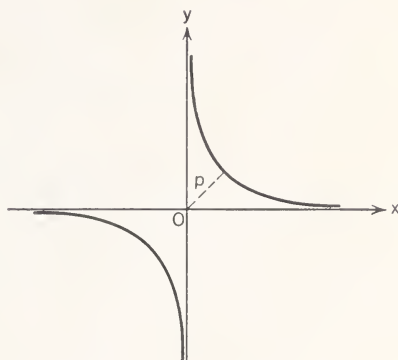
$$(b) \quad \frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

$$c^2 = a^2 + b^2$$

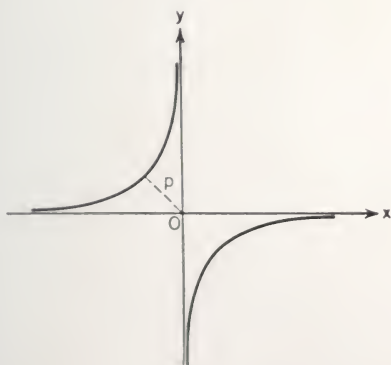


$$(c) \frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

$$c^2 = a^2 + b^2$$

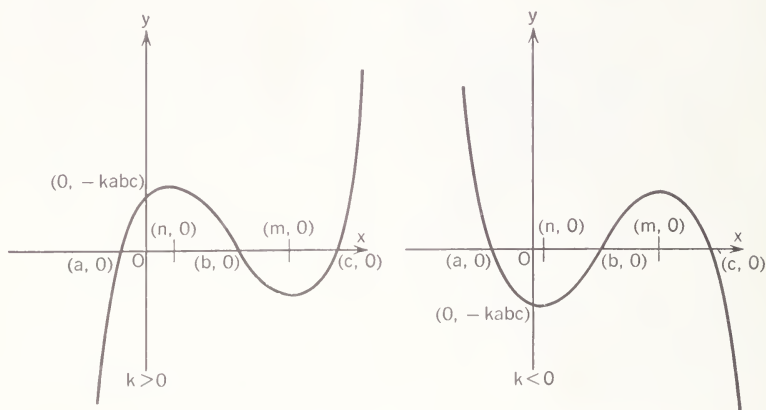


$$(d) xy = \frac{b^2}{2}$$



$$(e) xy = -\frac{b^2}{2}$$

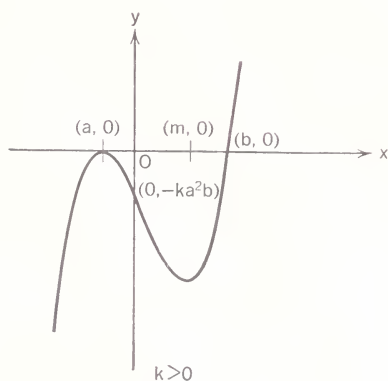
7.6 THE CUBIC



(a) $y = k(x - a)(x - b)(x - c)$, a, b, c all different

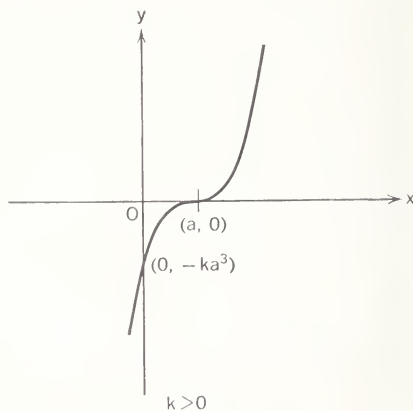
$$m = \frac{a + b + c + \sqrt{a^2 + b^2 + c^2 - ab - ac - bc}}{3}$$

$$n = \frac{a + b + c - \sqrt{a^2 + b^2 + c^2 - ab - ac - bc}}{3}$$

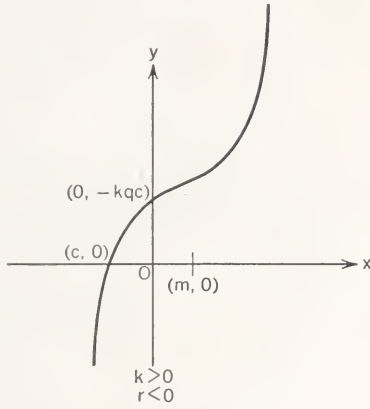
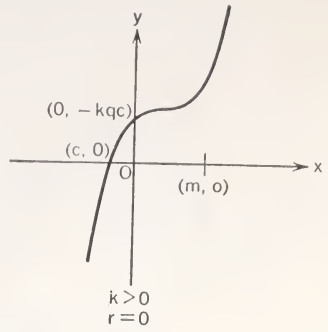
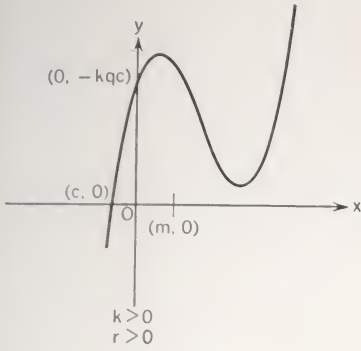


(b) $y = k(x - a)^2(x - b)$

$$m = \frac{a + b}{2}$$



(c) $y = k(x - a)^3$

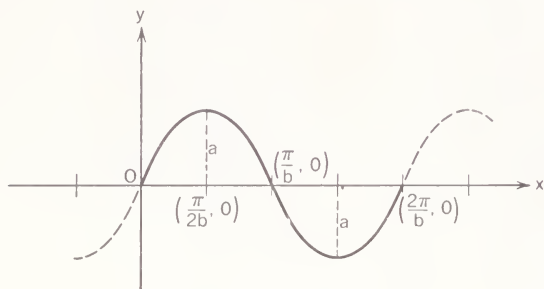
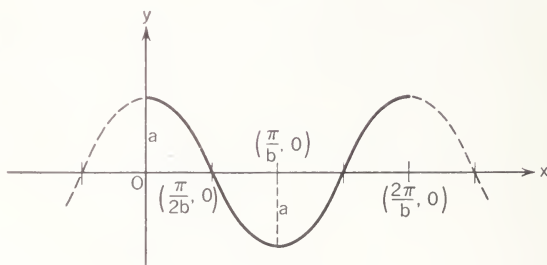
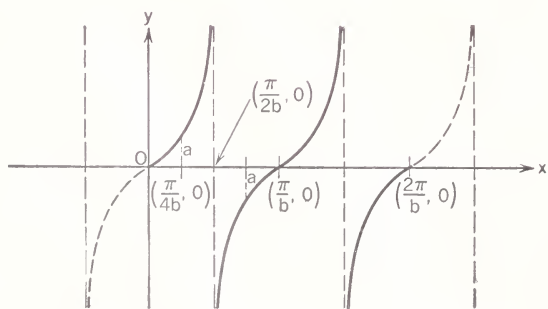


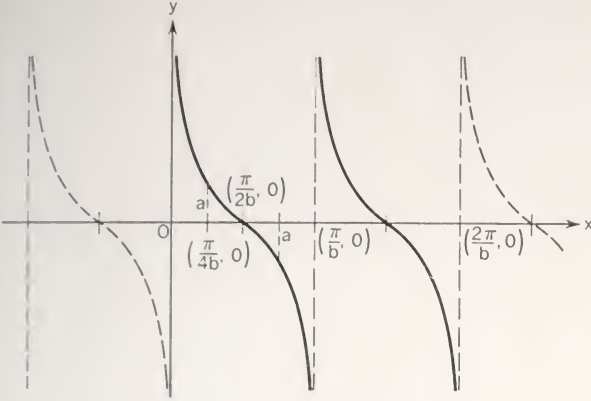
(d) $y = k(x^2 + px + q)(x - c)$, $p^2 - 4q < 0$

$$r = p^2 + c^2 + pc - 3q$$

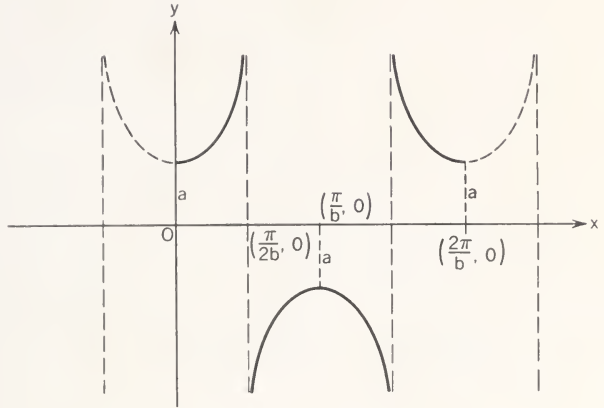
$$m = \frac{c - p}{3}$$

7.7 THE TRIGONOMETRIC FUNCTIONS

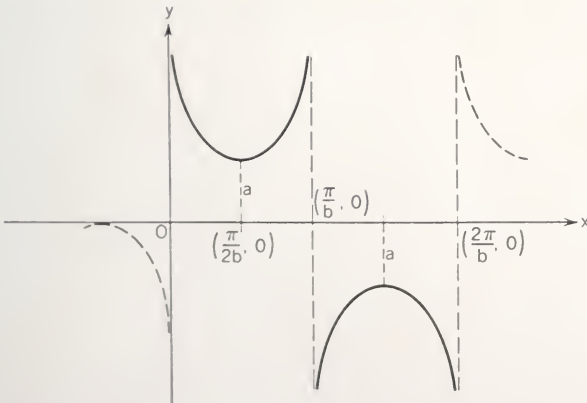
(a) $y = a \sin bx$ (b) $y = a \cos bx$ (c) $y = a \tan bx$



(d) $y = a \cot bx$

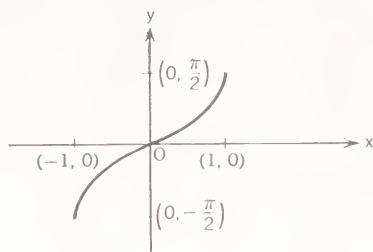
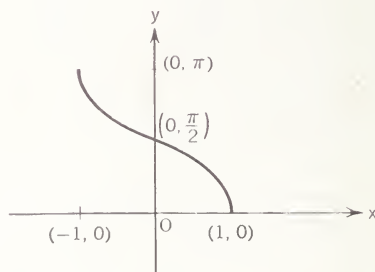
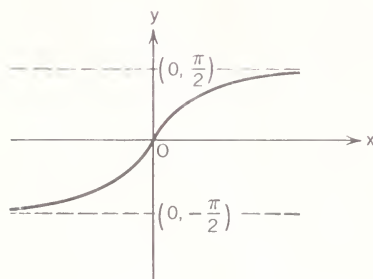
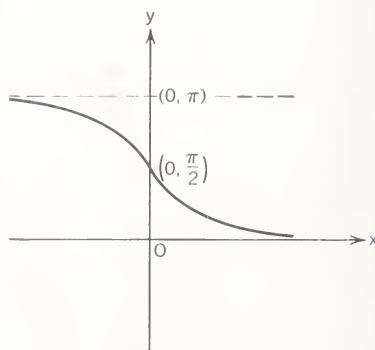


(e) $y = a \sec bx$

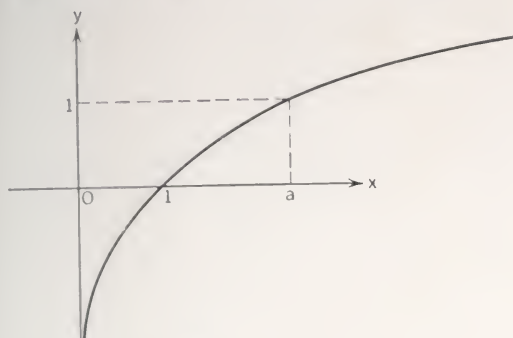


(f) $y = a \csc bx$

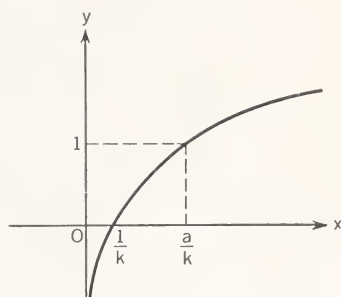
7.8 THE INVERSE TRIGONOMETRIC FUNCTIONS

(a) $y = \arcsin x$ (b) $y = \arccos x$ (c) $y = \arctan x$ (d) $y = \operatorname{arccot} x$

7.9 THE LOGARITHMIC FUNCTIONS

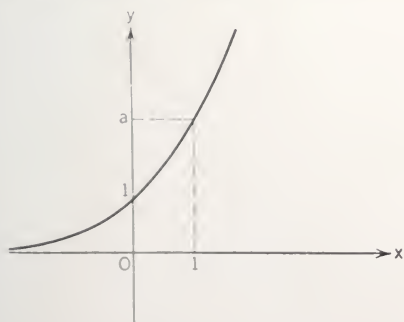


(a) $y = \log_a x, \quad a > 0, a \neq 1$

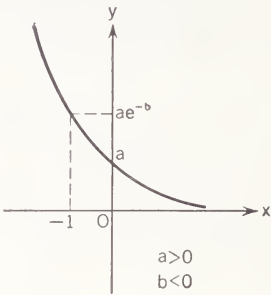
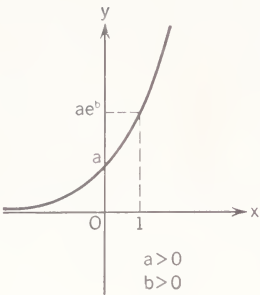


(b) $y = \log_a kx, \quad a > 0, a \neq 1, k \neq 0$

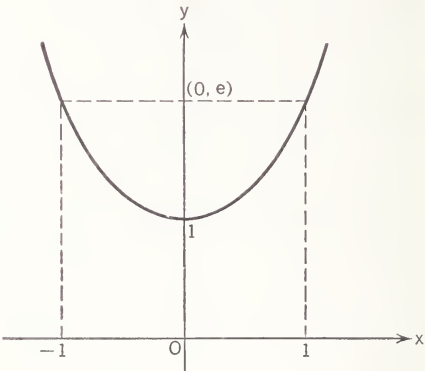
7.10 THE EXPONENTIAL FUNCTIONS



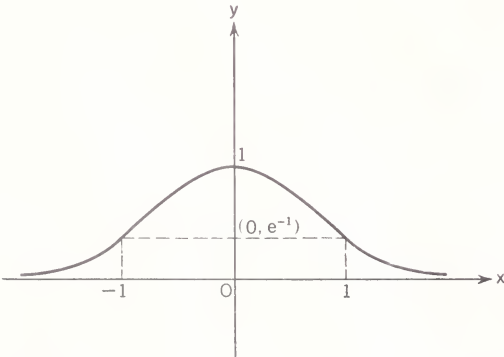
(a) $y = a^x, \quad a > 0, a \neq 1$



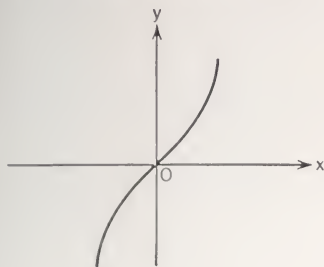
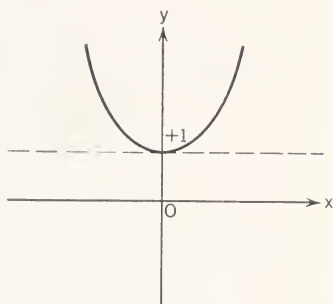
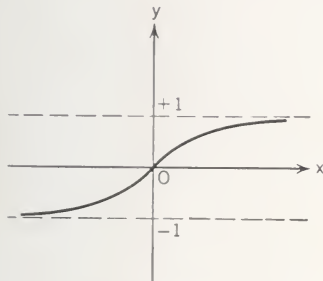
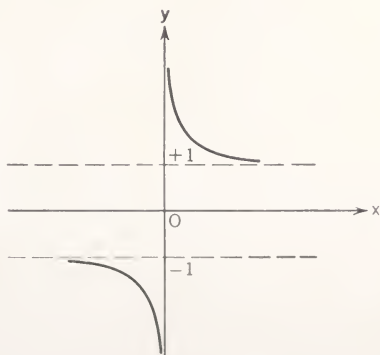
(b) $y = ae^{bx}$

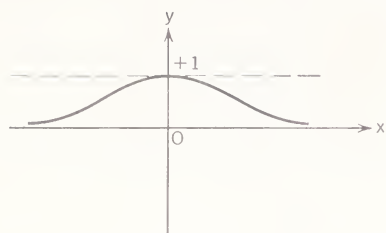
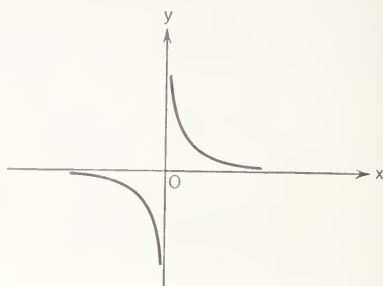


(c) $y = e^{x^2}$

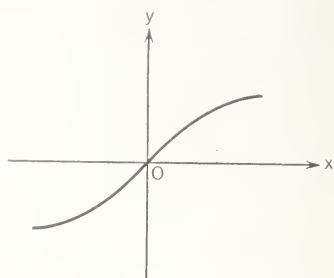
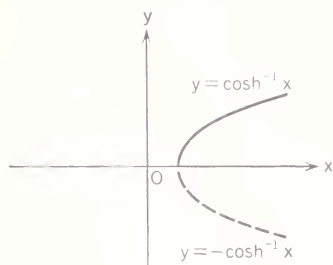
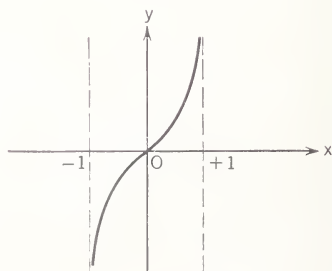


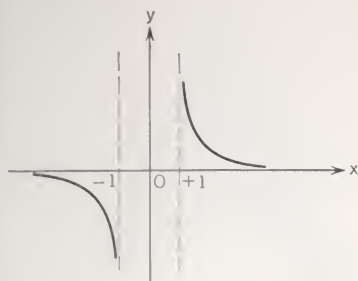
(d) $y = e^{-x^2}$

7.11 THE HYPERBOLIC FUNCTIONS(a) $y = \sinh x$ (b) $y = \cosh x$ (c) $y = \tanh x$ (d) $y = \coth x$

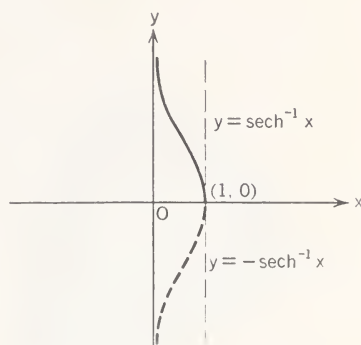
(e) $y = \operatorname{sech} x$ (f) $y = \operatorname{csch} x$

7.12 THE INVERSE HYPERBOLIC FUNCTIONS

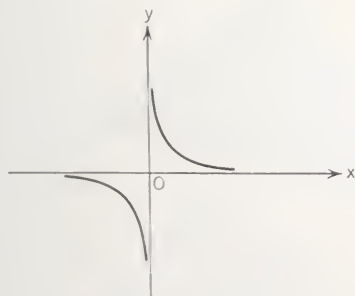
(a) $y = \sinh^{-1} x$ (b) $y = \cosh^{-1} x$ (c) $y = \tanh^{-1} x$



(d) $y = \coth^{-1} x$

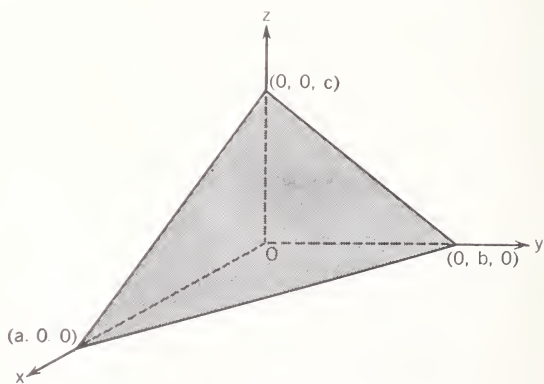
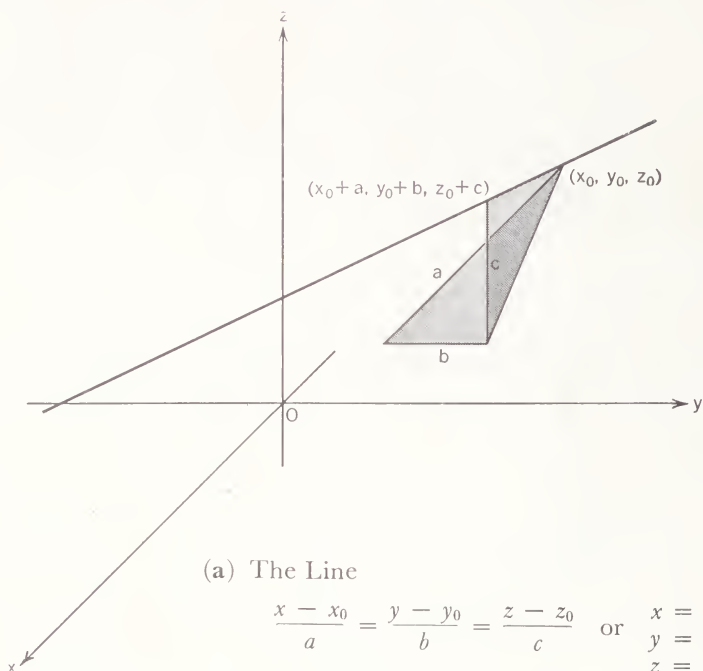


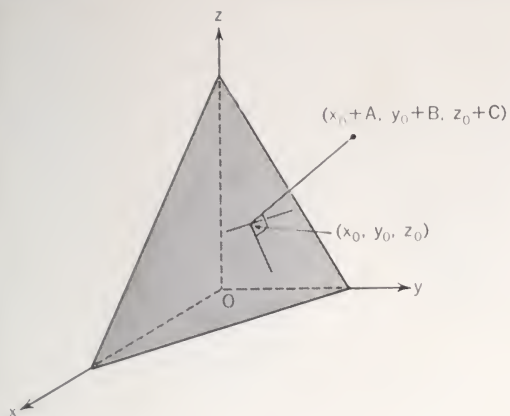
(e) $y = \operatorname{sech}^{-1} x$



(f) $y = \operatorname{csch}^{-1} x$

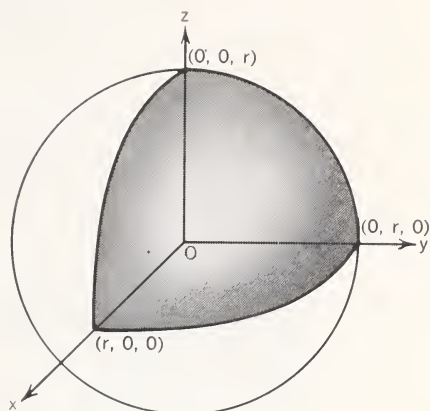
7.13 THREE-DIMENSIONAL GRAPHS





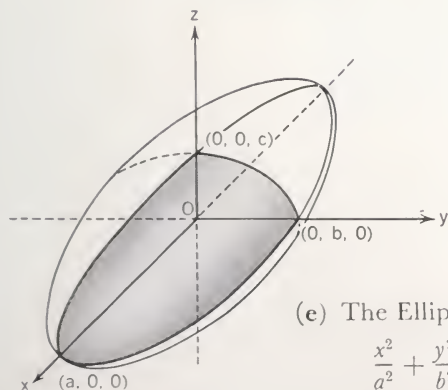
(c) The Plane

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$



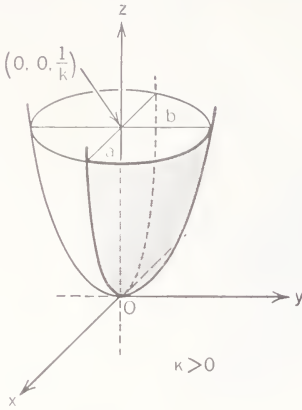
(d) The Sphere

$$x^2 + y^2 + z^2 = r^2$$



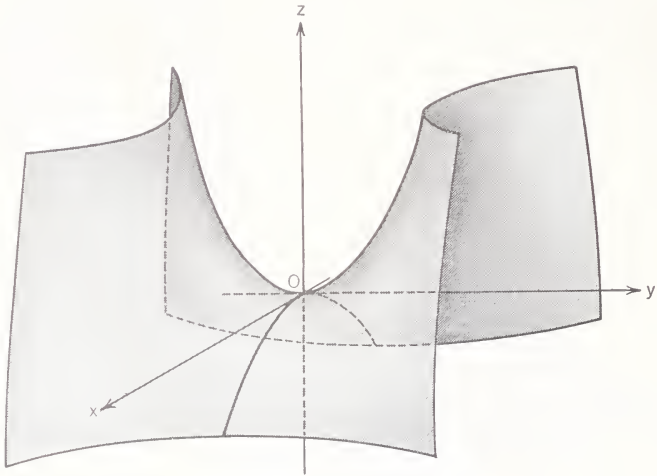
(e) The Ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



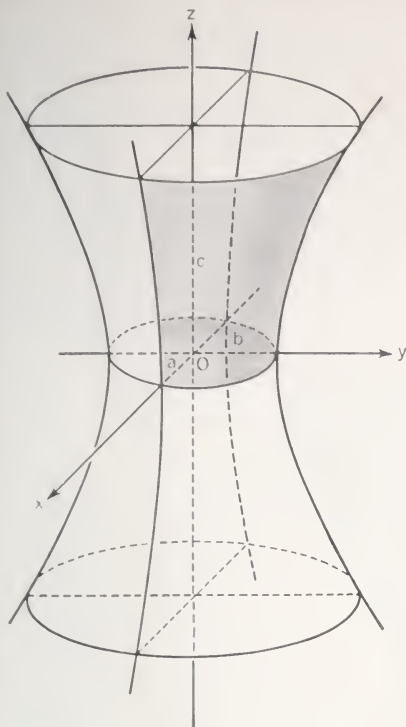
(f) Elliptic Paraboloid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = kz$$



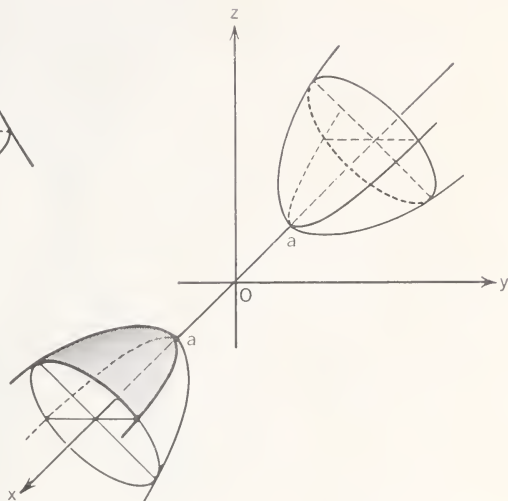
(g) Hyperbolic Paraboloid

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = kz, \quad k < 0$$



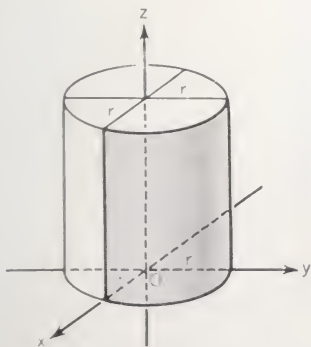
(h) Hyperboloid of One Sheet

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$



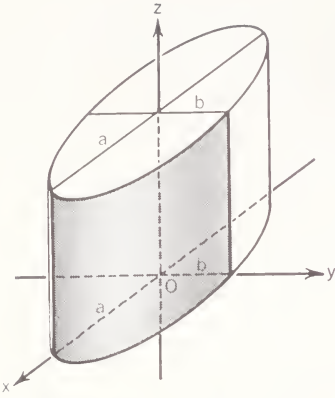
(i) Hyperboloid of Two Sheets

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$



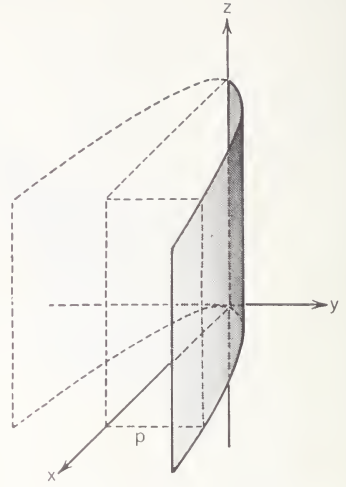
(j) Circular Cylinder

$$x^2 + y^2 = r^2$$



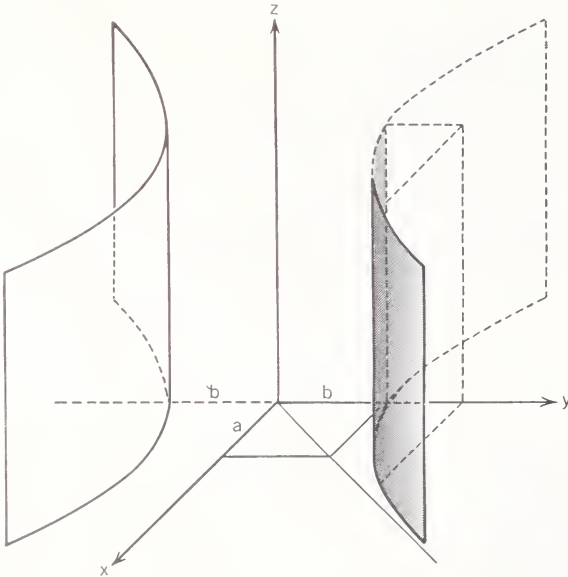
(k) Elliptic Cylinder

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



(l) Parabolic Cylinder

$$y = \frac{x^2}{2p}$$

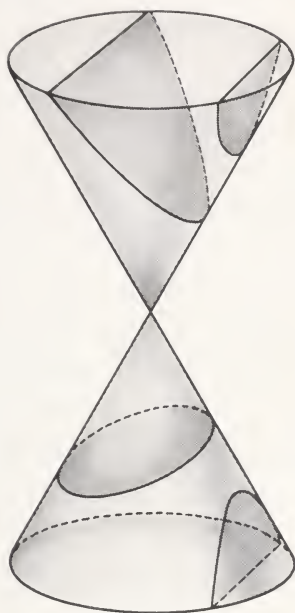


(m) Hyperbolic Cylinder

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

7.14 THE CONIC SECTIONS

The curves whose graphs appear in Secs. 7.2 to 7.5 are known classically as the **conic sections**, since each of them is the intersection of a plane with a circular cone of two nappes. Such a cone appears in the figure below. On its surface are sketched an ellipse, a parabola, and a hyperbola, each of which is obtained as the curve in which a plane cuts the cone.



The common point of the two nappes of the cone is called its **vertex**; a line through the vertex lying on the surface of the cone is called an **element** of the cone. The line through the vertex which forms equal angles at the vertex with all elements of the cone is called the **axis** of the cone. Let the angle between this line and an element be called α . Let P be a plane containing the axis of the cone, and let Q be a plane that intersects P in a line perpendicular to the axis at an angle β , with $0 \leq \beta \leq 90^\circ$.

(1) If $\alpha < \beta \leq 90^\circ$:

- (a) If $\beta \leq 90^\circ$ and Q does not contain the vertex, then the intersection of Q and the cone is an ellipse.
 - (b) If $\beta \leq 90^\circ$ and Q does contain the vertex, then the intersection of Q and the cone is a point, called a degenerate ellipse.
 - (c) If $\beta = 90^\circ$ and Q does not contain the vertex, then the intersection of Q and the cone is a circle. Thus, the circle is a special case of an ellipse.
 - (d) If $\beta = 90^\circ$ and Q does contain the vertex, then the intersection of Q and the cone is a point, called a degenerate circle.
- (2) If $\beta = \alpha$:
- (a) If Q does not contain the vertex, then the intersection of Q and the cone is a parabola.
 - (b) If Q does contain the vertex, then the intersection of Q and the cone is a straight line, called a degenerate parabola.

The degenerate parabola obtained here is a special case of the general degenerate parabola, which consists of two parallel lines. Here, the "parallel" lines are coincident. Although the list of conic sections in this section is complete, it does not include all second-degree loci. It omits only the general degenerate parabola and the null locus, both of which may be obtained as a plane section of a circular cylinder.

- (3) If $0 \leq \beta < \alpha$:
- (a) If Q does not contain the vertex, then the intersection of Q and the cone is a hyperbola.
 - (b) If Q does contain the vertex, then the intersection of Q and the cone is a pair of intersecting straight lines, called a degenerate hyperbola.

8

TYPES OF FUNCTIONAL EQUATIONS

THE TWO principal types of functional equations we shall take up are differential equations and difference equations, defined in Secs. 4.4 and 4.5.

8.1 DERIVATIVES, ANTIDERIVATIVES, AND INDEFINITE INTEGRALS

The simplest kind of differential equation is $\mathbf{Df} = \mathbf{g}$. If \mathbf{f} is given and we want to find \mathbf{g} , we take the derivative of \mathbf{f} . The basic laws of differentiation and the derivatives of basic functions are given in Chap. 9. If \mathbf{g} is given, then the general solution for \mathbf{f} is the indefinite integral of \mathbf{g} , while any particular solution is an antiderivative of \mathbf{g} . Basic properties of indefinite integrals are given in Chap. 10, which contains a table of integrals, Table 10.4.

8.2 DEFINITE INTEGRALS

From the fundamental theorem of the integral calculus, the evaluation of a definite integral

$$\int_a^b \mathbf{f}(x) \, dx$$

can sometimes be accomplished by first finding an antiderivative $\mathbf{F}(x)$

of $\mathbf{f}(x)$. Then

$$\int_a^b \mathbf{f}(x) dx = \mathbf{F}(b) - \mathbf{F}(a).$$

When it is possible to find an antiderivative, this procedure is usually the best way of evaluating the definite integral. Table 10.7 lists some definite integrals directly; many of these cannot be obtained conveniently by the method of first finding an antiderivative.

8.3 OTHER CLASSES OF DIFFERENTIAL EQUATIONS

A differential equation of the form

$$\mathbf{D}^n \mathbf{f} + \mathbf{h}_1(x) \mathbf{D}^{n-1} \mathbf{f} + \mathbf{h}_2(x) \mathbf{D}^{n-2} \mathbf{f} + \dots + \mathbf{h}_{n-1}(x) \mathbf{D} \mathbf{f} + \mathbf{h}_n(x) \mathbf{f} = \mathbf{g}(x),$$

where n is a positive integer and $\mathbf{h}_1(x) \dots \mathbf{h}_n(x)$ and $\mathbf{g}(x)$ are all given functions, is called a **linear differential equation**. A differential equation which cannot be written in this form is called **nonlinear**. If each of the functions $\mathbf{h}_i(x)$ is a constant, the equation is said to be **linear with constant coefficients**; these equations are treated in Chap. 11. The special case in which each $\mathbf{h}_i(x) = 0$ can be handled by n applications of Table 10.4; a few specific repeated indefinite integrals are given directly in Table 10.10. Some more general linear differential equations are discussed in Chap. 13.

Differential equations can also be classified according to their order. A general treatment of first- and second-order differential equations appears in Chap. 12.

8.4 DIFFERENCES, ANTIDIFFERENCES, AND INDEFINITE FINITE INTEGRALS

The simplest kind of difference equation is $\Delta \mathbf{f} = \mathbf{g}$. If \mathbf{f} is given and we want to find \mathbf{g} , we find the difference of \mathbf{f} . The basic laws of differences are given in Chap. 13, where a table of differences of basic functions is given. If \mathbf{g} is given, then the general solution for \mathbf{f} is the indefinite finite integral of \mathbf{g} , while any particular solution is an anti-difference of \mathbf{g} . Basic properties of indefinite finite integrals are given in Chap. 14, and Table 14.2 lists some of these finite integrals.

8.5 FINITE SUMS

In analogy with the application of the fundamental theorem of the integral calculus to the evaluation of definite integrals, it is sometimes possible to evaluate a finite sum with the aid of an antidifference.

Thus to evaluate $\sum_a^b f(x)$, if we can find a function $F(x)$ such that $\Delta F(x) = f(x)$, then

$$\sum_a^b f(x) = F(b+1) - F(a).$$

Some properties and a table of finite sums appear in Chap. 14.

8.6 OTHER CLASSES OF DIFFERENCE EQUATIONS

For most other classes of difference equations, it is more convenient to express the equation in terms of the operator E instead of Δ . Conversion from one form to the other can be accomplished by the relation $E = \Delta + 1$ (see Sec. 3.5). A difference equation of the form

$$E^n f + h_1(x) E^{n-1} f + h_2(x) E^{n-2} f + \dots + h_{n-1}(x) E f + h_n(x) f = g(x),$$

is called a **linear difference equation**. A difference equation which cannot be written in this form is called **nonlinear**. If each of the functions $h_i(x)$ is a constant, the equation is said to be **linear with constant coefficients**; these equations are treated in Chap. 15. Some more general linear difference equations are discussed in Chap. 16.

9

DERIVATIVES

9.1 TABLE: PROPERTIES OF DERIVATIVES

- (1) $Dcf(x) = cDf(x)$ for any real number c .
- (2) $D[f(x) \pm g(x)] = Df(x) \pm Dg(x)$.
- (3) $D[f(x)g(x)] = g(x) Df(x) + f(x) Dg(x)$.
- (4) $D \frac{f(x)}{g(x)} = \frac{g(x) Df(x) - f(x) Dg(x)}{[g(x)]^2}$.
- (5) $Df(g(x)) = f'(g(x)) \cdot Dg(x)$, where $f' = Df$.

9.2 TABLE: DERIVATIVES OF BASIC FUNCTIONS

- (1) $Dc = 0$, c a real number.
- (2) $Dx = 1$.
- (3) $Dx^p = px^{p-1}$, p any real number.
- (4) $D \sin x = \cos x$.
- (5) $D \cos x = -\sin x$.
- (6) $D \tan x = \sec^2 x$.
- (7) $D \cot x = -\csc^2 x$.
- (8) $D \sec x = \sec x \tan x$.
- (9) $D \csc x = -\csc x \cot x$.

$$(10) \quad D \arcsin x = \frac{1}{\sqrt{1-x^2}}.$$

$$(11) \quad D \arccos x = \frac{-1}{\sqrt{1-x^2}}.$$

$$(12) \quad D \arctan x = \frac{1}{1+x^2}.$$

$$(13) \quad D \operatorname{arccot} x = \frac{-1}{1+x^2}.$$

$$(14) \quad D \operatorname{arcsec} x = \frac{1}{x\sqrt{x^2-1}}.$$

$$(15) \quad D \operatorname{arccsc} x = \frac{-1}{|x|\sqrt{x^2-1}}.$$

$$(16) \quad D \ln x = \frac{1}{x}.$$

$$(17) \quad D \log_a x = \frac{1}{x \ln a}.$$

$$(18) \quad D e^x = e^x.$$

$$(19) \quad D a^x = a^x \ln a.$$

$$(20) \quad D \sinh x = \cosh x.$$

$$(21) \quad D \cosh x = \sinh x.$$

$$(22) \quad D \tanh x = \operatorname{sech}^2 x.$$

$$(23) \quad D \coth x = -\operatorname{csch}^2 x.$$

$$(24) \quad D \operatorname{sech} x = -\operatorname{sech} x \tanh x.$$

$$(25) \quad D \operatorname{csch} x = -\operatorname{csch} x \coth x.$$

$$(26) \quad D \sinh^{-1} x = \frac{1}{\sqrt{x^2+1}}.$$

$$(27) \quad D \cosh^{-1} x = \frac{1}{\sqrt{x^2-1}}.$$

$$(28) \quad D \tanh^{-1} x = \frac{1}{1-x^2}.$$

$$(29) \quad D \coth^{-1} x = \frac{1}{1-x^2}.$$

$$(30) \quad \mathbf{D} \operatorname{sech}^{-1} x = \frac{-1}{x\sqrt{1-x^2}}.$$

$$(31) \quad \mathbf{D} \operatorname{csch}^{-1} x = \frac{-1}{x\sqrt{1+x^2}}.$$

10

INTEGRALS

10.1 TABLE: PROPERTIES OF INDEFINITE INTEGRALS

- (1) $\int c f(x) dx = c \int f(x) dx$, c a real number.
- (2) $\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$.
- (3) $\int f(x) Dg(x) dx = f(x)g(x) - \int g(x) Df(x) dx$, where $g(x)$
is any antiderivative of $Dg(x)$.
- (4) $\int f(g(x)) Dg(x) dx = F(g(x)) + C$, where
 $\int f(x) dx = F(x) + C$.
- (5) $\int Df(x) dx = f(x) + C$.
- (6) $D \int f(x) dx = f(x)$.

10.2 TECHNIQUES OF INTEGRATION

The two basic techniques of integration are substitution and integration by parts [formulas (4) and (3) of Sec. 10.1]. The former is by far the more useful technique. According to (4), an integrand of the form $f(g(x)) Dg(x)$ for certain functions f and g may be treated as simply $f(x)$, provided that in the result of the integration x is replaced by $g(x)$. In order not to confuse the various uses of ' x '

in this process, we proceed formally as follows. To integrate a function $\mathbf{h}(x)$:

- (1) Attempt to find functions \mathbf{f} and \mathbf{g} such that

$$\mathbf{h}(x) = \mathbf{f}(\mathbf{g}(x)) \mathbf{D}\mathbf{g}(x).$$

- (2) Evaluate $\int \mathbf{f}(u) du$ as if ' u ' were ' x ,' to obtain a result of the form $\mathbf{F}(u) + C$.

- (3) Replace ' u ' in this result by $\mathbf{g}(x)$.

Example 1. Find $\int x^2 e^{2x^3} dx$. Let us choose $\mathbf{g}(x) = 2x^3$; then $\mathbf{D}\mathbf{g}(x) = 6x^2$. We may write the integral in the form

$$\frac{1}{6} \int e^{2x^3} (6x^2) dx,$$

hence the exponential function is a good choice for \mathbf{f} . We evaluate

$$\frac{1}{6} \int e^u du = \frac{1}{6} e^u + C,$$

so that

$$\int x^2 e^{2x^3} dx = \frac{1}{6} e^{2x^3} + C.$$

Example 2. Find $\int \frac{\sin 2x}{1 + \cos^2 x} dx$. Since $\sin 2x = 2 \sin x \cos x$, we shall evaluate $\int \frac{2 \sin x \cos x}{1 + \cos^2 x} dx$. We choose $\mathbf{g}(x) = \cos x$. Then $\mathbf{D}\mathbf{g}(x) = -\sin x$, and

$$\int \frac{2 \sin x \cos x}{1 + \cos^2 x} dx = \int \frac{-2\mathbf{D}\mathbf{g}(x) \cdot \mathbf{g}(x)}{1 + [\mathbf{g}(x)]^2} dx.$$

Now we find

$$\int \frac{-2u}{1 + u^2} du = -\ln(1 + u^2) + C.$$

Replacing u by $\cos x$,

$$\int \frac{\sin 2x}{1 + \cos^2 x} dx = -\ln(1 + \cos^2 x) + C.$$

Another valuable technique is that of integration by parts, making use of formula (3) of Sec. 10.1,

$$\int \mathbf{f}(x) \mathbf{D}\mathbf{g}(x) dx = \mathbf{f}(x)\mathbf{g}(x) - \int \mathbf{g}(x) \mathbf{D}\mathbf{f}(x) dx.$$

To make good use of this method of finding the indefinite integral of a function $\mathbf{h}(x)$, we must write $\mathbf{h}(x)$ as a product of two functions $\mathbf{f}(x)$ and $\mathbf{Dg}(x)$, where it is possible to find the indefinite integral of $\mathbf{Dg}(x)$, and where $\mathbf{g}(x) \mathbf{Df}(x)$ is easier to integrate than the given function. In many cases, more than one choice of \mathbf{f} and \mathbf{Dg} can be made; usually only one of these leads to an essential simplification of the problem. There are no rules which specify the choices to be made, but only a little practice is needed to learn to reject most bad choices.

Example 3. Find $\int x e^x dx$. We choose $\mathbf{f}(x) = x$ and $\mathbf{Dg}(x) = e^x$. Then $\mathbf{Df}(x) = 1$ and $\mathbf{g}(x) = e^x$. Applying formula (3) of Sec. 10.1, we obtain

$$\int x e^x dx = x e^x - \int e^x dx = x e^x - e^x + C.$$

If we had chosen $\mathbf{f}(x) = e^x$ and $\mathbf{Dg}(x) = x$, then $\mathbf{Df}(x) = e^x$ and $\mathbf{g}(x) = x^2/2$ and we would have

$$\int x e^x dx = \frac{1}{2} x^2 e^x - \frac{1}{2} \int x^2 e^x dx.$$

While this is a correct application of the rule, it is not a useful one, since the new integral is more complicated than the original.

There are cases in which two or more applications of the process of integration by parts yield a new integral which is a multiple of the original one. In some of these cases, we can solve for the integral by some elementary algebraic manipulations. In case this procedure results in a particular integral, the indefinite integral can be obtained by adding an arbitrary constant.

Example 4. We shall apply integration by parts to $\int e^x \sin x dx$. Let $\mathbf{f}(x) = e^x$ and $\mathbf{Dg}(x) = \sin x$; then $\mathbf{Df}(x) = e^x$, $\mathbf{g}(x) = -\cos x$, and

$$\int e^x \sin x dx = -e^x \cos x + \int e^x \cos x dx.$$

Applying the same process to $\int e^x \cos x dx$, let $\mathbf{f}(x) = e^x$ and $\mathbf{Dg}(x) = \cos x$; then $\mathbf{Df}(x) = e^x$, $\mathbf{g}(x) = \sin x$, and

$$\int e^x \cos x dx = e^x \sin x - \int e^x \sin x dx.$$

We now have two equations in the unknowns $\int e^x \sin x dx$ and

$\int e^x \cos x \, dx$. If we substitute the expression for $\int e^x \cos x \, dx$ from the second equation into the first and solve for $\int e^x \sin x \, dx$, we obtain

$$\int e^x \sin x \, dx = \frac{1}{2}e^x (\sin x - \cos x) + C,$$

where C was added to obtain the indefinite integral. Solving for the other integral, we get

$$\int e^x \cos x \, dx = \frac{1}{2}e^x (\sin x + \cos x) + C.$$

If an integrand involves x and one of the expressions $\sqrt{a^2 - x^2}$, $\sqrt{a^2 + x^2}$, $\sqrt{x^2 - a^2}$, then the integration may often be accomplished by the replacement of x by a trigonometric function $g(x)$ and by integrating an associated expression. In this expression, ' x ' is replaced by ' u ' in order not to confuse the various uses of ' x .' Such replacements are justified by the converse of formula (4) of Sec. 10.1, which holds if the inverse of g is a function. We have the rule: To evaluate $\int f(x) \, dx$, where f involves one of the above expressions, replace x by a function of the form $g(u)$, where g is chosen according to the rules below, and evaluate $\int f(g(u)) \, Dg(u) \, du$. In this result, replace ' u ' by $g^{-1}(x)$. The replacement procedures are given in detail in the following rules:

(1) If $f(x)$ involves x and $\sqrt{a^2 - x^2}$, substitute $a \sin u$ for x , $a \cos u$ for $\sqrt{a^2 - x^2}$, and let $Dg(u) = a \cos u$. After substitution, the integrated expression is in terms of u . To state it in terms of x , write

$$\begin{aligned} \arcsin \frac{x}{a} & \text{ for } u, \\ \frac{x}{a} & \text{ for } \sin u, \\ \frac{\sqrt{a^2 - x^2}}{a} & \text{ for } \cos u, \end{aligned}$$

to obtain the final result.

(2) If $f(x)$ involves x and $\sqrt{a^2 + x^2}$, substitute $a \tan u$ for x , $a \sec u$ for $\sqrt{a^2 + x^2}$ and let $Dg(u) = a \sec^2 u$. After substitution, the integrated expression is in terms of u . To state it in terms of x , write

$$\begin{aligned} \arctan \frac{x}{a} & \text{ for } u, \\ \frac{x}{a} & \text{ for } \tan u, \\ \frac{\sqrt{a^2 + x^2}}{a} & \text{ for } \sec u. \end{aligned}$$

(3) If $f(x)$ involves x and $\sqrt{x^2 - a^2}$, substitute $a \sec u$ for x , $a \tan u$ for $\sqrt{x^2 - a^2}$, and let $Dg(u) = a \sec u \tan u$. After substitution, the integrated expression is in terms of u . To state it in terms of x , write

$$\begin{aligned} \operatorname{arcsec} \frac{x}{a} & \text{ for } u, \\ \frac{x}{a} & \text{ for } \sec u, \\ \frac{\sqrt{x^2 - a^2}}{a} & \text{ for } \tan u. \end{aligned}$$

In each case, use trigonometric identities to find what expressions in x replace the trigonometric functions of u in the integrated expression.

Example 5. Evaluate

$$\int \frac{\sqrt{9 - x^2} dx}{x}$$

According to formula (1) of Sec. 10.1, let $g(u) = 3 \sin u$. Then $\sqrt{9 - 9g^2(u)} = 3 \cos u$, and $Dg(u) = 3 \cos u$. Pass to the integral

$$\int \frac{3 \cos u}{3 \sin u} (3 \cos u) du = 3 \int \frac{\cos^2 u du}{\sin u}.$$

Integrating by means of Table 10.4, we obtain

$$\cos u + \ln \left| \tan \frac{u}{2} \right| + C.$$

From the trigonometric identities,

$$\tan \frac{u}{2} = \frac{\sin (u/2)}{\cos (u/2)} = \sqrt{\frac{1 - \cos u}{1 + \cos u}} = \frac{1 - \cos u}{\sin u}.$$

Making the replacements indicated in formula (1) of Sec. 10.1,

$$\int \frac{\sqrt{9-x^2} dx}{x} = \frac{\sqrt{9-x^2}}{3} + \ln \left| \frac{3 - \sqrt{9-x^2}}{x} \right| + C.$$

10.3 SUGGESTIONS FOR THE USE OF THE TABLE OF INTEGRALS

A particular order for listing integrals has been used in Tables 10.4 and 10.7, so that the reader will be able to find any integrand quickly if he knows the order. The order of integrands is as follows: algebraic functions, trigonometric functions, inverse trigonometric functions, logarithms, exponentials, and hyperbolic functions. Integrands involving more than one function will be placed immediately following whichever function is listed last in this ordering.

In these tables we have adopted the notation that a , b , c , and p designate real numbers; m and n designate integers. C is an arbitrary (real) constant.

To integrate a rational function, it is almost always advisable to perform first a partial fractions decomposition of the integrand (see Sec. 2.8).

A number of integrals in Table 10.4 are recursion formulas; they reduce the problem of finding the integral of a function involving an n -th power to one involving a lower or more convenient power. After a sufficient number of applications of these recursion formulas, the integrand will be in a form which can be found elsewhere in the tables; a reference to such an entry will be found following the recursion formula.

Example 6. Find $\int \sin^7 x dx$. Use formula (74) of Table 10.4 with $n = 7$ to obtain

$$\int \sin^7 x dx = -\frac{1}{7}(\sin^6 x \cos x - 6 \int \sin^5 x dx).$$

Using the same formula with $n = 5$, we find

$$\int \sin^5 x dx = -\frac{1}{5}(\sin^4 x \cos x - 4 \int \sin^3 x dx).$$

Finally, since n is odd, we use formula (72) of Table 10.4 to obtain

$$\int \sin^3 x \, dx = -\frac{1}{3} \cos x (\sin^2 x + 2) + C.$$

Combining these results we obtain

$$\int \sin^7 x \, dx = -\frac{\sin^6 x \cos x}{7} - \frac{6 \sin^4 x \cos x}{35} - \frac{8 \cos x (\sin^2 x + 2)}{35} + C.$$

If an integrand is not found in the table, one should try to make a substitution which will form an integrand which is in the table. However, one should bear in mind that many integrands are of such a nature that no integral of them can be written in terms of a finite number of elementary functions.

10.4 TABLE: INDEFINITE INTEGRALS

Algebraic Functions

$$(1) \quad \int x^p \, dx = \frac{1}{p+1} x^{p+1} + C, \quad \text{if } p \neq -1.$$

$$(2) \quad \int \frac{dx}{x} = \ln |x| + C.$$

Algebraic Functions of x and $ax + b$

$$(3) \quad \int (ax + b)^p \, dx = \frac{(ax + b)^{p+1}}{a(p+1)} + C, \quad \text{if } p \neq -1.$$

$$(4) \quad \int \frac{dx}{ax + b} = \frac{1}{a} \ln |ax + b| + C.$$

$$(5) \quad \int \frac{x \, dx}{ax + b} = \frac{1}{a^2} [ax - b \ln |ax + b|] + C.$$

$$(6) \quad \int \frac{x \, dx}{(ax + b)^2} = \frac{1}{a^2} \left[\ln |ax + b| + \frac{b}{ax + b} \right] + C.$$

$$(7) \quad \int \frac{x \, dx}{(ax + b)^3} = \frac{1}{a^2} \left[\frac{b}{2(ax + b)^2} - \frac{1}{ax + b} \right] + C.$$

$$(8) \quad \int x(ax + b)^p \, dx = \frac{1}{a^2(p+2)} (ax + b)^{p+2} - \frac{b}{a^2(p+1)} (ax + b)^{p+1} + C \quad \text{if } p \neq -1, p \neq -2.$$

$$(9) \quad \int x^m(ax+b)^n dx = \frac{1}{m+n+1} \left[x^{m+1}(ax+b)^n + nb \int x^m(ax+b)^{n-1} dx \right], \quad \text{if } m+n+1 \neq 0.$$

See (1).

$$(10) \quad \int x^m(ax+b)^n dx = \frac{1}{a(m+n+1)} \left[x^m(ax+b)^{n+1} - mb \int x^{m-1}(ax+b)^n dx \right], \quad \text{if } m+n+1 \neq 0.$$

See (3).

$$(11) \quad \int \frac{x^2 dx}{ax+b} = \frac{1}{a^3} \left[\frac{1}{2} (ax+b)^2 - 2b(ax+b) + b^2 \ln |ax+b| \right] + C.$$

$$(12) \quad \int \frac{x^2 dx}{(ax+b)^2} = \frac{1}{a^3} \left[ax+b - 2b \ln |ax+b| - \frac{b^2}{ax+b} \right] + C.$$

$$(13) \quad \int \sqrt{ax+b} dx = \frac{2}{3a} \sqrt{(ax+b)^3} + C.$$

$$(14) \quad \int \frac{dx}{\sqrt{ax+b}} = \frac{2\sqrt{ax+b}}{a} + C.$$

$$(15) \quad \int x\sqrt{ax+b} dx = \frac{2(3ax-2b)\sqrt{(ax+b)^3}}{15a^2} + C.$$

$$(16) \quad \int \frac{\sqrt{ax+b}}{x} dx = 2\sqrt{ax+b} + b \int \frac{dx}{x\sqrt{ax+b}}.$$

See (17) if $b > 0$; (18) if $b < 0$.

$$(17) \quad \int \frac{dx}{x\sqrt{ax+b}} = \frac{1}{\sqrt{b}} \ln \left| \frac{\sqrt{ax+b} - \sqrt{b}}{\sqrt{ax+b} + \sqrt{b}} \right| + C \\ = -\frac{2}{\sqrt{b}} \tanh^{-1} \sqrt{\frac{ax+b}{b}} + C, \quad \text{if } b > 0.$$

$$(18) \quad \int \frac{dx}{x\sqrt{ax-b}} = \frac{2}{\sqrt{b}} \arctan \sqrt{\frac{ax-b}{b}} + C, \quad \text{if } b < 0.$$

$$(19) \quad \int x^2 \sqrt{ax+b} dx = \frac{2}{105a^3} \sqrt{(ax+b)^3} (15a^2x^2 - 12abx + 8b^2) + C.$$

$$(20) \quad \int \frac{x^n dx}{\sqrt{ax+b}} = \frac{2}{a(2n+1)} \left[x^n \sqrt{ax+b} - nb \int \frac{x^{n-1} dx}{\sqrt{ax+b}} \right].$$

See (14).

$$(21) \quad \int \frac{dx}{x^n \sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{(n-1)bx^{n-1}} - \frac{(2n-3)a}{(2n-2)b} \int \frac{dx}{x^{n-1} \sqrt{ax+b}}.$$

See (14).

Algebraic Functions of x and $ax^2 + b$

$$(22) \quad \int \frac{dx}{ax^2+b} = \frac{1}{\sqrt{ab}} \arctan \frac{x\sqrt{ab}}{b} + C, \quad \text{if } ab > 0.$$

$$(23) \quad \int \frac{dx}{ax^2+b} = \frac{1}{2\sqrt{-ab}} \ln \left| \frac{b+x\sqrt{-ab}}{b-x\sqrt{-ab}} \right| + C \\ = \frac{1}{\sqrt{-ab}} \tanh^{-1} \frac{x\sqrt{-ab}}{b} + C, \quad \text{if } ab < 0.$$

$$(24) \quad \int \frac{dx}{ax^2-b} = \frac{1}{2\sqrt{ab}} \ln \left| \frac{x\sqrt{a}-\sqrt{b}}{x\sqrt{a}+\sqrt{b}} \right| + C, \quad \text{if } a > 0, b > 0.$$

$$(25) \quad \int \frac{dx}{-ax^2+b} = \frac{1}{2\sqrt{ab}} \ln \left| \frac{\sqrt{b}+x\sqrt{a}}{\sqrt{b}-x\sqrt{a}} \right| + C, \quad \text{if } a > 0, b > 0.$$

$$(26) \quad \int \frac{dx}{(ax^2+b)^{m+1}} = \frac{1}{2mb} \left[\frac{x}{(ax^2+b)^m} + (2m-1) \int \frac{dx}{(ax^2+b)^m} \right].$$

See (23) or (24) if $b > 0$; (23) if $b < 0$.

$$(27) \quad \int x(ax^2+b)^p dx = \frac{1}{2a} \frac{(ax^2+b)^{p+1}}{p+1} + C, \quad \text{if } p \neq -1.$$

$$(28) \quad \int \frac{x dx}{ax^2+b} = \frac{1}{2a} \ln |ax^2+b| + C.$$

$$(29) \quad \int \frac{x dx}{(ax^2+b)^{p+1}} = \frac{-1}{2pa(ax^2+b)^p} + C, \quad \text{if } p > 0.$$

$$(30) \quad \int \frac{dx}{x(ax^n+b)} = \frac{1}{bn} \ln \left| \frac{x^n}{ax^n+b} \right| + C.$$

$$(31) \quad \int x^p(ax^n + b)^m dx = \frac{1}{nm + p + 1} [x^{p+1}(ax^n + b)^m + bnm \int x^p(ax^n + b)^{m-1} dx] \quad \text{if } m > 0 \text{ and } nm + p + 1 \neq 0.$$

See (1).

$$(32) \quad \int \sqrt{ax^2 + b} dx = \frac{x}{2} \sqrt{ax^2 + b} + \frac{b}{2\sqrt{a}} \ln |x\sqrt{a} + \sqrt{ax^2 + b}| + C, \quad \text{if } a > 0.$$

$$(33) \quad \int \sqrt{ax^2 + b} dx = \frac{x}{2} \sqrt{ax^2 + b} + \frac{b}{2\sqrt{-a}} \arcsin \left(x \sqrt{\frac{-a}{b}} \right) + C, \quad \text{if } a < 0.$$

Algebraic Functions of x and $ax^2 + bx + c$

$$X = ax^2 + bx + c, \quad q = 4ac - b^2.$$

$$(34) \quad \int \frac{dx}{ax^2 + bx + c} = \frac{2}{\sqrt{q}} \arctan \frac{2ax + b}{\sqrt{q}} + C, \quad \text{if } q > 0.$$

$$(35) \quad \int \frac{dx}{ax^2 + bx + c} = \frac{1}{\sqrt{-q}} \ln \left| \frac{2ax + b - \sqrt{-q}}{2ax + b + \sqrt{-q}} \right| + C \\ = \frac{-2}{\sqrt{-q}} \tanh^{-1} \frac{2ax + b}{\sqrt{-q}} + C, \quad \text{if } q < 0.$$

$$(36) \quad \int \frac{dx}{(ax^2 + bx + c)^{n+1}} = \frac{2ax + b}{nqX^n} + \frac{2(2n-1)a}{nq} \int \frac{dx}{X^n}.$$

See (33) if $q > 0$; (34) if $q < 0$.

$$(37) \quad \int \sqrt{ax^2 + bx + c} dx = \frac{2ax + b}{4a} \sqrt{X} + \frac{q}{8a} \int \frac{dx}{\sqrt{X}}.$$

See (37) if $a > 0$; (38) if $a < 0$.

$$(38) \quad \int \frac{dx}{\sqrt{ax^2 + bx + c}} = \frac{1}{\sqrt{a}} \ln \left| X + x\sqrt{a} + \frac{b}{2\sqrt{a}} \right| + C \\ = \frac{1}{\sqrt{a}} \sinh^{-1} \frac{2ax + b}{\sqrt{q}} + C, \quad \text{if } a > 0.$$

$$(39) \quad \int \frac{dx}{\sqrt{ax^2 + bx + c}} = \frac{1}{\sqrt{-a}} \arcsin \frac{-2ax - b}{\sqrt{-q}} + C, \quad \text{if } a < 0.$$

$$(40) \quad \int \frac{x \, dx}{ax^2 + bx + c} = \frac{1}{2a} \left[\ln |X| - b \int \frac{dx}{X} \right].$$

See (33) if $q > 0$; (34) if $q < 0$.

Algebraic Functions of x and $x^2 \pm a^2$

$$(41) \quad \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a} + C.$$

$$(42) \quad \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x - a}{x + a} \right| + C = -\frac{1}{a} \coth^{-1} \frac{x}{a} + C.$$

$$(43) \quad \int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln |x + \sqrt{x^2 \pm a^2}|] + C.$$

$$(44) \quad \int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln |x + \sqrt{x^2 \pm a^2}| + C.$$

$$(45) \quad \int x\sqrt{x^2 \pm a^2} \, dx = \frac{1}{3} \sqrt{(x^2 \pm a^2)^3} + C.$$

$$(46) \quad \int \frac{\sqrt{x^2 + a^2}}{x} \, dx = \sqrt{x^2 + a^2} - a \ln \left| \frac{a + \sqrt{x^2 + a^2}}{x} \right| + C.$$

$$(47) \quad \int \frac{\sqrt{x^2 - a^2}}{x} \, dx = \sqrt{x^2 - a^2} - a \operatorname{arcsec} \frac{x}{a} + C.$$

$$(48) \quad \int \frac{x \, dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2} + C.$$

$$(49) \quad \int \frac{dx}{x\sqrt{x^2 + a^2}} = -\frac{1}{a} \ln \left| \frac{\sqrt{x^2 + a^2} + a}{x} \right| + C \\ = -\frac{1}{a} \operatorname{arcsinh} \frac{a}{x} + C.$$

$$(50) \quad \int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{x}{a} + C = \frac{1}{a} \arccos \frac{a}{x} + C.$$

$$(51) \quad \int \sqrt{(x^2 \pm a^2)^3} \, dx = \frac{1}{8} [2x\sqrt{(x^2 \pm a^2)^3} \pm 3a^2x\sqrt{x^2 \pm a^2} \\ + 3a^4 \ln |x - \sqrt{x^2 \pm a^2}|] + C.$$

$$(52) \quad \int \frac{dx}{\sqrt{(x^2 \pm a^2)^3}} = \frac{\pm x}{a^2 \sqrt{x^2 \pm a^2}} + C.$$

$$(53) \quad \int x\sqrt{(x^2 \pm a^2)^3} \, dx = \frac{1}{5} \sqrt{(x^2 \pm a^2)^5} + C.$$

$$(54) \quad \int \frac{x \, dx}{\sqrt{(x^2 \pm a^2)^3}} = \frac{-1}{\sqrt{x^2 \pm a^2}} + C.$$

Algebraic Functions of x and $a^2 - x^2$

$$(55) \quad \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + C$$

$$= \frac{1}{a} \tanh^{-1} \frac{x}{a} + C.$$

$$(56) \quad \int \sqrt{a^2 - x^2} \, dx = \frac{1}{2} \left(x \sqrt{a^2 - x^2} + a^2 \arcsin \frac{x}{a} \right) + C.$$

$$(57) \quad \int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$$

$$= -\arccos \frac{x}{a} + C.$$

$$(58) \quad \int x \sqrt{a^2 - x^2} \, dx = -\frac{1}{3} \sqrt{(a^2 - x^2)^3} + C.$$

$$(59) \quad \int \frac{\sqrt{a^2 - x^2}}{x} \, dx = \sqrt{a^2 - x^2} - a \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C.$$

$$(60) \quad \int \frac{x \, dx}{\sqrt{a^2 - x^2}} = -\sqrt{a^2 - x^2} + C.$$

$$(61) \quad \int \frac{dx}{x \sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$$

$$= -\frac{1}{a} \cosh^{-1} \frac{a}{x} + C.$$

$$(62) \quad \int \sqrt{(a^2 - x^2)^3} \, dx = \frac{1}{8} \left[2x \sqrt{(a^2 - x^2)^3} + 3a^2 x \sqrt{a^2 - x^2} \right.$$

$$\left. + 3a^4 \arcsin \frac{x}{a} \right] + C.$$

$$(63) \quad \int \frac{dx}{\sqrt{(a^2 - x^2)^3}} = \frac{x}{a^2 \sqrt{a^2 - x^2}} + C.$$

$$(64) \quad \int x \sqrt{(a^2 - x^2)^3} dx = -\frac{1}{5} \sqrt{(a^2 - x^2)^5} + C.$$

$$(65) \quad \int \frac{x dx}{\sqrt{(a^2 - x^2)^3}} = \frac{1}{\sqrt{a^2 - x^2}} + C.$$

Other Algebraic Functions

$$(66) \quad \int \sqrt{2ax - x^2} dx = \frac{1}{2} \left[(x - a) \sqrt{2ax - x^2} + a^2 \arcsin \frac{x - a}{a} \right] + C.$$

$$(67) \quad \int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \frac{a - x}{a} + C.$$

$$(68) \quad \int \sqrt{\frac{1+x}{1-x}} dx = \arcsin x - \sqrt{1-x^2} + C.$$

Trigonometric Functions

$$(69) \quad \int \sin x dx = -\cos x + C.$$

$$(70) \quad \int \sin (ax + b) dx = -\frac{1}{a} \cos (ax + b) + C.$$

$$(71) \quad \begin{aligned} \int \sin^2 (ax + b) dx &= \frac{x}{2} - \frac{1}{2a} \cos (ax + b) \sin (ax + b) + C \\ &= \frac{x}{2} - \frac{\sin 2(ax + b)}{4a} + C_1. \end{aligned}$$

$$(72) \quad \begin{aligned} \int \sin^3 (ax + b) dx \\ = -\frac{1}{3a} \cos (ax + b) [\sin^2 (ax + b) + 2] + C. \end{aligned}$$

$$(73) \quad \begin{aligned} \int \sin^4 (ax + b) dx &= \frac{3x}{8} - \frac{3 \sin 2(ax + b)}{16a} \\ &\quad - \frac{\sin^3 (ax + b) \cos (ax + b)}{4a} + C. \end{aligned}$$

$$(74) \quad \int \sin^n(ax + b) dx = -\frac{1}{an} \left[\sin^{n-1}(ax + b) \cos(ax + b) - a(n-1) \int \sin^{n-2}(ax + b) dx \right].$$

See (73) if n is even; (72) if n is odd.

$$(75) \quad \int x \sin(ax + b) dx = \frac{1}{a^2} \sin(ax + b) - \frac{x}{a} \cos(ax + b) + C.$$

$$(76) \quad \int x \sin^2(ax + b) dx = \frac{x^2}{4} - \frac{x \sin 2(ax + b)}{4a} - \frac{\cos 2(ax + b)}{8a^2} + C.$$

$$(77) \quad \int x^2 \sin^2(ax + b) dx = \frac{x^3}{6} - \left(\frac{x^2}{4a} - \frac{1}{8a^3} \right) \sin 2(ax + b) - \frac{x \cos 2(ax + b)}{4a^2} + C.$$

$$(78) \quad \int x^n \sin(ax + b) dx = -\frac{1}{a} \left[x^n \cos(ax + b) - n \int x^{n-1} \cos(ax + b) dx \right].$$

See (91) if n is even; (75) if n is odd.

$$(79) \quad \int \frac{\sin ax dx}{x} = ax - \frac{(ax)^3}{3(3!)} + \frac{(ax)^5}{5(5!)} - + \dots + C.$$

$$(80) \quad \int \frac{\sin(ax + b) dx}{x^n} = \frac{-1}{n-1} \frac{\sin(ax + b)}{x^{n-1}} + \frac{a}{n-1} \int \frac{\cos(ax + b) dx}{x^{n-1}}, \quad \text{if } n > 1.$$

See (70) if n is even; (86) if n is odd.

$$(81) \quad \int \frac{dx}{1 \pm \sin(ax + b)} = \mp \frac{1}{a} \tan \left(\frac{\pi}{4} \pm \frac{ax + b}{2} \right) + C.$$

$$(82) \quad \int \sqrt{1 + \sin x} dx = \pm 2 \left(\sin \frac{x}{2} - \cos \frac{x}{2} \right) + C.$$

Use $+$ sign if $(8n-1)\frac{\pi}{2} < x \leq (8n+3)\frac{\pi}{2}$ for some integer n ; otherwise use $-$ sign.

$$(83) \quad \int \sqrt{1 - \sin x} \, dx = \pm 2 \left(\sin \frac{x}{2} + \cos \frac{x}{2} \right) + C.$$

Use $+$ sign if $(8n - 3) \frac{\pi}{2} < x \leq (8n + 1) \frac{\pi}{2}$ for some integer n ; otherwise use $-$ sign.

$$(84) \quad \int \sin ax \sin bx \, dx = \frac{\sin (a - b)x}{2(a - b)} - \frac{\sin (a + b)x}{2(a + b)} + C, \\ \text{if } a^2 \neq b^2.$$

$$(85) \quad \int \cos x \, dx = \sin x + C.$$

$$(86) \quad \int \cos (ax + b) \, dx = \frac{\sin (ax + b)}{a} + C.$$

$$(87) \quad \int \cos^2 (ax + b) \, dx = \frac{1}{2a} \sin (ax + b) \cos (ax + b) + \frac{x}{2} + C. \\ = \frac{1}{4a} \sin 2(ax + b) + \frac{x}{2} + C.$$

$$(88) \quad \int \cos^3 (ax + b) \, dx = \frac{1}{a} \sin (ax + b) - \frac{1}{3a} \sin^3 (ax + b) + C$$

$$(89) \quad \int \cos^4 (ax + b) \, dx = \frac{3x}{8} + \frac{3 \sin 2(ax + b)}{16a} \\ + \frac{\cos^3 (ax + b) \sin (ax + b)}{4a} + C.$$

$$(90) \quad \int \cos^n (ax + b) \, dx = \frac{1}{na} \left[\cos^{n-1} (ax + b) \sin (ax + b) \right. \\ \left. + a(n - 1) \int \cos^{n-2} (ax + b) \, dx \right].$$

See (89) if n is even; (88) if n is odd.

$$(91) \quad \int x \cos (ax + b) \, dx = \frac{1}{a^2} \cos (ax + b) + \frac{x}{a} \sin (ax + b) + C.$$

$$(92) \quad \int x \cos^2 (ax + b) \, dx = \frac{x^2}{4} + \frac{x \sin 2(ax + b)}{4a} \\ + \frac{\cos 2(ax + b)}{8a^2} + C.$$

$$(93) \quad \int x^2 \cos^2(ax + b) dx = \frac{x^3}{6} + \left(\frac{x^2}{4a} - \frac{1}{8a^3} \right) \sin 2(ax + b) \\ + \frac{x \cos 2(ax + b)}{4a^2} + C.$$

$$(94) \quad \int x^n \cos(ax + b) dx \\ = \frac{1}{a} \left[x^n \sin(ax + b) - n \int x^{n-1} \sin(ax + b) dx \right].$$

See (75) if n is even; (91) if n is odd.

$$(95) \quad \int \frac{\cos ax dx}{x} = \ln |ax| - \frac{(ax)^2}{2 \cdot 2!} + \frac{(ax)^4}{4 \cdot 4!} - + \dots + C.$$

$$(96) \quad \int \frac{\cos(ax + b)}{x^n} dx = \frac{-1}{n-1} \frac{\cos(ax + b)}{x^{n-1}} \\ - \frac{a}{n-1} \int \frac{\sin(ax + b)}{x^{n-1}} dx, \quad \text{if } n > 1.$$

See (86) if n is even; (70) if n is odd.

$$(97) \quad \int \frac{dx}{1 + \cos(ax + b)} = \frac{1}{a} \tan \frac{ax + b}{2} + C.$$

$$(98) \quad \int \frac{dx}{1 - \cos(ax + b)} = -\frac{1}{a} \cot \frac{ax + b}{2} + C.$$

$$(99) \quad \int \sqrt{1 + \cos x} dx = \pm 2\sqrt{2} \sin \frac{x}{2} + C.$$

Use $+$ sign if $(4n-1)\pi < x \leq (4n+1)\pi$ for some integer n ; otherwise use $-$ sign.

$$(100) \quad \int \sqrt{1 - \cos x} dx = \pm 2\sqrt{2} \cos \frac{x}{2} + C.$$

Use $+$ sign if $(4n-2)\pi < x \leq 4n\pi$ for some integer n ; otherwise use $-$ sign.

$$(101) \quad \int \cos ax \cos bx dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C, \\ \text{if } a^2 \neq b^2.$$

$$(102) \quad \int \sin ax \cos bx dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} + C, \\ \text{if } a^2 \neq b^2.$$

$$(103) \quad \int \sin(ax+b) \cos(ax+b) dx = \frac{1}{2a} \sin^2(ax+b) + C.$$

$$(104) \quad \int \sin^p(ax+b) \cos(ax+b) dx = \frac{\sin^{p+1}(ax+b)}{a(p+1)} + C, \\ \text{if } p \neq -1.$$

$$(105) \quad \int \sin(ax+b) \cos^p(ax+b) dx = -\frac{\cos^{p+1}(ax+b)}{a(p+1)} + C, \\ \text{if } p \neq -1.$$

$$(106) \quad \int \sin^2(ax+b) \cos^2(ax+b) dx \\ = -\frac{1}{32a} \sin 4(ax+b) + \frac{x}{8} + C.$$

$$(107) \quad \int \sin^m(ax+b) \cos^n(ax+b) dx \\ = -\frac{1}{a(m+n)} \left[\sin^{m-1}(ax+b) \cos^{n+1}(ax+b) \right. \\ \left. - (m-1)a \int \sin^{m-2}(ax+b) \cos^n(ax+b) dx \right], \\ \text{if } m > 0, m+n \neq 0.$$

See (90) if m is even; (105) if m is odd.

$$(108) \quad \int \sin^m(ax+b) \cos^n(ax+b) dx \\ = \frac{1}{a(m+n)} \left[\sin^{m+1}(ax+b) \cos^{n-1}(ax+b) \right. \\ \left. + (n-1)a \int \sin^m(ax+b) \cos^{n-2}(ax+b) dx \right], \\ \text{if } n > 0, m+n \neq 0.$$

See (74) if n is even; (104) if n is odd.

$$(109) \quad \int \frac{dx}{\sin(ax+b) \cos(ax+b)} = \frac{1}{a} \ln |\tan(ax+b)| + C.$$

$$(110) \quad \int \frac{dx}{\sin^n(ax+b) \cos(ax+b)} = \frac{-1}{a(n-1)} \frac{1}{\sin^{n-1}(ax+b)} \\ + \int \frac{dx}{\sin^{n-2}(ax+b) \cos(ax+b)}, \quad \text{if } n > 1.$$

See (128) if n is even; (109) if n is odd.

$$(111) \quad \int \frac{dx}{\sin(ax+b) \cos^n(ax+b)} = \frac{1}{a(n-1)} \frac{1}{\cos^{n-1}(ax+b)} + \int \frac{dx}{\sin(ax+b) \cos^{n-2}(ax+b)}, \quad \text{if } n > 1.$$

See (135) if n is even; (109) if n is odd.

$$(112) \quad \int \frac{\sin(ax+b)}{\cos^2(ax+b)} dx = \frac{1}{a \cos(ax+b)} + C \\ = \frac{1}{a} \sec(ax+b) + C.$$

$$(113) \quad \int \frac{\sin^2(ax+b)}{\cos(ax+b)} dx \\ = -\frac{1}{a} \left[\sin(ax+b) - \ln \left| \tan \left(\frac{ax+b}{2} + \frac{\pi}{4} \right) \right| \right] + C.$$

$$(114) \quad \int \frac{\cos(ax+b)}{\sin^2(ax+b)} dx = \frac{-1}{a \sin(ax+b)} + C \\ = -\frac{1}{a} \csc(ax+b) + C.$$

$$(115) \quad \int \frac{\cos^2(ax+b)}{\sin(ax+b)} dx = \frac{1}{a} \left[\cos(ax+b) + \ln \left| \tan \frac{ax+b}{2} \right| \right] + C.$$

$$(116) \quad \int \tan x dx = -\ln |\cos x| + C = \ln |\sec x| + C.$$

$$(117) \quad \int \tan(ax+b) dx = -\frac{1}{a} \ln |\cos(ax+b)| + C.$$

$$(118) \quad \int \tan^2(ax+b) dx = \frac{1}{a} \tan(ax+b) - x + C.$$

$$(119) \quad \int \tan^3(ax+b) dx \\ = \frac{1}{2a} [\tan^2(ax+b) + 2 \ln |\cos(ax+b)|] + C.$$

$$(120) \quad \int \tan^n(ax+b) dx = \frac{1}{a(n-1)} \tan^{n-1}(ax+b) \\ - \int \tan^{n-2}(ax+b) dx, \quad n \geq 2.$$

See (118) if n is even; (119) if n is odd.

$$(121) \quad \int \cot x \, dx = \ln |\sin x| + C = -\ln |\csc x| + C.$$

$$(122) \quad \int \cot (ax + b) \, dx = \frac{1}{a} \ln |\sin (ax + b)| + C.$$

$$(123) \quad \int \cot^2 (ax + b) \, dx = -\frac{1}{a} \cot (ax + b) - x + C.$$

$$(124) \quad \int \cot^3 (ax + b) \, dx \\ = -\frac{1}{2a} [\cot^2 (ax + b) + 2 \ln |\sin (ax + b)|] + C.$$

$$(125) \quad \int \cot^n (ax + b) \, dx = -\frac{1}{a(n-1)} \cot^{n-1} (ax + b) \\ - \int \cot^{n-2} (ax + b) \, dx, \quad n \geq 2.$$

See (123) if n is even; (124) if n is odd.

$$(126) \quad \int \sec x \, dx = \ln \left| \tan \left(\frac{x}{2} + \frac{\pi}{4} \right) \right| + C = \ln |\sec x + \tan x| + C.$$

$$(127) \quad \int \sec^2 x \, dx = \tan x + C.$$

$$(128) \quad \int \sec (ax + b) \, dx = \frac{1}{a} \ln \left| \tan \left(\frac{ax + b}{2} + \frac{\pi}{4} \right) \right| + C.$$

$$(129) \quad \int \sec^2 (ax + b) \, dx = \frac{1}{a} \tan (ax + b) + C.$$

$$(130) \quad \int \sec^3 (ax + b) \, dx = \frac{1}{2a} \left[\sec (ax + b) \tan (ax + b) \right. \\ \left. + \ln \left| \tan \left(\frac{ax + b}{2} + \frac{\pi}{4} \right) \right| \right] + C.$$

$$(131) \quad \int \sec^n (ax + b) \, dx = \frac{1}{a(n-1)} \frac{\sin (ax + b)}{\cos^{n-1} (ax + b)} \\ + \frac{n-2}{n-1} \int \sec^{n-2} (ax + b) \, dx, \quad n \geq 2.$$

See (129) if n is even; (130) if n is odd.

$$(132) \quad \int \sec x \tan x \, dx = \sec x + C.$$

$$(133) \quad \int \csc x \, dx = \ln \left| \tan \frac{x}{2} \right| + C = \ln |\csc x - \cot x| + C.$$

$$(134) \quad \int \csc^2 x \, dx = -\cot x + C.$$

$$(135) \quad \int \csc (ax + b) \, dx = \frac{1}{a} \ln \left| \tan \frac{ax + b}{2} \right| + C.$$

$$(136) \quad \int \csc^2 (ax + b) \, dx = -\frac{1}{a} \cot (ax + b) + C.$$

$$(137) \quad \int \csc^3 (ax + b) \, dx = \frac{1}{2a} \left[-\csc (ax + b) \cot (ax + b) + \ln \left| \tan \frac{ax + b}{2} \right| \right] + C.$$

$$(138) \quad \int \csc^n (ax + b) \, dx = \frac{-1}{a(n-1)} \frac{\cos (ax + b)}{\sin^{n-1} (ax + b)} + \frac{n-2}{n-1} \int \csc^{n-2} (ax + b) \, dx, \quad n \geq 2.$$

See (136) if n is even; (137) if n is odd.

$$(139) \quad \int \csc x \cot x \, dx = -\csc x + C.$$

Inverse Trigonometric Functions

$$(140) \quad \int \arcsin \frac{x}{a} \, dx = x \arcsin \frac{x}{a} + \sqrt{a^2 - x^2} + C.$$

$$(141) \quad \int (\arcsin ax)^2 \, dx = x(\arcsin ax)^2 - 2x + \frac{2}{a} \sqrt{1 - a^2 x^2} \arcsin ax + C.$$

$$(142) \quad \int x \arcsin ax \, dx = \frac{1}{4a^2} [(2a^2 x^2 - 1) \arcsin ax + ax \sqrt{1 - a^2 x^2}] + C.$$

$$(143) \quad \int \frac{\arcsin ax}{x^2} \, dx = a \ln \left| \frac{1 - \sqrt{1 - a^2 x^2}}{ax} \right| - \frac{\arcsin ax}{x} + C.$$

$$(144) \quad \int \arccos \frac{x}{a} \, dx = x \arccos \frac{x}{a} - \sqrt{a^2 - x^2} + C.$$

$$(145) \quad \int (\arccos ax)^2 dx = x(\arccos ax)^2 - 2x \\ - \frac{2}{a} \sqrt{1 - a^2 x^2} \arccos ax + C.$$

$$(146) \quad \int \arctan \frac{x}{a} dx = x \arctan \frac{x}{a} - \frac{a}{2} \ln(a^2 + x^2) + C.$$

$$(147) \quad \int \operatorname{arccot} \frac{x}{a} dx = x \operatorname{arccot} \frac{x}{a} + \frac{a}{2} \ln(a^2 + x^2) + C.$$

$$(148) \quad \int \operatorname{arcsec} \frac{x}{a} dx = x \operatorname{arcsec} \frac{x}{a} - a \ln |x + \sqrt{x^2 - a^2}| + C.$$

$$(149) \quad \int \operatorname{arccsc} \frac{x}{a} dx = x \operatorname{arccsc} \frac{x}{a} + a \ln |x + \sqrt{x^2 - a^2}| + C.$$

Logarithmic Functions

$$(150) \quad \int \ln |x| dx = x \ln |x| - x + C.$$

$$(151) \quad \int \log_a |x| dx = x \log_a |x| - \frac{x}{\ln a} + C, \quad \text{if } a \neq 1, a > 0.$$

$$(152) \quad \int \ln |ax + b| dx = \frac{ax + b}{a} \ln |ax + b| - x + C.$$

$$(153) \quad \int (\ln |x|)^2 dx = x(\ln |x|)^2 - 2x \ln |x| + 2x + C.$$

$$(154) \quad \int (\ln |ax + b|)^n dx = \frac{ax + b}{a} (\ln |ax + b|)^n \\ - n \int (\ln |ax + b|)^{n-1} dx.$$

See (152).

$$(155) \quad \int x \ln |x| dx = \frac{x^2}{2} \ln |x| - \frac{x^2}{4} + C.$$

$$(156) \quad \int \frac{dx}{x \ln |x|} = \ln |\ln |x|| + C.$$

$$(157) \quad \int x^p \ln |x| dx = x^{p+1} \left[\frac{\ln |x|}{p+1} - \frac{1}{(p+1)^2} \right] + C, \quad \text{if } p \neq -1.$$

$$(158) \quad \int \frac{(\ln |x|)^p}{x} dx = \frac{1}{p+1} (\ln |x|)^{p+1} + C, \quad \text{if } p \neq -1.$$

$$(159) \quad \int \sin \ln |x| dx = \frac{x}{2} [\sin \ln |x| - \cos \ln |x|] + C.$$

$$(160) \quad \int \cos \ln |x| dx = \frac{x}{2} [\sin \ln |x| + \cos \ln |x|] + C.$$

Exponential Functions

$$(161) \quad \int e^x dx = e^x + C.$$

$$(162) \quad \int e^{ax} dx = \frac{e^{ax}}{a} + C.$$

$$(163) \quad \int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1) + C.$$

$$(164) \quad \int x^m e^{ax} dx = \frac{x^m e^{ax}}{a} - \frac{m}{a} \int x^{m-1} e^{ax} dx, \quad m \geq 2.$$

See (163).

$$(165) \quad \int \frac{e^{ax} dx}{x} = \ln |x| + ax + \frac{(ax)^2}{2 \cdot 2!} + \frac{(ax)^3}{3 \cdot 3!} + \dots + C.$$

$$(166) \quad \int e^{ax} \sin bx dx = \frac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2} + C.$$

$$(167) \quad \int e^{ax} \cos bx dx = \frac{e^{ax}(a \cos bx + b \sin bx)}{a^2 + b^2} + C.$$

$$(168) \quad \int \frac{dx}{1 + e^x} = x - \ln(1 + e^x) + C.$$

$$(169) \quad \int \frac{dx}{ae^{px} + b} = \frac{x}{b} - \frac{1}{bp} \ln |ae^{px} + b| + C, \quad \text{if } b \neq 0, p \neq 0.$$

$$(170) \quad \int \frac{dx}{ae^{px} + be^{-px}} = \frac{1}{p\sqrt{ab}} \arctan \left(e^{px} \sqrt{\frac{a}{b}} \right) + C, \text{ if } ab > 0.$$

$$(171) \quad \int e^{ax} \ln |bx| dx = \frac{1}{a} e^{ax} \ln |bx| - \frac{1}{a} \int \frac{e^{ax}}{x} dx.$$

See (165).

$$(172) \quad \int a^x dx = \frac{a^x}{\ln a} + C, \text{ if } a > 0, a \neq 1.$$

$$(173) \quad \int a^{bx} dx = \frac{a^{bx}}{b \ln a} + C, \text{ if } a > 0, a \neq 1.$$

$$(174) \quad \int x a^{bx} = \frac{x a^{bx}}{b \ln a} - \frac{a^{bx}}{b^2 (\ln a)^2} + C, \quad \text{if } a > 0, a \neq 1.$$

Hyperbolic Functions

$$(175) \quad \int \sinh ax \, dx = \frac{1}{a} \cosh ax + C.$$

$$(176) \quad \int \sinh^2 ax \, dx = \frac{1}{4a} \sinh 2ax - \frac{1}{2}x + C.$$

$$(177) \quad \int \cosh ax \, dx = \frac{1}{a} \sinh ax + C.$$

$$(178) \quad \int \cosh^2 ax \, dx = \frac{1}{4a} \sinh 2ax + \frac{1}{2}x + C.$$

$$(179) \quad \int \tanh ax \, dx = \frac{1}{a} \ln |\cosh ax| + C.$$

$$(180) \quad \int \tanh^2 ax \, dx = x - \frac{1}{a} \tanh ax + C.$$

$$(181) \quad \int \coth ax \, dx = \frac{1}{a} \ln |\sinh ax| + C.$$

$$(182) \quad \int \coth^2 ax \, dx = x - \frac{1}{a} \coth ax + C.$$

$$(183) \quad \int \operatorname{sech} ax \, dx = \frac{1}{a} \arctan (\sinh ax) + C.$$

$$(184) \quad \int \operatorname{sech}^2 ax \, dx = \frac{1}{a} \tanh ax + C.$$

$$(185) \quad \int \operatorname{csch} ax \, dx = -\frac{1}{a} \ln |\coth ax + \operatorname{csch} ax| + C$$

$$= \frac{1}{a} \ln \left| \tanh \frac{ax}{2} \right| + C.$$

$$(186) \quad \int \operatorname{csch}^2 ax \, dx = -\frac{1}{a} \coth ax + C.$$

$$(187) \quad \int \operatorname{sech} ax \tanh ax \, dx = -\frac{1}{a} \operatorname{sech} ax + C.$$

$$(188) \quad \int \operatorname{csch} ax \coth ax \, dx = -\frac{1}{a} \operatorname{csch} ax + C.$$

10.5 DEFINITE INTEGRALS

Table 10.7 contains some definite integrals, many of which cannot be obtained by the computation of the antiderivative of the integrand. Many of these are **improper integrals**, that is, integrals whose intervals of integration are infinite or for which the integrand is undefined at an end point of the interval of integration. Provided the limit exists in each case, we have

$$\begin{aligned}\int_a^\infty \mathbf{f}(x) \, dx &= \lim_{b \rightarrow \infty} \int_a^b \mathbf{f}(x) \, dx; \\ \int_{-\infty}^b \mathbf{f}(x) \, dx &= \lim_{a \rightarrow -\infty} \int_a^b \mathbf{f}(x) \, dx; \\ \int_{-\infty}^\infty \mathbf{f}(x) \, dx &\doteq \lim_{a \rightarrow -\infty} \int_a^c \mathbf{f}(x) \, dx + \lim_{b \rightarrow \infty} \int_c^b \mathbf{f}(x) \, dx.\end{aligned}$$

If $\mathbf{f}(x)$ is undefined at $x = b$, then if $b > a$,

$$\int_a^b \mathbf{f}(x) \, dx = \lim_{h \rightarrow 0} \int_a^{b-h} \mathbf{f}(x) \, dx, \quad \text{where } h > 0.$$

If $\mathbf{f}(x)$ is undefined at $x = a$, then if $b > a$,

$$\int_a^b \mathbf{f}(x) \, dx = \lim_{h \rightarrow 0} \int_{a+h}^b \mathbf{f}(x) \, dx, \quad \text{where } h > 0.$$

10.6 TABLE: PROPERTIES OF DEFINITE INTEGRALS

- (1) $\int_a^b c\mathbf{f}(x) \, dx = c \int_a^b \mathbf{f}(x) \, dx, \quad c \text{ any real number.}$
- (2) $\int_a^b [\mathbf{f}(x) \pm \mathbf{g}(x)] \, dx = \int_a^b \mathbf{f}(x) \, dx \pm \int_a^b \mathbf{g}(x) \, dx.$
- (3) $\int_a^a \mathbf{f}(x) \, dx = 0.$
- (4) $\int_a^c \mathbf{f}(x) \, dx = \int_a^b \mathbf{f}(x) \, dx + \int_b^c \mathbf{f}(x) \, dx.$
- (5) $\int_a^b \mathbf{f}(x) \, dx = -\int_b^a \mathbf{f}(x) \, dx.$

10.7 TABLE: DEFINITE INTEGRALS

$$(1) \int_0^{\infty} \frac{\sin x}{\sqrt{x}} dx = \int_0^{\infty} \frac{\cos x}{\sqrt{x}} dx = \sqrt{\frac{\pi}{2}}.$$

$$(2) \int_0^{\pi/2} \sin^n x dx = \int_0^{\pi/2} \cos^n x dx$$

$$= \frac{1 \cdot 3 \cdot 5 \cdots (n-1)}{2 \cdot 4 \cdot 6 \cdots n} \frac{\pi}{2}, \quad \text{if } n \text{ is even}$$

$$= \frac{2 \cdot 4 \cdot 6 \cdots (n-1)}{1 \cdot 3 \cdot 5 \cdots n}, \quad \text{if } n \text{ is odd.}$$

$$(3) \int_0^{\pi} \sin^2 ax dx = \int_0^{\pi} \cos^2 ax dx = \frac{\pi}{2}.$$

$$(4) \int_0^{\infty} \frac{\sin ax}{x} dx = \frac{\pi}{2}, \quad \text{if } a > 0,$$

$$= 0, \quad \text{if } a = 0,$$

$$= -\frac{\pi}{2}, \quad \text{if } a < 0.$$

$$(5) \int_0^{\pi} \sin ax \sin bx dx = \int_0^{\pi} \cos ax \cos bx dx = 0, \quad a \neq b.$$

$$(6) \int_0^{\infty} \frac{\cos ax}{1+x^2} dx = \frac{\pi}{2} e^{-a}, \quad a > 0$$

$$= \frac{\pi}{2} e^a, \quad a < 0.$$

$$(7) \int_0^{\infty} \cos(x^2) dx = \int_0^{\infty} \sin(x^2) dx = \frac{1}{2} \sqrt{\frac{\pi}{2}}.$$

$$(8) \int_0^{\pi/2} \frac{dx}{1+a \cos x} = \frac{\arccos a}{\sqrt{1-a^2}}, \quad a^2 < 1.$$

$$(9) \int_0^{\pi} \frac{dx}{1+a \cos x} = \frac{\pi}{\sqrt{1-a^2}}, \quad a^2 < 1.$$

$$(10) \int_0^{\infty} \frac{\tan x}{x} dx = \frac{\pi}{2}.$$

$$(11) \int_0^{\infty} e^{-ax} \cos bx dx = \frac{a}{a^2 + b^2}, \quad a > 0.$$

$$(12) \quad \int_0^{\infty} e^{-ax} \sin bx \, dx = \frac{b}{a^2 + b^2}, \quad a > 0.$$

$$(13) \quad \int_0^{\infty} e^{-a^2x^2} \cos bx \, dx = \frac{\sqrt{\pi} e^{-b^2/4a^2}}{2a}, \quad a > 0.$$

10.8 TABLE: DEFINITE INTEGRALS OF SPECIAL INTEREST IN PROBABILITY

$$\text{Let } f(x) = \frac{1}{a} e^{-x/a}.$$

$$(1a) \quad \int_0^{\infty} f(x) \, dx = 1.$$

$$(1b) \quad \int_0^{\infty} xf(x) \, dx = a.$$

$$(1c) \quad \int_0^{\infty} x^2 f(x) \, dx = 2a^2.$$

$$\text{Let } f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}.$$

$$(2a) \quad \int_{-\infty}^{\infty} f(x) \, dx = 1.$$

$$(2b) \quad \int_{-\infty}^{\infty} xf(x) \, dx = \mu.$$

$$(2c) \quad \int_{-\infty}^{\infty} (x - \mu)^2 f(x) \, dx = \sigma^2.$$

10.9 ANTIDERIVATIVES OF ORDER n

Given a function $g(x)$, the general solution form of the equation $D^n f = g(x)$ is the sum of a function $h(x)$ and an expression of the form

$$C_1 x^n + C_2 x^{n-1} + \dots + C_{n-1} x + C_n.$$

The function $h(x)$ is obtained by integrating successive particular integrals of $g(x)$, the process continuing for n steps. The function $h(x)$ is called the **antiderivative of $g(x)$ of order n** and is denoted by $I^n g(x)$. Algebraic properties of the repeated indefinite integral are:

$$(1) \quad \mathbf{I}^n(c\mathbf{g}(x)) = c \cdot \mathbf{I}^n\mathbf{g}(x), \quad c \text{ a constant,}$$

and

$$(2) \quad \mathbf{I}^n[\mathbf{g}_1(x) \pm \mathbf{g}_2(x)] = \mathbf{I}^n\mathbf{g}_1(x) \pm \mathbf{I}^n\mathbf{g}_2(x).$$

10.10 TABLE: ANTIDERIVATIVES OF ORDER n

$$(1) \quad \mathbf{I}^n(x^p) = \frac{x^{p+n}}{(p+1)(p+2)\dots(p+n)}, \quad p \neq -1, -2, \dots, -n.$$

$$(2) \quad \mathbf{I}^n(\sin ax) = \frac{\sin ax}{a^n}, \quad \text{if } n = 4k.$$

$$(3) \quad \mathbf{I}^n(\sin ax) = \frac{-\cos ax}{a^n}, \quad \text{if } n = 4k + 1.$$

$$(4) \quad \mathbf{I}^n(\sin ax) = \frac{-\sin ax}{a^n}, \quad \text{if } n = 4k + 2.$$

$$(5) \quad \mathbf{I}^n(\sin ax) = \frac{\cos ax}{a^n}, \quad \text{if } n = 4k + 3.$$

$$(6) \quad \mathbf{I}^n(\cos ax) = \frac{\cos ax}{a^n}, \quad \text{if } n = 4k.$$

$$(7) \quad \mathbf{I}^n(\cos ax) = \frac{\sin ax}{a^n}, \quad \text{if } n = 4k + 1.$$

$$(8) \quad \mathbf{I}^n(\cos ax) = \frac{-\cos ax}{a^n}, \quad \text{if } n = 4k + 2.$$

$$(9) \quad \mathbf{I}^n(\cos ax) = \frac{-\sin ax}{a^n}, \quad \text{if } n = 4k + 3.$$

$$(10) \quad \mathbf{I}^n(e^{ax}) = \frac{e^{ax}}{a^n}.$$

LINEAR DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS

THE GENERAL form of such an equation is

$$\mathbf{D}^n \mathbf{f}(x) + a_1 \mathbf{D}^{n-1} \mathbf{f}(x) + \dots + a_{n-1} \mathbf{D} \mathbf{f}(x) + a_n \mathbf{f}(x) = \mathbf{g}(x),$$

which we rewrite

$$[\mathbf{D}^n + a_1 \mathbf{D}^{n-1} + \dots + a_{n-1} \mathbf{D} + a_n] \mathbf{f}(x) = \mathbf{g}(x).$$

The quantity in brackets is a polynomial in \mathbf{D} which we shall denote by $\mathbf{p}(\mathbf{D})$. The differential equation now takes the form

$$[\mathbf{p}(\mathbf{D})] \mathbf{f}(x) = \mathbf{g}(x).$$

If $\mathbf{g}(x) = 0$, the equation is said to be **homogeneous**. If $\mathbf{g}(x) \neq 0$, then the homogeneous equation $[\mathbf{p}(\mathbf{D})] \mathbf{f}(x) = 0$ obtained by replacing $\mathbf{g}(x)$ by 0 is called the **reduced equation** of the given equation.

If $\mathbf{f}^*(x)$ is a solution of the given equation, then, for any other solution $\mathbf{f}_1^*(x)$, the function $\mathbf{f}_1(x) = \mathbf{f}_1^*(x) - \mathbf{f}^*(x)$ is a solution of its reduced equation, since

$$[\mathbf{p}(\mathbf{D})](\mathbf{f}_1^*(x) - \mathbf{f}^*(x)) = \mathbf{g}(x) - \mathbf{g}(x) = 0.$$

Thus the general solution form (GSF) of the given equation is found by adding any particular solution (PS) of the given equation to the

general solution form of the reduced equation. There are two major steps to the solution of the problem:

- (1) Find the general solution of the reduced equation.
- (2) Find a particular solution for the given equation.

11.1 THE AUXILIARY EQUATION

To obtain the general solution of a homogeneous linear differential equation with constant coefficients $[\mathbf{p}(\mathbf{D})]\mathbf{f}(x) = 0$, we first solve the auxiliary equation

$$\mathbf{p}(m) = 0,$$

where $\mathbf{p}(m)$ is the polynomial in m obtained by replacing \mathbf{D} by m in $\mathbf{p}(\mathbf{D})$.

11.2 HOMOGENEOUS EQUATIONS OF FIRST ORDER

The first-order equation takes the form

$$[\mathbf{p}(\mathbf{D})]\mathbf{f}(x) = [\mathbf{D} - a]\mathbf{f}(x) = 0, \quad a \text{ a real number.}$$

The general solution form is

$$\mathbf{f}(x) = C\mathbf{e}^{ax}.$$

11.3 HOMOGENEOUS EQUATIONS OF SECOND ORDER

The general solution form of the second-order equation $[\mathbf{p}(\mathbf{D})]\mathbf{f}(x) = 0$ is tabulated in Table 11.4 according to the nature of the roots of the auxiliary equation $\mathbf{p}(m) = 0$.

11.4 TABLE: GENERAL SOLUTION FORM OF THE LINEAR HOMOGENEOUS EQUATION OF SECOND ORDER WITH CONSTANT COEFFICIENTS

Let m_1 and m_2 be the roots of $\mathbf{p}(m) = 0$.

- (1) $m_1 \neq m_2$, both real:

$$\text{GSF: } \mathbf{f}(x) = C_1\mathbf{e}^{m_1x} + C_2\mathbf{e}^{m_2x}.$$

$$(2) \quad m_1 = m_2:$$

$$\text{GSF: } \mathbf{f}(x) = (C_1 + C_2 x) \mathbf{e}^{m_1 x}.$$

$$(3) \quad \text{The roots of the auxiliary equation are complex:}$$

$$m_1 = a + bi, \quad m_2 = a - bi.$$

$$\text{GSF: } \mathbf{f}(x) = \mathbf{e}^{ax}(C_1 \sin bx + C_2 \cos bx),$$

$$\text{or} \quad \mathbf{f}(x) = \mathbf{e}^{ax} C_1 \sin (bx + C_2),$$

$$\text{or} \quad \mathbf{f}(x) = \mathbf{e}^{ax} C_1 \cos (bx + C_2),$$

$$\text{or} \quad \mathbf{f}(x) = \mathbf{e}^{ax}(C_1 \mathbf{e}^{ibx} + C_2 \mathbf{e}^{-ibx}).$$

The equivalence of the first three of these forms is discussed in Example 8 of Sec. 4.9.

11.5 EXAMPLES

(1) $[\mathbf{D}^2 + 2\mathbf{D} + 5]\mathbf{f}(x) = 0$. We first solve the auxiliary equation $m^2 + 2m + 5 = 0$, obtaining $m = -1 \pm 2i$. Then $a = -1$, $b = 2$, and the general solution form is

$$\mathbf{f}(x) = \mathbf{e}^{-x}(C_1 \sin 2x + C_2 \cos 2x).$$

(2) $[\mathbf{D}^2 - 3\mathbf{D} - 4]\mathbf{f} = 0$. Solving $m^2 - 3m - 4 = 0$, we obtain $m_1 = -1$ and $m_2 = +4$. The general solution form is

$$\mathbf{f}(x) = C_1 \mathbf{e}^{-x} + C_2 \mathbf{e}^{4x}.$$

(3) $[\mathbf{D}^2 + 6\mathbf{D} + 9]\mathbf{f} = 0$. The roots of $m^2 + 6m + 9 = 0$ are both -3 . Hence, the general solution form is

$$\mathbf{f}(x) = (C_1 + C_2 x) \mathbf{e}^{-3x}.$$

11.6 HOMOGENEOUS EQUATIONS OF ORDER n

To find the general solution form for the homogeneous equation of order n , $[\mathbf{p}(\mathbf{D})]\mathbf{f} = 0$, one should first find the roots of the auxiliary equation $\mathbf{p}(m) = 0$. Let the distinct real roots of this equation be denoted by m_1, m_2, \dots, m_k , and let their multiplicities be r_1, r_2, \dots, r_k , respectively. Let the pairs of distinct complex roots be denoted by

$a_{k+1} \pm b_{k+1}i, a_{k+2} \pm b_{k+2}i, \dots, a_{k+l} \pm b_{k+l}i$, with multiplicities $r_{k+1}, r_{k+2}, \dots, r_{k+l}$, respectively. Then the general solution form of the equation is the sum of the general solution forms of the equations

$$[(\mathbf{D} - \mathbf{m}_j)^{r_j}]\mathbf{f}(x) = 0, \quad 1 \leq j \leq k,$$

and

$$[(\mathbf{D} - \mathbf{a}_j - \mathbf{b}_ji)^{r_j}(\mathbf{D} - \mathbf{a}_j + \mathbf{b}_ji)^{r_j}]\mathbf{f}(x) = 0, \quad k+1 \leq j \leq k+l,$$

where the arbitrary constants appearing in the solutions will be renumbered so that all have different subscripts. These general solution forms are listed in Table 11.7.

11.7 TABLE: GENERAL SOLUTION FORMS FOR HOMOGENEOUS DIFFERENTIAL EQUATIONS OF ORDER n WITH CONSTANT COEFFICIENTS (See Sec. 11.6)

- (1) m_j is real, and $r_j = 1$:

$$\text{GSF: } \mathbf{f}_j(x) = C\mathbf{e}^{m_jx}.$$

- (2) m_j is real, and $r_j > 1$:

$$\text{GSF: } \mathbf{f}_j(x) = (C_1 + C_2x + \dots + C_{r_j}x^{r_j-1})\mathbf{e}^{m_jx}.$$

- (3) $a_j \pm b_ji$ are complex roots, and $r_j = 1$:

$$\text{GSF: } \mathbf{f}_j(x) = \mathbf{e}^{a_jx}(C_1 \sin b_jx + C_2 \cos b_jx),$$

$$\text{or } \mathbf{f}_j(x) = C_1\mathbf{e}^{a_jx} \sin(b_jx + C_2),$$

$$\text{or } \mathbf{f}_j(x) = C_1\mathbf{e}^{a_jx} \cos(b_jx + C_2),$$

$$\text{or } \mathbf{f}_j(x) = \mathbf{e}^{a_jx}(C_1\mathbf{e}^{ib_jx} + C_2\mathbf{e}^{-ib_jx}).$$

- (4) $a_j \pm b_ji$ are complex roots, and $r_j > 1$.

$$\begin{aligned} \text{GSF: } \mathbf{f}_j(x) = & (C_1 + C_2x + \dots + C_{r_j}x^{r_j-1})\mathbf{e}^{a_jx} \sin b_jx \\ & + (C_{r_j+1} + C_{r_j+2}x + \dots + C_{2r_j}x^{r_j-1})\mathbf{e}^{a_jx} \cos b_jx. \end{aligned}$$

11.8 EXAMPLE

$[\mathbf{D}(\mathbf{D} - 2)^3(\mathbf{D} + 1)]\mathbf{f} = 0$. The roots (and their respective multiplicities) of $m(m - 2)^3(m + 1) = 0$ are

$$m_1 = 0, \quad r_1 = 1,$$

$$m_2 = -1, \quad r_2 = 1,$$

$$m_3 = 2, \quad r_3 = 3.$$

$$\text{GSF: } \mathbf{f}(x) = C_1 + C_2 \mathbf{e}^{-x} + (C_3 + C_4 x + C_5 x^2) \mathbf{e}^{2x}.$$

11.9 PARTICULAR SOLUTIONS OF LINEAR DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS

A particular solution of the equation $[\mathbf{p}(\mathbf{D})]\mathbf{f}(x) = \mathbf{g}(x)$ can often be found if one can guess the form such a solution is likely to take. Table 11.10 lists a number of aids to good guesses for particular solutions. The constants A and B (with or without subscripts) are real numbers to be determined by substituting the particular solution $\mathbf{f}^*(x)$ into the given equation. The examples of Sec. 11.12 illustrate how the table is used.

It will appear that it is necessary to check whether certain numbers are roots of the auxiliary equation $\mathbf{p}(m) = 0$. One should recall, however, that all roots of this equation were obtained in finding the general solution of the reduced equation. In particular, in each case, the number in question is a root if and only if $\mathbf{g}(x)$ is a solution of the reduced equation.

11.10 TABLE: PARTICULAR SOLUTIONS OF LINEAR DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS

- (1) $\mathbf{g}(x) = k$, k a real number:

PS: $\mathbf{f}^*(x) = A$ unless 0 is a root of $\mathbf{p}(m) = 0$;

$\mathbf{f}^*(x) = Ax^r$ if 0 is a root of $\mathbf{p}(m) = 0$ of multiplicity r .

- (2) $\mathbf{g}(x) = \mathbf{X}$, where \mathbf{X} is a polynomial in x of degree s :

PS: $\mathbf{f}^*(x) = A_0 + A_1x + \dots + A_sx^s$ unless 0 is a root of $\mathbf{p}(m) = 0$;

$\mathbf{f}^*(x) = (A_0 + A_1x + \dots + A_sx^s)x^r$ if 0 is a root of $\mathbf{p}(m) = 0$ of multiplicity r .

$$(3) \quad g(x) = p \sin qx:$$

$$\text{PS: } f^*(x) = A \sin qx + B \cos qx \text{ unless } iq \text{ is a root of } p(m) = 0;$$

$$f^*(x) = (A \sin qx + B \cos qx)x^r \text{ if } iq \text{ is a root of } p(m) = 0 \text{ of multiplicity } r.$$

$$(4) \quad g(x) = p \cos qx:$$

$$\text{PS: } f^*(x) = A \sin qx + B \cos qx \text{ unless } iq \text{ is a root of } p(m) = 0;$$

$$f^*(x) = (A \sin qx + B \cos qx)x^r \text{ if } iq \text{ is a root of } p(m) = 0 \text{ of multiplicity } r.$$

$$(5) \quad g(x) = p e^{qx}:$$

$$\text{PS: } f^*(x) = A e^{qx} \text{ unless } q \text{ is a root of } p(m) = 0;$$

$$f^*(x) = A x^r e^{qx} \text{ if } q \text{ is a root of } p(m) = 0 \text{ of multiplicity } r.$$

$$(6) \quad g(x) = k e^{px} \sin qx:$$

$$\text{PS: } f^*(x) = A e^{px} \sin qx + B e^{px} \cos qx \text{ unless } p + iq \text{ is a root of } p(m) = 0;$$

$$f^*(x) = (A e^{px} \sin qx + B e^{px} \cos qx)x^r \text{ if } p + iq \text{ is a root of } p(m) = 0 \text{ of multiplicity } r.$$

$$(7) \quad g(x) = k e^{px} \cos qx:$$

$$\text{PS: } f^*(x) = A e^{px} \sin qx + B e^{px} \cos qx \text{ unless } p + iq \text{ is a root of } p(m) = 0;$$

$$f^*(x) = (A e^{px} \sin qx + B e^{px} \cos qx)x^r \text{ if } p + iq \text{ is a root of } p(m) = 0 \text{ of multiplicity } r.$$

$$(8) \quad g(x) = X e^{qx}, \text{ where } X \text{ is a polynomial in } x \text{ of degree } s:$$

$$\text{PS: } f^*(x) = (A_0 + A_1 x + \dots + A_s x^s) e^{qx} \text{ unless } q \text{ is a root of } p(m) = 0;$$

$$f^*(x) = (A_0 + A_1 x + \dots + A_s x^s) x^r e^{qx} \text{ if } q \text{ is a root of } p(m) = 0 \text{ of multiplicity } r.$$

(9) $g(x) = \mathbf{X} \sin qx$, where \mathbf{X} is a polynomial in x of degree s :

$$\text{PS: } \mathbf{f}^*(x) = (A_0 + A_1x + \dots + A_sx^s) \sin qx \\ + (B_0 + B_1x + \dots + B_sx^s) \cos qx \\ \text{unless } iq \text{ is a root of } \mathbf{p}(m) = 0;$$

$$\mathbf{f}^*(x) = (A_0 + A_1x + \dots + A_sx^s)x^r \sin qx \\ + (B_0 + B_1x + \dots + B_sx^s)x^r \cos qx \\ \text{if } iq \text{ is a root of } \mathbf{p}(m) = 0 \text{ of multiplicity } r.$$

(10) $g(x) = \mathbf{X} \cos qx$, where \mathbf{X} is a polynomial in x of degree s :

$$\text{PS: } \mathbf{f}^*(x) = (A_0 + A_1x + \dots + A_sx^s) \sin qx \\ + (B_0 + B_1x + \dots + B_sx^s) \cos qx \\ \text{unless } iq \text{ is a root of } \mathbf{p}(m) = 0;$$

$$\mathbf{f}^*(x) = (A_0 + A_1x + \dots + A_sx^s)x^r \sin qx \\ + (B_0 + B_1x + \dots + B_sx^s)x^r \cos qx \\ \text{if } iq \text{ is a root of } \mathbf{p}(m) = 0 \text{ of multiplicity } r.$$

(11) $g(x) = \mathbf{X}e^{px} \sin qx$, where \mathbf{X} is a polynomial in x of degree s :

$$\text{PS: } \mathbf{f}^*(x) = (A_0 + A_1x + \dots + A_sx^s)e^{px} \sin qx \\ + (B_0 + B_1x + \dots + B_sx^s)e^{px} \cos qx \\ \text{unless } p + iq \text{ is a root of } \mathbf{p}(m) = 0;$$

$$\mathbf{f}^*(x) = (A_0 + A_1x + \dots + A_sx^s)x^r e^{px} \sin qx \\ + (B_0 + B_1x + \dots + B_sx^s)x^r e^{px} \cos qx \\ \text{if } p + iq \text{ is a root of } \mathbf{p}(m) = 0 \text{ of multiplicity } r.$$

(12) $g(x) = \mathbf{X}e^{px} \cos qx$, where \mathbf{X} is a polynomial in x of degree s :

$$\text{PS: } \mathbf{f}^*(x) = (A_0 + A_1x + \dots + A_sx^s)e^{px} \sin qx \\ + (B_0 + B_1x + \dots + B_sx^s)e^{px} \cos qx \\ \text{unless } p + iq \text{ is a root of } \mathbf{p}(m) = 0;$$

$$\mathbf{f}^*(x) = (A_0 + A_1x + \dots + A_sx^s)x^r e^{px} \sin qx \\ + (B_0 + B_1x + \dots + B_sx^s)x^r e^{px} \cos qx \\ \text{if } p + iq \text{ is a root of } \mathbf{p}(m) = 0 \text{ of multiplicity } r.$$

11.11 REMARK ON THE EXTENSION OF TABLE 11.10

Suppose that

$$g(x) = g_1(x) + g_2(x) + \dots + g_n(x),$$

where particular solutions for $[\mathbf{p}(\mathbf{D})]\mathbf{f}(x) = g_i(x)$ can be found

in Table 11.10 (or elsewhere). Then a particular solution for $[\mathbf{p}(\mathbf{D})]\mathbf{f}(x) = \mathbf{g}(x)$ can be obtained by adding the particular solutions for each of the equations $[\mathbf{p}(\mathbf{D})]\mathbf{f}(x) = \mathbf{g}_i(x)$.

11.12 EXAMPLES

(1) $[\mathbf{D}^2 + 2\mathbf{D} + 5]\mathbf{f}(x) = 5x^2 + 8x + 6$. Since $\mathbf{g}(x)$ is a polynomial of degree 2, and since 0 is not a root of $m^2 + 2m + 5 = 0$, formula (2) of Table 11.10 tells us that a particular solution of the given equation has the form $A_0 + A_1x + A_2x^2$. Substituting this in the given equation, we obtain

$$2A_2 + 2A_1 + 4A_2x + 5A_0 + 5A_1x + 5A_2x^2 = 5x^2 + 8x + 6.$$

Equating coefficients of like powers of x ,

$$5A_2 = 5,$$

$$4A_2 + 5A_1 = 8,$$

$$2A_2 + 2A_1 + 5A_0 = 6.$$

Solving, we get $A_2 = 1$, $A_1 = \frac{4}{5}$, $A_0 = \frac{1}{5}$. Adding our particular solution to the general solution form found in Example 1 of Sec. 11.5, we find that the general solution form for the given equation is

$$\mathbf{f}(x) = \mathbf{e}^{-x}(C_1 \sin 2x + C_2 \cos 2x) + x^2 + \frac{4}{5}x + \frac{1}{5}.$$

(2) $[\mathbf{D}^2 + 6\mathbf{D} + 9]\mathbf{f}(x) = \mathbf{e}^{-3x}$. In Example 3 of Sec. 11.5, we found that -3 is a double root of $m^2 + 6m + 9 = 0$. Thus, a particular solution, according to (5) of Table 11.10, has the form $\mathbf{f}^*(x) = Ax^2\mathbf{e}^{-3x}$. Substituting this for $\mathbf{f}(x)$ in the given equation, we get

$$A(9x^2 - 12x + 2)\mathbf{e}^{-3x} + 6A(-3x^2 + 2x)\mathbf{e}^{-3x} + 9Ax^2\mathbf{e}^{-3x} = \mathbf{e}^{-3x}.$$

The coefficients of $x^2\mathbf{e}^{-3x}$ and $x\mathbf{e}^{-3x}$ are both 0. Equating coefficients of \mathbf{e}^{-3x} , we get $A = \frac{1}{2}$. Using this particular solution and the general solution form found in Example 3 of Sec. 11.5 for the reduced equation, the general solution form for the given equation is

$$\mathbf{f}(x) = (C_1 + C_2x)\mathbf{e}^{-3x} + \frac{1}{2}x^2\mathbf{e}^{-3x}.$$

(3) $[\mathbf{D}^2 - 3\mathbf{D} - 4]\mathbf{f}(x) = 25 \sin 3x$. Since $3i$ is not a root of $m^2 - 3m - 4$, we use (3) of Table 11.10 and find that a particular

solution has the form $A \sin 3x + B \cos 3x$. Substituting this for $\mathbf{f}(x)$ in the given equation, we get

$$(-9A - 9B - 4A) \sin 3x + (-9B + 9A - 4B) \cos 3x = 25 \sin 3x.$$

Equating coefficients of $\sin 3x$ and $\cos 3x$, we get

$$9A - 13B = 0,$$

$$-13A - 9B = 25.$$

Solving these two equations together, we find $A = -1.3$, $B = -0.9$. Adding the general solution form of the reduced equation found in Example 2 of Sec. 11.5 to the particular solution found here, we get

$$\mathbf{f}(x) = C_1 e^{-x} + C_2 e^{4x} - 1.3 \sin 3x - 0.9 \cos 3x.$$

12

DIFFERENTIAL EQUATIONS OF FIRST AND SECOND ORDER

12.1 FUNCTIONS OF MORE THAN ONE ARGUMENT

A function of more than one argument is a mapping from sets of real numbers to real numbers. A value of a function of n arguments is determined by specifying n real numbers, one for each argument. Functions of more than one argument will be denoted by \mathbf{g} or a capital letter. Arguments, which can be values of functions, will be denoted by w, x, y, z, \dots , or by the symbols which denote functions. In particular, if \mathbf{f} is a function of a single argument x , we shall frequently consider a function $\mathbf{g}(x, \mathbf{f})$ of the two arguments x and $\mathbf{f}(x)$.

12.2 PARTIAL DERIVATIVES

Let \mathbf{F} be a function of n arguments x_1, \dots, x_n . From this function, we can form a new function of any one of its arguments (say, x_i) by keeping all the other arguments fixed. If this function has a derivative, its derivative is called the **partial derivative** of \mathbf{F} with respect to x_i , written $\partial\mathbf{F}/\partial x_i$. In general, $\partial\mathbf{F}/\partial x_i$ can be regarded as a function of the same n arguments as \mathbf{F} , since the values of $\partial\mathbf{F}/\partial x_i$ are determined not only by x_i but also by the other arguments.

12.3 PARTIAL INTEGRATION

If \mathbf{G} is a function for which $\partial \mathbf{G} / \partial x_i = \mathbf{F}$, then \mathbf{G} is called a **partial integral** of \mathbf{F} with respect to x_i , written

$$\mathbf{G}(x_1, \dots, x_n) = \int \mathbf{F}(x_1, \dots, x_n) dx_i.$$

12.4 EXAMPLES

(1) If $\mathbf{F}(x, y) = x \sin y - y \cos x$, then

$$\frac{\partial \mathbf{F}}{\partial x} = \sin y + y \sin x \quad \text{and} \quad \frac{\partial \mathbf{F}}{\partial y} = x \cos y - \cos x.$$

(2) If $\mathbf{g}(x, \mathbf{f}) = x^2 \mathbf{f} - 5 \mathbf{f}^3 + 3x$, then

$$\frac{\partial \mathbf{g}}{\partial x} = 2x\mathbf{f} + 3 \quad \text{and} \quad \frac{\partial \mathbf{g}}{\partial \mathbf{f}} = x^2 - 15\mathbf{f}^2.$$

12.5 LINEAR DEPENDENCE AND INDEPENDENCE OF FUNCTIONS

Let $\mathbf{f}_1, \mathbf{f}_2, \dots, \mathbf{f}_n$ be a set of n functions. If there exists a set of n constants, k_1, k_2, \dots, k_n , not all zero such that the sum

$$k_1 \mathbf{f}_1 + k_2 \mathbf{f}_2 + \dots + k_n \mathbf{f}_n = 0,$$

then the functions of the set are said to be **linearly dependent**. On the other hand, if the equation

$$k_1 \mathbf{f}_1 + k_2 \mathbf{f}_2 + \dots + k_n \mathbf{f}_n = 0$$

implies that $k_1 = k_2 = \dots = k_n = 0$, then the functions of the set are said to be **linearly independent**.

Any set of functions from among the following is a linearly independent set:

A single nonzero constant function;

x, x^2, x^3, \dots ;

$e^x, e^{2x}, e^{3x}, \dots$;

$e^{-x}, e^{-2x}, e^{-3x}, \dots$;

$\sin x, \sin 2x, \sin 3x, \dots$;

$\cos x, \cos 2x, \cos 3x, \dots$;

$xe^{kx}, x^2e^{kx}, x^3e^{kx}, \dots$;

$x \sin kx, x^2 \sin kx, x^3 \sin kx, \dots$;

$x \cos kx, x^2 \cos kx, x^3 \cos kx, \dots$, k any real number, $k \neq 0$.

The functions $\cos 2x$, $\sin^2 x$, and 4 form an example of a linearly dependent set of functions, since

$$4 \cos 2x - 8 \sin^2 x + 1 \cdot 4 = 0.$$

12.6 USE OF THE INTEGRAL TABLE

In the tables which follow, the procedures which lead to general solution forms of certain types of differential equations require the evaluation of one or more integrals. These integrals can often be found in Table 10.4. The constant C of integration listed with the formula in that table should be omitted when the integral is used here, since the formulas given below supply arbitrary constants where they are needed.

12.7 LINEAR DIFFERENTIAL EQUATIONS OF FIRST ORDER

A differential equation of first order expresses a condition to be satisfied by a function \mathbf{f} and its first derivative \mathbf{Df} . If such an equation is linear, then it can be written in the form

$$(1) \quad \mathbf{Df} = \mathbf{g}(x, \mathbf{f}),$$

where \mathbf{g} is a function of at most two arguments x and \mathbf{f} . Conversely, if a differential equation of first order cannot be written in the form (1) (that is, solved explicitly for \mathbf{Df}) then it is a nonlinear equation.

If an equation can be written in the form (1), the technique of solving it, if one exists, depends on the character of the function $\mathbf{g}(x, \mathbf{f})$. The procedure for using Table 12.9 to solve a linear differential equation of first order is:

(1) Solve the equation for \mathbf{Df} ; that is, write the equation in the form $\mathbf{Df} = \mathbf{g}(x, \mathbf{f})$.

(2) Locate the form of the function $\mathbf{g}(x, \mathbf{f})$ in the classification list in Sec. 12.8. This refers to an entry in Table 12.9.

(3) Follow the procedure outlined in the appropriate entry of Table 12.9 to obtain the general solution form of the equation.

12.8 CLASSIFICATION LIST FOR EQUATIONS OF THE FORM

$$Df = g(x, f)$$

(1) $g(x, f)$ is a function of x alone (does not involve f). Write $g(x)$ for $g(x, f)$. See formula (1) of Table 12.9.

(2) $g(x, f)$ is a function of f alone (does not involve x). Write $g(f)$ for $g(x, f)$. See formula (2) of Table 12.9.

(3) $g(x, f)$ is the product of a function of x alone and a function of f alone (separable equation). Write $h(x)j(f)$ for $g(x, f)$. See formula (3) of Table 12.9.

(4) $g(x, f) = u(x) + fv(x)$, $u(x)$, and $v(x)$ are functions of x alone (equation linear in f). See formula (4) of Table 12.9.

(5) $g(x, f) = f^n u(x) + fv(x)$, $u(x)$, and $v(x)$ are functions of x alone (Bernoulli equation). See formula (5) of Table 12.9.

(6) $g(x, f) = \frac{p(x, f)}{q(x, f)}$, where $p(tx, tf) = t^n p(x, f)$ and $q(tx, tf) = t^n q(x, f)$ for some number n (homogeneous equation). See formula (6) of Table 12.9.

[Note: If $p(x, f)$ and $q(x, f)$ are polynomials in which the sums of the powers of x and f in each term of both are equal, the equation is homogeneous.]

$$(7) \quad g(x, f) = \frac{a_1 x + b_1 f + c_1}{a_2 x + b_2 f + c_2} \quad (\text{linear fractional form}). \text{ See formula (7) of Table 12.9.}$$

(8) $g(x, f) = \frac{M(x, f)}{N(x, f)}$, $\frac{\partial M}{\partial f} + \frac{\partial N}{\partial x} = 0$ (equation in exact form). See formula (8) of Table 12.9.

$$(9) \quad g(x, f) = \frac{P(x, f)}{Q(x, f)}, \quad \frac{\partial P}{\partial f} + \frac{\partial Q}{\partial x} \neq 0.$$

See formula (9) of Table 12.9 for methods of finding a function

$R(x, f)$, called an **integrating factor**, so that the equation

$$Df = \frac{P(x, f)R(x, f)}{Q(x, f)R(x, f)}$$

is exact. Then apply (8) of Table 12.9. In general, the problem of finding an integrating factor is just as difficult as solving the differential equation itself. Though the entries of formula (9) will yield an integrating factor in some cases, it is more efficient to try to find one by inspection before consulting the table.

12.9 TABLE: GENERAL SOLUTION FORMS FOR LINEAR DIFFERENTIAL EQUATIONS OF FIRST ORDER

(1) Equation: $Df = g(x)$.

GSF: $f(x) = \int g(x) dx + C$.

(2) Equation: $Df = g(f)$.

GSF: $\int \frac{df}{g(f)} = x + C$.

(3) Equation: $Df = h(x)j(f)$.

GSF: $\int \frac{df}{j(f)} = \int h(x) dx + C$.

(4) Equation: $Df = u(x) + fv(x)$.

Let $w(x) = -\int v(x) dx$.

GSF: $f(x) = e^{-w(x)} [\int u(x)e^{w(x)} dx + C]$.

(5) Equation $Df = f^n u(x) + fv(x)$.

Let $w(x) = (n-1) \int v(x) dx$.

GSF: $f(x) = \{e^{-w(x)} [(1-n) \int u(x)e^{w(x)} dx + C]\}^{1/(1-n)}$.

(6) Equation:

$$Df = \frac{p(x, f)}{q(x, f)} \quad (\text{homogeneous}).$$

Let

$$F(v) = \int \frac{q(1, v) dv}{p(1, v) + vq(1, v)}.$$

GSF: $\mathbf{f}(x)$ is defined implicitly by

$$\mathbf{F}\left(\frac{y}{x}\right) + \ln x = C,$$

where $y = \mathbf{f}(x)$.

(7) Equation:

$$\mathbf{D}\mathbf{f} = \frac{a_1x + b_1\mathbf{f} + c_1}{a_2x + b_2\mathbf{f} + c_2}.$$

Case 1: $a_1/a_2 = b_1/b_2$.

Let $k = a_1/a_2$, $m = b_1 + a_1k$, and $n = b_1c_1 + a_1c_2$.

GSF: $\mathbf{f}(x)$ is defined implicitly by

$$km(a_1x + b_1y) + (mc_2 - kn) \ln |m(a_1x + b_1y) + n| = m^2x + C,$$

where $y = \mathbf{f}(x)$.

Case 2: $a_1/a_2 \neq b_1/b_2$.

Let m and n be simultaneous solutions of the linear equations

$$a_1m + b_1n + c_1 = 0,$$

$$a_2m + b_2n + c_2 = 0.$$

Let

$$\mathbf{F}(v) = \int \frac{(a_2 + b_2v) dv}{a_1 + (a_2 + b_1)v + b_2v^2}.$$

GSF: $\mathbf{f}(x)$ is defined implicitly by

$$\mathbf{F}\left(\frac{y-n}{x-m}\right) + \ln |x-m| = C, \quad \text{where } y = \mathbf{f}(x).$$

(8) Equation:

$$\mathbf{D}\mathbf{f} = \frac{\mathbf{M}(x, \mathbf{f})}{\mathbf{N}(x, \mathbf{f})} \quad (\text{exact}).$$

Let

$$\mathbf{F}(x, \mathbf{f}) = \frac{\partial}{\partial \mathbf{f}} \int \mathbf{M}(x, \mathbf{f}) d\mathbf{f},$$

$$\mathbf{G}(x, \mathbf{f}) = \frac{\partial}{\partial x} \int \mathbf{N}(x, \mathbf{f}) d\mathbf{f}.$$

GSF:

$$(1) \int \mathbf{M}(x, \mathbf{f}) d\mathbf{f} - \int [\mathbf{N}(x, \mathbf{f}) + \mathbf{F}(x, \mathbf{f})] d\mathbf{f} = C.$$

$$(2) \int \mathbf{N}(x, \mathbf{f}) d\mathbf{f} - \int [\mathbf{M}(x, \mathbf{f}) + \mathbf{G}(x, \mathbf{f})] dx = C.$$

(Use either form.)

(9) Equation of the form

$$Df = \frac{P(x, f)}{Q(x, f)}, \quad \frac{\partial P}{\partial f} + \frac{\partial Q}{\partial x} \neq 0.$$

Integrating Factors

$$(9a) \quad \frac{\partial P}{\partial f} + \frac{\partial Q}{\partial x} = kQ, \quad k \text{ a real number.}$$

$$\text{IF: } R(x, f) = e^{-kx}.$$

$$(9b) \quad \frac{\partial P}{\partial f} + \frac{\partial Q}{\partial x} = -\frac{kQ}{x}, \quad k \text{ a real number.}$$

$$\text{IF: } R(x, f) = x^k.$$

$$(9c) \quad \frac{\partial P}{\partial f} + \frac{\partial Q}{\partial x} = -\frac{kQ}{f}, \quad k \text{ a real number.}$$

$$\text{IF: } R(x, f) = f^k.$$

$$(9d) \quad \frac{\partial P}{\partial f} + \frac{\partial Q}{\partial x} = -aQ \cot ax.$$

$$\text{IF: } R(x, f) = \sin ax.$$

$$(9e) \quad \frac{\partial P}{\partial f} + \frac{\partial Q}{\partial x} = aQ \tan ax.$$

$$\text{IF: } R(x, f) = \cos ax.$$

$$(9f) \quad \frac{\frac{\partial P}{\partial f} + \frac{\partial Q}{\partial x}}{Q} = h(x),$$

where h is a function of the single argument x .

$$\text{IF: } R(x, f) = e^{-\int h(x) dx}.$$

$$(9g) \quad \frac{\frac{\partial P}{\partial f} + \frac{\partial Q}{\partial x}}{P} = h(f),$$

where h is a function of the single argument f .

$$\text{IF: } R(x, f) = e^{-\int h(f) df}.$$

$$(9h) \quad \frac{\frac{\partial P}{\partial f} + \frac{\partial Q}{\partial x}}{P + Q} = h(x + f),$$

where \mathbf{h} is a function of the single argument $x + \mathbf{f}$.

$$\text{IF: } \mathbf{R}(x, \mathbf{f}) = \mathbf{e}^{-\int \mathbf{h}(u) du},$$

where $u = x + \mathbf{f}$.

$$(9i) \quad \frac{\frac{\partial \mathbf{P}}{\partial \mathbf{f}} + \frac{\partial \mathbf{Q}}{\partial x}}{\mathbf{Q} - \mathbf{P}} = \mathbf{h}(x - \mathbf{f}),$$

where \mathbf{h} is a function of the single argument $x - \mathbf{f}$.

$$\text{IF: } \mathbf{R}(x, \mathbf{f}) = \mathbf{e}^{-\int \mathbf{h}(u) du},$$

where $u = x - \mathbf{f}$.

$$(9j) \quad \frac{\frac{\partial \mathbf{P}}{\partial \mathbf{f}} + \frac{\partial \mathbf{Q}}{\partial x}}{\mathbf{P} + k\mathbf{Q}} = \mathbf{h}(kx + \mathbf{f}),$$

where \mathbf{h} is a function of the single argument $kx + \mathbf{f}$.

$$\text{IF: } \mathbf{R}(x, \mathbf{f}) = \mathbf{e}^{-\int \mathbf{h}(u) du},$$

where $u = kx + \mathbf{f}$.

$$(9k) \quad \frac{\frac{\partial \mathbf{P}}{\partial \mathbf{f}} + \frac{\partial \mathbf{Q}}{\partial x}}{x\mathbf{P} + \mathbf{f}\mathbf{Q}} = \mathbf{h}(x\mathbf{f}),$$

where \mathbf{h} is a function of the single argument $x\mathbf{f}$.

$$\text{IF: } \mathbf{R}(x, \mathbf{f}) = \mathbf{e}^{-\int \mathbf{h}(u) du},$$

where $u = x\mathbf{f}$.

$$(9l) \quad \frac{x^2 \frac{\partial \mathbf{P}}{\partial \mathbf{f}} + \frac{\partial \mathbf{Q}}{\partial x}}{\mathbf{P}x - \mathbf{Q}\mathbf{f}} = \mathbf{h}\left(\frac{\mathbf{f}}{x}\right),$$

where \mathbf{h} is a function of the single argument \mathbf{f}/x .

$$\text{IF: } \mathbf{R}(x, \mathbf{f}) = \mathbf{e}^{-\int \mathbf{h}(u) du},$$

where $u = \mathbf{f}/x$.

$$(9m) \quad \mathbf{P}(x, \mathbf{f}) = \mathbf{f}h_1(x\mathbf{f}), \quad \text{and} \quad \mathbf{Q}(x, \mathbf{f}) = xh_2(x\mathbf{f}),$$

where h_1 and h_2 are functions of the single argument $x\mathbf{f}$.

$$\text{IF: } \mathbf{R}(x, \mathbf{f}) = \frac{1}{x\mathbf{f}[h_1(x\mathbf{f}) + h_2(x\mathbf{f})]}.$$

$$(9n) \quad \mathbf{P}(x, \mathbf{f}) = ax^p\mathbf{f} + b\mathbf{f}^{q+1} = (ax^p + b\mathbf{f}^q)\mathbf{f},$$

$$\mathbf{Q}(x, \mathbf{f}) = cx^{p+1} + d\mathbf{f}^q = (cx^p + d\mathbf{f}^q)x,$$

where $ad \neq bc$.

Let

$$m = -1 + \frac{c(pb - qa)}{ad - bc}, \quad n = -1 + \frac{b(qc - pd)}{ad - bc}.$$

$$\text{IF: } \mathbf{R}(x, \mathbf{f}) = x^m\mathbf{f}^n.$$

12.10 EXAMPLES

$$(1) \quad \frac{D\mathbf{f}}{\mathbf{f}} = \frac{x - 1}{x^2(1 - \mathbf{f})}.$$

Solve for $D\mathbf{f}$, obtaining

$$\begin{aligned} D\mathbf{f} &= \mathbf{g}(x, \mathbf{f}) = \frac{(x - 1)\mathbf{f}}{x^2(1 - \mathbf{f})} \\ &= \mathbf{h}(x)\mathbf{j}(\mathbf{f}) = \frac{x - 1}{x^2} \frac{\mathbf{f}}{1 - \mathbf{f}}, \end{aligned}$$

which falls under (3) of Sec. 12.8. Hence, use (3) of Table 12.9. The general solution form is

$$\begin{aligned} \int \frac{1 - \mathbf{f}}{\mathbf{f}} d\mathbf{f} &= \int \frac{x - 1}{x^2} dx + C, \\ \ln |\mathbf{f}| - \mathbf{f} &= \ln |x| + \frac{1}{x} + C. \end{aligned}$$

$$(2) \quad 3xD\mathbf{f} = 3x\mathbf{f}^4 \ln x + \mathbf{f}.$$

Solve for $D\mathbf{f}$, obtaining

$$D\mathbf{f} = \mathbf{g}(x, \mathbf{f}) = \frac{3x\mathbf{f}^4 \ln x + \mathbf{f}}{3x} = \mathbf{f}^4 \ln x + \frac{\mathbf{f}}{3x},$$

which falls under (5) of Sec. 12.8. Hence, use (5) of Table 12.9. Let

$$\mathbf{u}(x) = \ln x, \quad \mathbf{v}(x) = \frac{1}{3x}, \quad n = 4.$$

Let

$$\mathbf{w}(x) = -3 \int \frac{1}{3x} dx = -\int \frac{dx}{x} = -\ln |x|;$$

then,

$$\mathbf{e}^{\mathbf{w}(x)} = \frac{1}{x}.$$

The general solution form is

$$\begin{aligned} \mathbf{f}(x) &= \left[\frac{1}{x} - 3 \left(\int x \ln x dx + C \right) \right]^{-1/3} \\ &= \left[\frac{1}{x} - 3 \left(\frac{x^2}{2} \ln |x| - \frac{x^2}{4} + C \right) \right]^{-1/3}, \\ \mathbf{f}(x) &= \left(\frac{3}{4}x - \frac{3}{2}x \ln |x| + \frac{C}{x} \right)^{-1/3}. \end{aligned}$$

$$(3) \quad \mathbf{Df} = \mathbf{g}(x, \mathbf{f}) = \frac{x\mathbf{f} - \mathbf{f}^2}{x^2 - 2x\mathbf{f}}.$$

This equation falls under (6) of Sec. 12.8 with $n = 2$. Hence, use (6) of Table 12.9 with $\mathbf{p}(x, \mathbf{f}) = x\mathbf{f} - \mathbf{f}^2$ and $\mathbf{q}(x, \mathbf{f}) = x^2 - 2x\mathbf{f}$. Evaluate

$$\mathbf{F}(v) = \int \frac{(1 - 2v) dv}{v - v^2 + v(1 - 2v)} = \int \frac{1 - 2v}{v(2 - 3v)} dv.$$

This integral is evaluated by expressing the integrand as a sum of simpler fractions, using the technique of partial fractions.

$$\frac{1 - 2v}{v(2 - 3v)} = \frac{A}{v} + \frac{B}{2 - 3v}.$$

$$1 - 2v = 2A - 3Av + Bv.$$

$$A = \frac{1}{2}, \quad B = -\frac{1}{2}.$$

Thus,

$$\begin{aligned} \mathbf{F}(v) &= \int \left(\frac{1}{2v} - \frac{1}{2(2 - 3v)} \right) dv \\ &= \frac{1}{2} \ln |v| + \frac{1}{6} \ln |2 - 3v|. \end{aligned}$$

The solution is given implicitly by the equation

$$\frac{1}{2} \ln \left| \frac{y}{x} \right| + \frac{1}{6} \ln \left| 2 - 3 \frac{y}{x} \right| + \ln x = C,$$

where $y = f(x)$.

$$(4) \quad Df = g(x, f) = \frac{\cos x - 2xf}{x^2 - 1}.$$

Let $M(x, f) = \cos x - 2xf$ and $N(x, f) = x^2 - 1$. Note that

$$\frac{\partial M}{\partial f} + \frac{\partial N}{\partial x} = -2x + 2x = 0.$$

Thus, the equation falls under (8) of Sec. 12.8. Its solution is found in (8) of Table 12.9. Since

$$\int M(x, f) dx = \sin x - fx^2,$$

$$F(x, f) = \frac{\partial}{\partial f} (\sin x - fx^2) = -x^2.$$

The solution can be written

$$\begin{aligned} \int M(x, f) dx - \int [N(x, f) + F(x, f)] df &= \sin x - fx^2 \\ &\quad - \int (x^2 - 1 - x^2) df \\ &= \sin x - fx^2 + f = C. \end{aligned}$$

Solving for f ,

$$f(x) = \frac{\sin x - C}{x^2 - 1} = \frac{\sin x + C_1}{x^2 - 1}.$$

$$(5) \quad 2xf^3 Df + x^3 f^2 Df = x^3 + f^4.$$

Solving for Df , we obtain

$$Df = g(x, f) = \frac{x^3 + f^4}{2xf^3 + x^3 f^2}.$$

This differential equation satisfies none of the first eight conditions. Therefore, we look at (9) of Table 12.9 and seek an integrating factor. Letting $P(x, f) = x^3 + f^4$ and $Q(x, f) = 2xf^3 + x^3 f^2$, we find that condition (9f) is satisfied; for

$$\begin{aligned} \frac{\frac{\partial P}{\partial f} + \frac{\partial Q}{\partial x}}{Q} &= \frac{4f^3 + 2f^3 + 3x^2 f^2}{2xf^3 + x^3 f^2} \\ &= \frac{3f^2(2f + x^2)}{xf^2(2f + x^2)} = \frac{3}{x} \end{aligned}$$

is a function of x alone. Thus, $e^{-\int (3/x) dx} = x^{-3}$ is an integrating factor. Letting $\mathbf{M}(x, \mathbf{f}) = x^{-3}\mathbf{P}(x, \mathbf{f})$ and $\mathbf{N}(x, \mathbf{f}) = x^{-3}\mathbf{Q}(x, \mathbf{f})$, we have

$$\mathbf{Df} = \frac{\mathbf{M}(x, \mathbf{f})}{\mathbf{N}(x, \mathbf{f})} = \frac{1 + \mathbf{f}^4 x^{-3}}{2\mathbf{f}^3 x^{-2} + \mathbf{f}^2},$$

which is exact. We solve for \mathbf{f} by means of (10) of Table 12.9.

$$\int \mathbf{N}(x, \mathbf{f}) d\mathbf{f} = \int (2x^{-2}\mathbf{f}^3 + \mathbf{f}^2) d\mathbf{f} = \frac{x^{-2}\mathbf{f}^4}{2} + \frac{\mathbf{f}^3}{3}.$$

Then,
$$\mathbf{G}(x, \mathbf{f}) = \frac{\partial}{\partial x} \left(\frac{x^{-2}\mathbf{f}^4}{2} + \frac{\mathbf{f}^3}{3} \right) = -\mathbf{f}^4 x^{-3}.$$

The solution may now be written in the implicit form

$$\int \mathbf{N} d\mathbf{f} - \int (\mathbf{M} + \mathbf{G}) dx = C.$$

$$\frac{-x^{-2}\mathbf{f}^4}{2} + \frac{\mathbf{f}^3}{3} - \int (1 + \mathbf{f}^4 x^{-3} - \mathbf{f}^4 x^{-3}) dx = \frac{-\mathbf{f}^4}{2x^2} + \frac{\mathbf{f}^3}{3} - x = C,$$

or
$$3\mathbf{f}^4 + 6x^3 + 6Cx^2 - 2x^2\mathbf{f}^3 = 0.$$

12.11 LINEAR DIFFERENTIAL EQUATIONS OF SECOND ORDER

Linear differential equations of second order are written in the form

$$(1) \quad [\mathbf{u}\mathbf{D}^2 + \mathbf{v}\mathbf{D} + \mathbf{w}]\mathbf{f} = \mathbf{t},$$

where \mathbf{u} , \mathbf{v} , \mathbf{w} , and \mathbf{t} are given functions of x alone. The quadratic operator polynomials have coefficients which are functions of x . Such operators are handled in a way analogous to that in which those with constant coefficients are used.

The general solution of an equation of type (1) depends on the functions \mathbf{u} , \mathbf{v} , \mathbf{w} , and \mathbf{t} .

The entries of Table 12.12 are techniques of solving equations in this form for various classes of given functions. An equation must first be written in the above form. Next, it must be classified according to the types considered in Table 12.12. Finally, the procedure which leads to a general solution form as outlined in the appropriate entry of Table 12.12 should be followed.

Table 12.12 is far from exhaustive. Only the more elementary types of equations are considered here.

12.12 TABLE: GENERAL SOLUTION FORMS FOR LINEAR DIFFERENTIAL EQUATIONS OF SECOND ORDER

(1) Equation: $[D^2 + vD + w]f = 0$, v and w constant functions.

GSF: See Table 11.4.

(2) Equation: $[D^2 + vD + w]f = 0$, v is a polynomial of degree m , and w is a polynomial of degree n .

Let $r =$ the greater of m and n . Use the method of undetermined coefficients to determine constants $A_0, A_1, \dots, A_{r-1}, A_r, B_1$, and B_2 , so that

$$u_1 = A_0 x^r + A_1 x^{r-1} + \dots + A_{r-1} + A_r$$

and

$$u_2 = B_1 e^{B_2 x}$$

are particular solutions of the equation. This is not always possible. If it can be done, then u_1 and u_2 are linearly independent.

$$\text{GSF: } f(x) = C_1 u_1(x) + C_2 u_2(x).$$

(3) Equation: $[uD^2 + vD + w]f = 0$.

Attempt to express $uD^2 + vD + w$ as $[pD + q][rD + s]$, with the factors in that order, and p, q, r , and s functions of x alone. These functions must satisfy the equations

$$pr = u,$$

$$qr + ps + p \cdot Dr = v,$$

$$qs + p \cdot Ds = w.$$

This cannot always be done. If it can, let

$$h(x) = - \int \frac{q(x)}{p(x)} dx.$$

GSF: Use (4) of Table 12.9 to solve for $f(x)$ the equation

$$Df = C_1 \frac{e^{h(x)}}{r(x)} - \frac{s(x)}{r(x)} f,$$

where C_1 is an arbitrary constant. Call the new arbitrary constant C_2 .

(4) Equation: $[\mathbf{D}^2 + \mathbf{vD} + \mathbf{w}]\mathbf{f} = \mathbf{t}$, where \mathbf{v} , \mathbf{w} , and \mathbf{t} are functions of x alone.

Use (1) or (2) above, or good guessing to find a nonzero particular solution $\mathbf{u}(x)$ of the reduced equation $[\mathbf{D}^2 + \mathbf{vD} + \mathbf{w}]\mathbf{f} = 0$. Let $\mathbf{H}(x) = 2 \mathbf{D}\mathbf{u}(x) + \mathbf{v}(x)\mathbf{u}(x)$. Use (4) of Table 12.9 to solve for $\mathbf{g}(x)$ the equation

$$\mathbf{D}\mathbf{g} = \frac{\mathbf{t}(x)}{\mathbf{u}(x)} - \frac{\mathbf{H}(x)}{\mathbf{u}(x)} \mathbf{g},$$

calling the arbitrary constant C_1 .

$$\text{GSF: } \mathbf{f}(x) = \mathbf{u}(x) \left[\int \mathbf{g}(x) dx + C_2 \right].$$

(5) Equation: $[\mathbf{D}^2 + \mathbf{vD} + \mathbf{w}]\mathbf{f} = \mathbf{t}$, where \mathbf{v} , \mathbf{w} , and \mathbf{t} are functions of x alone (method of variation of parameters).

Use (1) or (2) above or good guessing to find two linearly independent solutions $\mathbf{u}_1(x)$ and $\mathbf{u}_2(x)$ of $[\mathbf{D}^2 + \mathbf{vD} + \mathbf{w}]\mathbf{f} = 0$.

$$\text{Let } \mathbf{W}(x) = \mathbf{u}_1(x) \cdot \mathbf{D}\mathbf{u}_2(x) - \mathbf{u}_2(x) \cdot \mathbf{D}\mathbf{u}_1(x).$$

$$\begin{aligned} \text{GSF: } \mathbf{f}(x) = \mathbf{u}_2(x) \left[\int \frac{\mathbf{t}(x)\mathbf{u}_1(x)}{\mathbf{W}(x)} dx + C_1 \right] \\ - \mathbf{u}_1(x) \left[\int \frac{\mathbf{t}(x)\mathbf{u}_2(x)}{\mathbf{W}(x)} dx + C_2 \right]. \end{aligned}$$

(6) Equation: $[\mathbf{uD}^2 + \mathbf{vD} + \mathbf{w}]\mathbf{f} = \mathbf{t}$, where \mathbf{u} , \mathbf{v} , \mathbf{w} , and \mathbf{t} are functions of x alone, with $\mathbf{D}^2\mathbf{u} - \mathbf{Dv} - \mathbf{w} = 0$ (exact equation).

$$\text{Let } \mathbf{H}(x) = \mathbf{v}(x) - \mathbf{D}\mathbf{u}(x) \text{ and } \mathbf{J}(x) = \int \mathbf{t}(x) dx + C_1.$$

GSF: Solve by (4) of Table 12.9 for $\mathbf{f}(x)$ the equation

$$\mathbf{D}\mathbf{f} = \mathbf{J}(x) + \mathbf{H}(x)\mathbf{f},$$

calling the new arbitrary constant C_2 .

(7) Equation: $[\mathbf{D}^2 + \mathbf{vD}]\mathbf{f} = \mathbf{t}$, where \mathbf{v} and \mathbf{t} are functions of x alone.

Use (4) of Table 12.9 to solve for $\mathbf{g}(x)$ the equation

$$\mathbf{D}\mathbf{g} = \mathbf{t}(x) - \mathbf{v}(x)\mathbf{g},$$

calling the arbitrary constant C_1 .

$$\text{GSF: } \mathbf{f}(x) = \int \mathbf{g}(x) dx + C_2.$$

12.13 NONLINEAR DIFFERENTIAL EQUATIONS OF FIRST ORDER

The general form of the nonlinear equation of first degree is

$$\mathbf{F}(x, \mathbf{f}, \mathbf{Df}) = 0$$

where \mathbf{F} is a function of three arguments. In general use of the techniques of Table 12.14, the argument character of \mathbf{Df} is emphasized by replacing \mathbf{Df} wherever it occurs by the single letter \mathbf{p} . The technique of solving these equations depends on the form of the function \mathbf{F} . The different forms of \mathbf{F} considered in Table 12.16 are:

- (1) $\mathbf{F}(x, \mathbf{f}, \mathbf{p})$ is a polynomial of degree n in \mathbf{p} . Write

$$[\mathbf{p} - \mathbf{G}_1(x, \mathbf{f})][\mathbf{p} - \mathbf{G}_2(x, \mathbf{f})] \dots [\mathbf{p} - \mathbf{G}_n(x, \mathbf{f})]$$

for $\mathbf{F}(x, \mathbf{f}, \mathbf{p})$ [see (1) of Table 12.14].

- (2) $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ may be solved for \mathbf{f} . Write $\mathbf{f} = \mathbf{G}(x, \mathbf{p})$ for $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ [see (2) of Table 12.14].

- (3) $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ may be solved for \mathbf{f} and written $\mathbf{f} = \mathbf{G}(x, \mathbf{p})$ and $\mathbf{G}(x, \mathbf{p}) = \mathbf{p}x + \mathbf{H}(\mathbf{p})$ (Clairaut's equation). Write $\mathbf{f} = \mathbf{p}x + \mathbf{H}(\mathbf{p})$ for $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ [see (3) of Table 12.14].

- (4) $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ may be solved for \mathbf{f} and written $\mathbf{f} = \mathbf{G}(x, \mathbf{p})$ and $\mathbf{G}(x, \mathbf{p}) = x\mathbf{H}(\mathbf{p}) + \mathbf{J}(\mathbf{p})$ (D'Alembert's equation). Write $\mathbf{f} = x\mathbf{H}(\mathbf{p}) + \mathbf{J}(\mathbf{p})$ for $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ [see (4) of Table 12.14].

- (5) $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ may be solved for x . Write $x = \mathbf{G}(\mathbf{f}, \mathbf{p})$ for $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ [see (5) of Table 12.14].

- (6) $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ contains no x and may be solved for \mathbf{p} . Write $\mathbf{p} = \mathbf{G}(\mathbf{f})$ for $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ [see (6) of Table 12.14].

- (7) $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ contains no \mathbf{f} and can be solved for \mathbf{p} . Write $\mathbf{p} = \mathbf{G}(x)$ for $\mathbf{F}(x, \mathbf{f}, \mathbf{p}) = 0$ [see (7) of Table 12.14].

For some equations of this type, the general solution form fails to represent all the particular solutions. A particular solution not included in the general solution form is called a **singular solution** (abbreviated SS). Singular solutions are tabulated for those equations in Table 12.14 for which the general solution form given does not include all particular solutions.

12.14 TABLE: GENERAL SOLUTION FORMS OF NONLINEAR DIFFERENTIAL EQUATIONS OF FIRST ORDER

(1) Equation:

$$[\mathbf{p} - \mathbf{G}_1(x, \mathbf{f})][\mathbf{p} - \mathbf{G}_2(x, \mathbf{f})] \dots [\mathbf{p} - \mathbf{G}_n(x, \mathbf{f})] = 0.$$

Use Table 12.9 to solve for \mathbf{f} each of the equations

$$\mathbf{D}\mathbf{f} = \mathbf{G}_i(x, \mathbf{f}), \quad 1 \leq i \leq n,$$

and write each solution in the form $\mathbf{H}_i(x, \mathbf{f}, C) = 0$, using the same arbitrary constant in each solution. This defines \mathbf{f} implicitly.

$$\text{GSF: } \mathbf{H}_1(x, y, C)\mathbf{H}_2(x, y, C) \dots \mathbf{H}_n(x, y, C) = 0,$$

where $y = \mathbf{f}(x)$.

(2) Equation: $\mathbf{f} = \mathbf{G}(x, \mathbf{p})$.

Let $\mathbf{M}(x, \mathbf{p}) = (\partial/\partial x)\mathbf{G}(x, \mathbf{p})$ and $\mathbf{N}(x, \mathbf{p}) = (\partial/\partial \mathbf{p})\mathbf{G}(x, \mathbf{p})$. Use Table 12.9 to solve for \mathbf{p} the equation

$$\mathbf{D}\mathbf{p} = \frac{\mathbf{p} - \mathbf{M}(x, \mathbf{p})}{\mathbf{N}(x, \mathbf{p})},$$

writing the solution in the form $\mathbf{H}(x, \mathbf{p}, C) = 0$.

GSF: Eliminate \mathbf{p} between the equations

$$\mathbf{f} = \mathbf{G}(x, \mathbf{p}),$$

$$\mathbf{H}(x, \mathbf{p}, C) = 0.$$

(3) Equation: $\mathbf{f} = \mathbf{p}x + \mathbf{H}(\mathbf{p})$.

$$\text{GSF: } \mathbf{f}(x) = Cx + \mathbf{H}(C).$$

SS: Eliminate C between the equations

$$\mathbf{f}(x) = Cx + \mathbf{H}(C),$$

$$x + \mathbf{D}\mathbf{H}(C) = 0.$$

(4) Equation: $\mathbf{f} = x\mathbf{H}(\mathbf{p}) + \mathbf{J}(\mathbf{p})$.

$$\text{Let } \mathbf{M}(\mathbf{p}) = \frac{\mathbf{D}\mathbf{H}(\mathbf{p})}{\mathbf{p} - \mathbf{H}(\mathbf{p})} \text{ and } \mathbf{N}(\mathbf{p}) = \frac{\mathbf{D}\mathbf{J}(\mathbf{p})}{\mathbf{p} - \mathbf{H}(\mathbf{p})}. \text{ Solve for } \mathbf{g}$$

by (4) of Table 12.9 the equation

$$\mathbf{Dg} = \mathbf{N}(x) - \mathbf{M}(x)\mathbf{g},$$

writing the solution in the form $\mathbf{K}(x, \mathbf{g}, C) = 0$

GSF: Eliminate \mathbf{p} between the equations

$$\mathbf{f}(x) = x\mathbf{H}(\mathbf{p}) + \mathbf{J}(\mathbf{p}),$$

$$\mathbf{K}(\mathbf{p}, x, C) = 0.$$

(5) Equation: $x = \mathbf{G}(\mathbf{f}, \mathbf{p})$.

Let $\mathbf{M}(\mathbf{f}, \mathbf{p}) = x(\partial/\partial\mathbf{f})\mathbf{G}(\mathbf{f}, \mathbf{p})$ and $\mathbf{N}(\mathbf{f}, \mathbf{p}) = (\partial/\partial\mathbf{p})\mathbf{G}(\mathbf{f}, \mathbf{p})$.

Solve for \mathbf{p} by the methods of Table 12.9 the equation

$$\mathbf{Dp} = \frac{1 - \mathbf{pM}(x, \mathbf{p})}{\mathbf{p}^2\mathbf{N}(x, \mathbf{p})},$$

writing the solution in the form $\mathbf{H}(x, \mathbf{p}, C) = 0$.

GSF: Eliminate \mathbf{p} between the equations

$$x = \mathbf{G}(\mathbf{f}, \mathbf{p}),$$

$$\mathbf{H}(\mathbf{f}, \mathbf{p}, C) = 0.$$

(6) Equation: $\mathbf{p} = \mathbf{G}(\mathbf{f})$.

$$\text{GSF: } \int \frac{d\mathbf{f}}{\mathbf{G}(\mathbf{f})} = x + C.$$

(7) Equation: $\mathbf{p} = \mathbf{G}(x)$.

$$\text{GSF: } \mathbf{f}(x) = \int \mathbf{G}(x) dx + C.$$

13

DIFFERENCES OF FUNCTIONS

DIFFERENCES OF many functions whose domains are nonnegative integers may be found by consulting Sec. 13.1 and Table 13.2. The first of these furnishes formulas for differencing algebraic combinations of simpler functions; the second furnishes the differences of some basic functions from which algebraic combinations can be formed. For a discussion of the factorial function $x^{(n)}$ and the general procedure for differencing polynomials, see Sec. 5.6.

13.1 TABLE: PROPERTIES OF DIFFERENCES

- (1) $\Delta(cf(x)) = c\Delta f(x), \quad c \text{ a real number.}$
- (2) $\Delta(f(x) \pm g(x)) = \Delta f(x) \pm \Delta g(x).$
- (3) $\Delta(f(x) \cdot g(x)) = Ef(x) \cdot \Delta g(x) + g(x) \cdot \Delta f(x).$
- (4) $\Delta\left(\frac{f(x)}{g(x)}\right) = \frac{g(x) \cdot \Delta f(x) - f(x) \cdot \Delta g(x)}{g(x) \cdot Eg(x)}.$

13.2 TABLE: FIRST DIFFERENCES

- (1) $\Delta x^{(n)} = nx^{(n-1)}.$
- (2) $\Delta(ax + b)^{(n)} = an(ax + b)^{(n-1)}.$
- (3) $\Delta \frac{1}{(x + n - 1)^{(n)}} = \frac{-n}{(x + n)^{(n+1)}}.$

$$(4) \quad \Delta \binom{x}{n} = \binom{x}{n-1}.$$

$$(5) \quad \Delta(x!) = x(x!).$$

$$(6) \quad \Delta \sin(ax + b) = 2 \sin \frac{a}{2} \cos \left(ax + b + \frac{a}{2} \right).$$

$$(7) \quad \Delta \cos(ax + b) = -2 \sin \frac{a}{2} \sin \left(ax + b + \frac{a}{2} \right).$$

$$(8) \quad \Delta \tan(ax + b) = \frac{\sin a}{\cos(ax + b) \cos(ax + a + b)}.$$

$$(9) \quad \Delta \cot(ax + b) = \frac{-\sin a}{\sin(ax + b) \sin(ax + a + b)}.$$

$$(10) \quad \Delta \sec(ax + b) = \frac{2 \sin(a/2) \sin(ax + b + a/2)}{\cos(ax + b) \cos(ax + a + b)}.$$

$$(11) \quad \Delta \csc(ax + b) = \frac{-2 \sin(a/2) \cos(ax + b + a/2)}{\sin(ax + b) \sin(ax + a + b)}.$$

$$(12) \quad \Delta c^x = (c - 1)c^x.$$

$$(13) \quad \Delta 2^x = 2^x.$$

$$(14) \quad \Delta c^{ax+b} = (c^a - 1)c^{ax+b}.$$

$$(15) \quad \Delta \log_c x = \log_c \left(1 + \frac{1}{x} \right).$$

$$(16) \quad \Delta \log_c(ax + b) = \log_c \left(1 + \frac{a}{ax + b} \right).$$

14

INDEFINITE FINITE INTEGRALS, FINITE SUMS, INFINITE SERIES

INDEFINITE FINITE integrals of many functions whose domains are nonnegative integers may be found by consulting Tables 14.1 and 14.2. The first of these lists formulas for finding indefinite finite integrals of certain algebraic combinations of simpler functions; the second furnishes the indefinite finite integrals of some basic functions from which algebraic combinations can be formed.

A **finite sum** of a function $g(x)$, defined for nonnegative integers, is an expression of the form

$$\sum_a^b g(x) = g(a) + g(a+1) + \dots + g(n), \quad n \geq a.$$

It is sometimes possible to evaluate a finite sum with the aid of an antidifference of the function being summed (see Sec. 8.5). Table 14.3 gives algebraic properties of finite sums, and specific finite sums appear in Table 14.4.

14.1 TABLE: ALGEBRAIC PROPERTIES OF INDEFINITE FINITE INTEGRALS

- (1) $\Delta^{-1}(cf(x)) = c\Delta^{-1}f(x)$, c a real number.
- (2) $\Delta^{-1}(f(x) \pm g(x)) = \Delta^{-1}f(x) \pm \Delta^{-1}g(x)$.
- (3) $\Delta^{-1}(f(x) \cdot \Delta g(x)) = f(x) \cdot g(x) - \Delta^{-1}(Eg(x) \cdot \Delta f(x))$.

14.2 TABLE: INDEFINITE FINITE INTEGRALS

- $$(1) \quad \Delta^{-1}x^{(n)} = \frac{x^{(n+1)}}{n+1} + C, \quad n \neq -1.$$
- $$(2) \quad \Delta^{-1}(ax+b)^{(n)} = \frac{(ax+b)^{(n+1)}}{a(n+1)} + C, \quad n \neq -1.$$
- $$(3) \quad \Delta^{-1} \frac{1}{(x+n-1)^{(n)}} = \frac{1}{(1-n)(x+n-2)^{(n-1)}} + C, \quad n \neq 1.$$
- $$(4) \quad \Delta^{-1} \binom{x}{n} = \binom{x}{n+1} + C.$$
- $$(5) \quad \Delta^{-1}x(x!) = x! + C.$$
- $$(6) \quad \Delta^{-1} \sin(ax+b) = \frac{-\cos(ax+b-a/2)}{2 \sin(a/2)} + C, \quad a \neq 2n\pi.$$
- $$(7) \quad \Delta^{-1} \cos(ax+b) = \frac{\sin(ax+b-a/2)}{2 \sin a/2} + C, \quad a \neq 2n\pi.$$
- $$(8) \quad \Delta^{-1}(\sec(ax+b) \sec(ax+a+b)) = \csc a \tan(ax+b) + C, \\ a \neq n\pi.$$
- $$(9) \quad \Delta^{-1}(\csc(ax+b) \csc(ax+a+b)) = -\csc a \cot(ax+b) + C, \\ a \neq n\pi.$$
- $$(10) \quad \Delta^{-1} \frac{\sin(ax+b+a/2)}{\cos(ax+b) \cos(ax+a+b)} = \frac{\sec(ax+b)}{2 \sin(a/2)} + C, \\ a \neq 2n\pi.$$
- $$(11) \quad \Delta^{-1} \frac{\cos(ax+b+a/2)}{\sin(ax+b) \sin(ax+a+b)} = \frac{-\csc(ax+b)}{2 \sin(a/2)} + C, \\ a \neq 2n\pi.$$
- $$(12) \quad \Delta^{-1}c^x = \frac{c^x}{c-1} + C, \quad c \neq 1.$$
- $$(13) \quad \Delta^{-1}2^x = 2^x + C.$$
- $$(14) \quad \Delta^{-1}c^{ax+b} = \frac{c^{ax+b}}{c^a-1} + C, \quad c \neq 1, a \neq 0.$$

14.3 TABLE: PROPERTIES OF FINITE SUMS

- (1) $\sum_a^b c g(x) = c \sum_a^b g(x), \quad c \text{ a real number.}$
- (2) $\sum_a^b [g_1(x) \pm g_2(x)] = \sum_a^b g_1(x) \pm \sum_a^b g_2(x).$
- (3) $\sum_a^a g(x) = g(a).$
- (4) $\sum_a^c g(x) = \sum_a^b g(x) + \sum_{b+1}^c g(x), \quad a \leq b < c.$

14.4 TABLE: FINITE SUMS

- (1) $\sum_1^n c = nc, \quad c \text{ a real number.}$
- (2) $\sum_1^n x = \frac{n(n+1)}{2}.$
- (3) $\sum_1^n x^2 = \frac{n(n+1)(2n+1)}{6}.$
- (4) $\sum_1^n x^3 = \frac{n^2(n+1)^2}{4}.$
- (5) $\sum_1^n \frac{1+2+\dots+x}{x} = \frac{n(n+3)}{4}.$
- (6) $\sum_1^n \frac{1+2^2+\dots+x^2}{x} = \frac{n(4n^2+15n+17)}{36}.$
- (7) $\sum_1^n \frac{1}{1+2+\dots+x} = \frac{2n}{n+1}.$
- (8) $\sum_1^n \frac{1}{x(x+1)} = \frac{n}{n+1}.$
- (9) $\sum_1^n x(x!) = (n+1)! - 1.$
- (10) $\sum_1^n (x^2+1)x! = n(n+1)!.$
- (11) $\sum_1^n \binom{n}{x} = 2^n - 1.$

$$(12) \quad \sum_{a+1}^n \binom{x}{a} = \binom{n+1}{a+1} - 1.$$

$$(13) \quad \sum_1^n c^x = \frac{c}{c-1} (c^n - 1).$$

$$(14) \quad \sum_1^n x c^x = \frac{n c^{n+1}}{c-1} - \frac{c(c^n - 1)}{(c-1)^2}.$$

$$(15) \quad \sum_1^n x^2 c^x = \frac{n^2 c^{n+1}}{c-1} - \frac{2n c^{n+1}}{(c-1)^2} + \frac{c(c+1)(c^n - 1)}{(c-1)^3}.$$

$$(16) \quad \sum_1^n c^{x+p} = \frac{c^{p+1}}{c-1} (c^n - 1).$$

$$(17) \quad \sum_1^n x c^{x+p} = \frac{n c^{n+p+1}}{c-1} - \frac{c^{p+1}(c^n - 1)}{(c-1)^2}.$$

$$(18) \quad \sum_1^n x^2 c^{x+p} = \frac{n^2 c^{n+p+1}}{c-1} - \frac{2n c^{n+p+1}}{(c-1)^2} + \frac{c^{p+1}(c+1)(c^n - 1)}{(c-1)^3}.$$

$$(19) \quad \sum_1^n 2^x = 2(2^n - 1).$$

$$(20) \quad \sum_1^n c^{ax} = \frac{c^a}{c^a - 1} (c^{an} - 1).$$

$$(21) \quad \sum_1^n \sin(ax + b) = \frac{\sin\left(\frac{n+1}{2}a + b\right) \sin \frac{n}{2}a}{\sin(a/2)}, \quad a \neq 2n\pi.$$

$$(22) \quad \sum_1^n \cos(ax + b) = \frac{\cos\left(\frac{n+1}{2}a + b\right) \sin \frac{n}{2}a}{\sin(a/2)}, \quad a \neq 2n\pi.$$

$$(23) \quad \sum_0^n \binom{n}{x} a^x b^{n-x} = (a+b)^n \quad (\text{the binomial theorem}).$$

14.5 INFINITE SERIES

An infinite series defined for a function $g(x)$ whose domain consists of nonnegative integers is

$$\sum_a^\infty g(x) = \lim_{n \rightarrow \infty} \sum_a^n g(x).$$

We shall consider two problems for such series. The first is to determine whether this limit exists; if it does, the series is said to be **convergent**. (A series which is not convergent is called **divergent**.) The other problem is to evaluate the infinite sum if the series is convergent.

Two types of convergence arise, absolute and conditional. A series $\sum_a^\infty g(x)$ is **absolutely convergent** if $\sum_a^\infty |g(x)|$ is convergent. A convergent series which is not absolutely convergent is **conditionally convergent**.

Table 14.6 lists six tests for convergence or divergence. In some cases, the test may fail to indicate whether the series converges or diverges; one should then try another test.

14.6 TABLE: TESTS FOR CONVERGENCE OR DIVERGENCE

OF $\sum_a^\infty g(x)$

(1) *Term Test*

(a) If $\lim_{x \rightarrow \infty} g(x) \neq 0$, then $\sum_a^\infty g(x)$ is divergent.

(b) If $\lim_{x \rightarrow \infty} g(x) = 0$, the test fails.

(2) *Comparison Test*. Let $\sum_a^\infty h(x)$ be a convergent series of positive terms and $\sum_a^\infty k(x)$ be a divergent series of positive terms.

(a) If there exists a positive integer N such that

$$|g(x)| \leq h(x) \quad \text{whenever } x > N,$$

then $\sum_a^\infty g(x)$ converges absolutely.

(b) If there exists a positive integer N such that

$$g(x) \geq k(x) \quad \text{whenever } x > N,$$

then $\sum_a^\infty g(x)$ diverges.

(c) If there exists a positive integer N such that

$$|\mathbf{g}(x)| \geq \mathbf{k}(x) \quad \text{whenever } x > N,$$

then $\sum_a^\infty \mathbf{g}(x)$ does not converge absolutely, though it may converge conditionally.

(3) *Ratio Test.* Let $r = \lim_{x \rightarrow \infty} \left| \frac{\mathbf{g}(x+1)}{\mathbf{g}(x)} \right|$.

(a) If $r < 1$, then $\sum_a^\infty \mathbf{g}(x)$ converges absolutely.

(b) If $r > 1$, then $\sum_a^\infty \mathbf{g}(x)$ diverges.

(c) If $r = 1$, the test fails.

(4) *Root Test.* Let $r = \lim_{x \rightarrow \infty} [\mathbf{g}(x)]^{1/x}$.

(a) If $r < 1$, then $\sum_a^\infty \mathbf{g}(x)$ converges absolutely.

(b) If $r > 1$, then $\sum_a^\infty \mathbf{g}(x)$ diverges.

(c) If $r = 1$, the test fails.

(5) *Integral Test.* Let $\mathbf{h}(x)$ be a real-valued function such that $\mathbf{h}(x) = |\mathbf{g}(x)|$ for each x in the domain of \mathbf{g} . If $\mathbf{h}(x)$ has the property that, for some positive integer N , $\mathbf{h}(x+c) < \mathbf{h}(x)$ whenever $x > N$ and $c > 0$, determine whether $\lim_{b \rightarrow \infty} \int_a^b \mathbf{h}(x) dx$ exists, where $a > N$.

(a) If the limit exists, then $\sum_a^\infty \mathbf{g}(x)$ is absolutely convergent.

(b) If $\mathbf{g}(x)$ is a series of positive terms and the limit fails to exist, then $\sum_a^\infty \mathbf{g}(x)$ is divergent.

(c) If $\mathbf{g}(x)$ is not a series of positive terms and the limit fails to exist, then $\sum_a^\infty \mathbf{g}(x)$ is not absolutely convergent, though it may be conditionally convergent.

(6) *Alternating Series Test*(a) If there exists a positive integer N such that(i) the signs of $\mathbf{g}(x)$ and $\mathbf{g}(x+1)$ are opposite for every $x > N$ (ii) $|\mathbf{g}(x+1)| < |\mathbf{g}(x)|$ for every $x > N$, and if(iii) $\lim_{x \rightarrow \infty} \mathbf{g}(x) = 0$,then $\sum_a^\infty \mathbf{g}(x)$ converges.(b) If at least one of the conditions (ii) and (iii) fails when (i) holds, then the series $\sum_a^\infty \mathbf{g}(x)$ diverges.**14.7 TABLE: INFINITE SERIES***Convergent Series*

- | | |
|---|-------------------------------|
| (1) $\sum_1^\infty \frac{(-1)^{x-1}}{x}.$ | Sum: $\ln 2.$ |
| (2) $\sum_1^\infty \frac{(-1)^{x-1}}{2x-1}.$ | Sum: $\frac{\pi}{4}.$ |
| (3) $\sum_1^\infty \frac{1}{x(x+1)}.$ | Sum: $1.$ |
| (4) $\sum_1^\infty \frac{1}{x(x+1)(x+2)}.$ | Sum: $\frac{1}{4}.$ |
| (5) $\sum_0^\infty \frac{1}{x!}.$ | Sum: $e.$ |
| (6) $\sum_1^\infty \frac{1}{x^r}, \quad r > 1.$ | |
| (7) $\sum_1^\infty \frac{1}{1+x^2}.$ | |
| (8) $\sum_2^\infty \frac{1}{(\ln x)^{\ln x}}.$ | |
| (9) $\sum_0^\infty \mathbf{a}^x, \quad a < 1.$ | Sum: $\frac{1}{1-a}.$ |
| (10) $\sum_0^\infty x \mathbf{a}^x, \quad a < 1.$ | Sum: $\frac{a}{(1-a)^2}.$ |
| (11) $\sum_0^\infty x^2 \mathbf{a}^x, \quad a < 1.$ | Sum: $\frac{a^2+a}{(1-a)^3}.$ |

$$(12) \quad \sum_0^{\infty} a^{x+p}, \quad |a| < 1.$$

$$\text{Sum: } \frac{a^p}{1-a}.$$

$$(13) \quad \sum_0^{\infty} xa^{x+p}, \quad |a| < 1.$$

$$\text{Sum: } \frac{a^{p+1}}{(1-a)^2}.$$

$$(14) \quad \sum_0^{\infty} x^2 a^{x+p}, \quad |a| < 1.$$

$$\text{Sum: } \frac{a^{p+2} + a^{p+1}}{(1-a)^3}.$$

$$(15) \quad \sum_2^{\infty} \frac{1}{x(\ln x)^r}, \quad r > 1.$$

$$(16) \quad \sum_2^{\infty} \frac{1}{x \ln x (\ln \ln x)^r}, \quad r > 1.$$

Divergent Series

$$(17) \quad \sum_1^{\infty} \frac{1}{x}.$$

$$(18) \quad \sum_1^{\infty} \frac{1}{x^r}, \quad r \leq 1.$$

$$(19) \quad \sum_0^{\infty} a^x, \quad |a| \geq 1.$$

$$(20) \quad \sum_1^{\infty} \frac{1}{ax+b}.$$

$$(21) \quad \sum_2^{\infty} \frac{1}{x(\ln x)^r}, \quad r \leq 1.$$

$$(22) \quad \sum_2^{\infty} \frac{1}{x(\ln x)(\ln \ln x)^r}, \quad r \leq 1.$$

14.8 PROBABILITY SUMS

$$\text{Let } g(x) = \frac{e^{-m} m^x}{x!}.$$

$$(1a) \quad \sum_0^{\infty} g(x) = 1.$$

$$(1b) \quad \sum_0^{\infty} xg(x) = m.$$

$$(1c) \quad \sum_0^{\infty} x^2 g(x) = m + m^2.$$

Let $\mathbf{g}(x) = (1 - p)\mathbf{p}^x$, $0 \leq p < 1$.

$$(2a) \quad \sum_0^{\infty} \mathbf{g}(x) = 1.$$

$$(2b) \quad \sum_0^{\infty} x\mathbf{g}(x) = \frac{p}{1-p}.$$

$$(2c) \quad \sum_0^{\infty} x^2\mathbf{g}(x) = \frac{p^2 + p}{(1-p)^2}.$$

Let $\mathbf{g}(x) = \binom{n}{x} \mathbf{p}^x (1 - \mathbf{p})^{n-x}$, $0 < p < 1$.

$$(3a) \quad \sum_0^{\infty} \mathbf{g}(x) = 1.$$

$$(3b) \quad \sum_0^{\infty} x\mathbf{g}(x) = np.$$

$$(3c) \quad \sum_0^{\infty} x^2\mathbf{g}(x) = n^2p^2 + np(1-p).$$

14.9 ANTIDIFFERENCES OF ORDER n

Antidifferences of order n , denoted by $\mathbf{J}^n \mathbf{g}(x)$, are listed in Table 14.10. The integrals are particular solutions of equations of the form $\Delta^n \mathbf{g} = 0$. The general solution form of such an equation is

$$\mathbf{J}^n \mathbf{g}(x) + C_0 x^n + C_1 x^{n-1} + \dots + C_{n-1} x + C_n.$$

14.10 TABLE: ANTIDIFFERENCES OF ORDER n

$$(1) \quad \mathbf{J}^n \mathbf{c} = \frac{c x^{(n)}}{n!}, \quad c \text{ a real number.}$$

$$(2) \quad \mathbf{J}^n x^{(k)} = \frac{x^{(k+n)}}{(k+n)^{(n)}}.$$

$$(3) \quad \mathbf{J}^n (ax + b)^{(k)} = \frac{(ax + b)^{(k+n)}}{a^n (k+n)^{(n)}}.$$

$$(4) \quad \mathbf{J}^n \frac{1}{(x + k - 1)^{(k)}} = \frac{(-1)^n}{(n - k)^{(n)} (x + k - n - 1)^{(k-n)}}, \quad n \leq k.$$

$$(5) \quad \mathbf{J}^n \binom{x}{k} = \binom{x}{n+k}.$$

$$(6) \quad J^n \sin(ax + b) = \frac{\sin(ax + b - na/2)}{2^n \sin^n(a/2)}, \quad \text{if } n = 4m, a \neq 2n\pi.$$

$$(7) \quad J^n \sin(ax + b) = \frac{-\cos(ax + b - na/2)}{2^n \sin^n(a/2)},$$

if $n = 4m + 1, a \neq 2n\pi$.

$$(8) \quad J^n \sin(ax + b) = \frac{-\sin(ax + b - na/2)}{2^n \sin^n(a/2)},$$

if $n = 4m + 2, a \neq 2n\pi$.

$$(9) \quad J^n \sin(ax + b) = \frac{\cos(ax + b - na/2)}{2^n \sin^n(a/2)},$$

if $n = 4m + 3, a \neq 2n\pi$.

$$(10) \quad J^n \cos(ax + b) = \frac{\cos(ax + b - na/2)}{2^n \sin^n(a/2)}, \quad \text{if } n = 4m, a \neq 2n\pi.$$

$$(11) \quad J^n \cos(ax + b) = \frac{\sin(ax + b - na/2)}{2^n \sin^n(a/2)},$$

if $n = 4m + 1, a \neq 2n\pi$.

$$(12) \quad J^n \cos(ax + b) = \frac{-\cos(ax + b - na/2)}{2^n \sin^n(a/2)},$$

if $n = 4m + 2, a \neq 2n\pi$.

$$(13) \quad J^n \cos(ax + b) = \frac{-\sin(ax + b - na/2)}{2^n \sin^n(a/2)},$$

if $n = 4m + 3, a \neq 2n\pi$.

$$(14) \quad J^n c^x = \frac{c^x}{(c - 1)^n}.$$

$$(15) \quad J^n 2^x = 2^x.$$

$$(16) \quad J^n c^{ax} = \frac{c^{ax}}{(c^a - 1)^n}.$$

14.11 TAYLOR'S SERIES

Certain functions whose domains are real numbers can be written as infinite power series, called **Taylor's series** or **Taylor's expansions**, which may be used to approximate values of these functions for certain arguments. In general, the Taylor's expansion of a function does not converge over the whole domain of the function. The set of arguments for which the Taylor's expansion does converge occupies an interval of the real axis. This interval is called the **interval of convergence** of the expansion. The Taylor's series for a number of important functions are listed in Table 14.12, along with their intervals of convergence.

14.12 TABLE: TAYLOR'S SERIES

$$(1) \quad \frac{1}{1 \pm x} = 1 \mp x + x^2 \mp x^3 + \mp \dots, \quad |x| < 1.$$

$$(2) \quad \frac{1}{(1 \pm x)^2} = 1 \mp 2x + 3x^2 \mp 4x^3 + \mp \dots, \quad |x| < 1.$$

$$(3) \quad \sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{2 \cdot 4}x^2 + \frac{1 \cdot 3}{2 \cdot 4 \cdot 6}x^3 \\ - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8}x^4 + \dots, \quad |x| < 1.$$

$$(4) \quad \frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4}x^2 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^3 \\ + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8}x^4 - \dots, \quad |x| < 1.$$

$$(5) \quad (a+x)^n = a^n + na^{n-1}x + \frac{n(n-1)}{2!}a^{n-2}x^2 \\ + \frac{n(n-1)(n-2)}{3!}a^{n-3}x^3 + \dots, \quad x^2 < a^2.$$

$$(6) \quad \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots, \quad \text{for all values of } x.$$

$$(7) \quad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots, \quad \text{for all values of } x.$$

- (8) $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \dots, \quad |x| < \frac{\pi}{2}.$
- (9) $\arcsin x = x + \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{x^5}{5} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{x^7}{7} + \dots,$
 $|x| \leq 1.$
- (10) $\arccos x = \frac{\pi}{2} - \arcsin x.$
- (11) $\arctan x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots, \quad |x| \leq 1.$
- (12) $\ln x = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \dots,$
 $0 < x \leq 2.$
- (13) $\ln(1 \pm x) = \pm x - \frac{x^2}{2} \pm \frac{x^3}{3} - \frac{x^4}{4} \pm \dots, \quad -1 < x \leq 1.$
- (14) $\ln \frac{1+x}{1-x} = 2 \left(x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots \right), \quad |x| < 1.$
- (15) $\ln(x + \sqrt{1+x^2}) = x - \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \frac{x^5}{5}$
 $- \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{x^7}{7} + \dots, \quad |x| \leq 1.$
- (16) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, \quad \text{for all values of } x.$
- (17) $e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \dots, \quad \text{for all values of } x.$
- (18) $a^x = 1 + x \ln a + \frac{(x \ln a)^2}{2!} + \frac{(x \ln a)^3}{3!} + \dots,$
for all values of $x.$
- (19) $\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots, \quad \text{for all values of } x.$
- (20) $\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots, \quad \text{for all values of } x.$
- (21) $\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \dots, \quad \text{for all values of } x.$
- (22) $\sinh^{-1} x = x - \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \frac{x^5}{5} - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{x^7}{7} + \dots,$
 $|x| \leq 1.$
- (23) $\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots, \quad |x| < 1.$

15

LINEAR DIFFERENCE EQUATIONS WITH CONSTANT COEFFICIENTS

THE GENERAL form of such an equation is

$$\mathbf{E}^n \mathbf{f}(x) + a_1 \mathbf{E}^{n-1} \mathbf{f}(x) + \dots + a_{n-1} \mathbf{E} \mathbf{f}(x) + a_n \mathbf{f}(x) = \mathbf{g}(x),$$

which we rewrite

$$[\mathbf{E}^n + a_1 \mathbf{E}^{n-1} + \dots + a_{n-1} \mathbf{E} + a_n] \mathbf{f}(x) = \mathbf{g}(x).$$

The expression in brackets is a polynomial in \mathbf{E} which we shall denote by $\mathbf{p}(\mathbf{E})$. The difference equation now takes the form

$$[\mathbf{p}(\mathbf{E})] \mathbf{f}(x) = \mathbf{g}(x).$$

Any equation written in terms of the difference operator Δ may be rewritten in terms of \mathbf{E} , using the relations of Sec. 3.5.

If $\mathbf{g}(x) = 0$, the equation is said to be **homogeneous**. If $\mathbf{g}(x) \neq 0$, then the homogeneous equation $[\mathbf{p}(\mathbf{E})] \mathbf{f}(x) = 0$ obtained by replacing $\mathbf{g}(x)$ by 0 is called the **reduced equation** of the given equation.

If $\mathbf{f}^*(x)$ is a solution of the given equation, then, for any other solution $\mathbf{f}_1^*(x)$, the function $\mathbf{f}_1(x) = \mathbf{f}_1^*(x) - \mathbf{f}^*(x)$ is a solution of its reduced equation, since

$$[\mathbf{p}(\mathbf{E})](\mathbf{f}_1^*(x) - \mathbf{f}^*(x)) = \mathbf{g}(x) - \mathbf{g}(x) = 0.$$

Thus, the general solution form of the given equation is found by add-

ing any particular solution of the given equation to the general solution form of the reduced equation. There are two major steps to the solution of the problem:

- (1) Finding the general solution of the reduced equation.
- (2) Finding a particular solution of the given equation.

15.1 THE AUXILIARY EQUATION

To obtain the general solution of a homogeneous linear difference equation with constant coefficients $[p(E)]f(x) = 0$, we first solve the auxiliary equation

$$p(m) = 0$$

where $p(m)$ is the polynomial in m obtained by replacing E by m in $p(E)$.

15.2 HOMOGENEOUS EQUATIONS OF FIRST ORDER

The first-order equation takes the form

$$[p(E)]f(x) = [E - a]f(x) = 0, \quad a \text{ a real number.}$$

The general solution form is

$$f(x) = Ca^x.$$

15.3 HOMOGENEOUS EQUATIONS OF SECOND ORDER

The general solution form of the second-order equation

$$[p(E)]f(x) = 0$$

is tabulated in Table 15.4 according to the nature of the roots of the auxiliary equation $p(m) = 0$.

15.4 TABLE: GENERAL SOLUTION FORM OF THE LINEAR HOMOGENEOUS EQUATION OF SECOND ORDER WITH CONSTANT COEFFICIENTS

Let m_1 and m_2 be the roots of $p(m) = 0$.

- (1) $m_1 \neq m_2$, both real:

$$\text{GSF: } f(x) = C_1 m_1^x + C_2 m_2^x.$$

$$(2) \quad m_1 = m_2:$$

$$\text{GSF: } \mathbf{f}(x) = (C_1 + C_2x)\mathbf{m}_1^x.$$

$$(3) \quad \text{The roots of the auxiliary equation are complex: } m_1 = a + bi, \\ m_2 = a - bi. \text{ Let } \rho = \sqrt{a^2 + b^2} \text{ and } \theta = \arctan(b/a).$$

$$\text{GSF: } \mathbf{f}(x) = \rho^x (C_1 \sin x\theta + C_2 \cos x\theta)$$

$$\text{or } \mathbf{f}(x) = C_1(\mathbf{a} + bi)^x + C_2(\mathbf{a} - bi)^x.$$

15.5 EXAMPLES

(1) $[\mathbf{E}^2 + 2\mathbf{E} + 5]\mathbf{f} = 0$. We first solve the auxiliary equation $m^2 + 2m + 5 = 0$, obtaining $m = -1 \pm 2i$. Then $a = -1$, $b = 2$, $\rho = \sqrt{5}$, and $\theta = \arctan(-2)$. The general solution form is

$$\mathbf{f}(x) = (\sqrt{5})^x (C_1 \sin x(\arctan - 2) + C_2 \cos x(\arctan - 2)).$$

(2) $[\mathbf{E}^2 + 3\mathbf{E} - 4]\mathbf{f} = 0$. Solving $m^2 + 3m - 4 = 0$, we obtain $m_1 = 1$ and $m_2 = -4$. The general solution form is

$$\mathbf{f}(x) = C_1(\mathbf{1})^x + C_2(-4)^x = C_1 + C_2(-4)^x.$$

(3) $[\mathbf{E}^2 + 6\mathbf{E} + 9]\mathbf{f} = 0$. The roots of $m^2 + 6m + 9 = 0$ are both -3 . The general solution form is

$$\mathbf{f}(x) = (C_1 + C_2x)(-3)^x.$$

15.6 HOMOGENEOUS EQUATIONS OF ORDER n

To find the general solution form for the homogeneous equation of order n , $[\mathbf{p}(\mathbf{E})]\mathbf{f} = 0$, one should first find the roots of the auxiliary equation $\mathbf{p}(m) = 0$. Let the distinct real roots of this equation be denoted by m_1, m_2, \dots, m_k , and let their multiplicities be r_1, r_2, \dots, r_k , respectively. Let the pairs of distinct complex roots be denoted by $a_{k+1} \pm b_{k+1}i, a_{k+2} \pm b_{k+2}i, \dots, a_{k+l} \pm b_{k+l}i$, with multiplicities $r_{k+1}, r_{k+2}, \dots, r_{k+l}$, respectively. The general solution form of the equation is the sum of the general solution forms of the equations

$$[(\mathbf{E} - \mathbf{m}_j)^{r_j}]\mathbf{f}(x) = 0, \quad 1 \leq j \leq k$$

$$[(\mathbf{E} - \mathbf{a}_j - \mathbf{b}_j i)^{r_j}(\mathbf{E} - \mathbf{a}_j + \mathbf{b}_j i)^{r_j}]\mathbf{f}(x) = 0, \quad k+1 \leq j \leq k+l$$

where the arbitrary constants appearing in the solutions will be numbered so that all have different subscripts. These general solution forms are listed in Table 15.7.

15.7 TABLE: GENERAL SOLUTION FORMS FOR HOMOGENEOUS DIFFERENCE EQUATIONS OF ORDER n WITH CONSTANT COEFFICIENTS (See Sec. 15.6)

- (1) m_j is real, and $r_j = 1$:
GSF: $\mathbf{f}_j(x) = C\mathbf{m}_j^x$.
- (2) m_j is real, and $r_j > 1$:
GSF: $\mathbf{f}_j(x) = (C_1 + C_2x + \dots + C_jx^{r_j-1})\mathbf{m}_j^x$.
- (3) $a_j \pm b_ji$ are complex roots, and $r_j = 1$. Let $\rho_j = \sqrt{a_j^2 + b_j^2}$ and $\theta_j = \arctan(b_j/a_j)$.
GSF: $\mathbf{f}_j(x) = \rho_j^x (C_1 \sin x\theta_j + C_2 \cos x\theta_j)$.
or $\mathbf{f}_j(x) = C_1(\mathbf{a}_j + \mathbf{b}_ji)^x + C_2(\mathbf{a}_j - \mathbf{b}_ji)^x$.
- (4) $a_j \pm b_ji$ are complex roots, and $r_j > 1$. Let $\rho_j = \sqrt{a_j^2 + b_j^2}$ and $\theta_j = \arctan(b_j/a_j)$.
GSF: $\mathbf{f}_j(x) = (C_1 + C_2x + \dots + C_{r_j}x^{r_j-1})\rho_j^x \sin x\theta_j$
 $+ (C_{r_j+1} + C_{r_j+2}x + \dots + C_{2r_j}x^{r_j-1})\rho_j^x \cos x\theta_j$
or $\mathbf{f}_j(x) = (C_1 + C_2x + \dots + C_{r_j}x^{r_j-1})(\mathbf{a}_j + \mathbf{b}_ji)^x$
 $+ (C_{r_j+1} + C_{r_j+2}x + \dots + C_{2r_j}x^{r_j-1})(\mathbf{a}_j - \mathbf{b}_ji)^x$.

15.8 EXAMPLE

$[\mathbf{E}(\mathbf{E} - 2)^3(\mathbf{E} + 1)]\mathbf{f} = 0$. The roots (and their respective multiplicities) of $m(m - 2)^3(m + 1) = 0$ are

$$\begin{array}{ll} m_1 = 0, & r_1 = 1, \\ m_2 = -1, & r_2 = 1, \\ m_3 = 2, & r_3 = 3. \end{array}$$

GSF: $\mathbf{f}(x) = C(\mathbf{0})^x + C_1(-\mathbf{1})^x + (C_2 + C_3x + C_4x^2)2^x$. Note that m_1 makes no contribution to the general solution form of the equation. This is because there is no solution for the equation $\mathbf{E}\mathbf{f} = 0$ except the function $\mathbf{f}(x) = 0$.

15.9 PARTICULAR SOLUTIONS OF LINEAR DIFFERENCE EQUATIONS WITH CONSTANT COEFFICIENTS

A particular solution of the equation $[p(E)]f(x) = g(x)$ can often be found if one can guess the form such a solution is likely to take. Table 15.10 lists a number of aids to good guesses for particular solutions. The constants A and B (with or without subscripts) are real numbers to be determined by substituting the particular solution $f^*(x)$ into the given equation. The examples of Sec. 15.12 will illustrate how the table is used.

It will appear that it is necessary to check whether certain numbers are roots of the auxiliary equation $p(m) = 0$. One should recall, however, that all roots of this equation were obtained in finding the general solution of the reduced equation. In particular, in each case, the number in question is a root if and only if $g(x)$ is a solution of the reduced equation.

15.10 TABLE: PARTICULAR SOLUTIONS OF LINEAR DIFFERENCE EQUATIONS WITH CONSTANT COEFFICIENTS

(1) $g(x) = k$, k a real number:

PS: $f^*(x) = A$, unless 1 is a root of $p(m) = 0$;

$f^*(x) = Ax^r$, if 1 is a root of $p(m) = 0$ of multiplicity r .

(2) $g(x) = X$, where X is a polynomial in x of degree s :

PS: $f^*(x) = A_0 + A_1x + \dots + A_sx^s$, unless 1 is a root of $p(m) = 0$;

$f^*(x) = (A_0 + A_1x + \dots + A_sx^s)x^r$, if 1 is a root of $p(m) = 0$ of multiplicity r .

(3) $g(x) = p \sin qx$:

PS: $f^*(x) = A \sin qx + B \cos qx$, unless $a + bi$ [with $\tan q = \pm(b/a)$] is a root of $p(m) = 0$;

$f^*(x) = (A \sin qx + B \cos qx)x^r$, if $a + bi$ [with $\tan q = \pm(b/a)$] is a root of $p(m) = 0$ of multiplicity r .

$$(4) \quad g(x) = p \cos qx:$$

PS: $f^*(x) = A \sin qx + B \cos qx$, unless $a + bi$ [with $\tan q = \pm(b/a)$] is a root of $p(m) = 0$;

$f^*(x) = (A \sin qx + B \cos qx)x^r$, if $a + bi$ [with $\tan q = \pm(b/a)$] is a root of $p(m) = 0$ of multiplicity r .

$$(5) \quad g(x) = pq^x:$$

PS: $f^*(x) = Aq^x$, unless q is a root of $p(m) = 0$;

$f^*(x) = Ax^r q^x$, if q is a root of $p(m) = 0$ of multiplicity r .

$$(6) \quad g(x) = Xq^x, \text{ where } X \text{ is a polynomial in } x \text{ of degree } s:$$

PS: $f^*(x) = (A_0 + A_1x + \dots + A_sx^s)q^x$, unless q is a root of $p(m) = 0$;

$f^*(x) = (A_0 + A_1x + \dots + A_sx^s)x^r q^x$, if q is a root of $p(m) = 0$ of multiplicity r .

$$(7) \quad g(x) = X \sin qx, \text{ where } X \text{ is a polynomial in } x \text{ of degree } s:$$

PS: $f^*(x) = (A_0 + A_1x + \dots + A_sx^s) \sin qx + (B_0 + B_1x + \dots + B_sx^s) \cos qx$, unless $a + bi$ [with $\tan q = \pm(b/a)$] is a root of $p(m) = 0$;

$f^*(x) = (A_0 + A_1x + \dots + A_sx^s)x^r \sin qx + (B_0 + B_1x + \dots + B_sx^s)x^r \cos qx$, if $a + bi$ [with $\tan q = \pm(b/a)$] is a root of $p(m) = 0$ of multiplicity r .

$$(8) \quad g(x) = X \cos qx, \text{ where } X \text{ is a polynomial in } x \text{ of degree } s:$$

PS: $f^*(x) = (A_0 + A_1x + \dots + A_sx^s) \sin qx + (B_0 + B_1x + \dots + B_sx^s) \cos qx$, unless $a + bi$ [with $\tan q = \pm(b/a)$] is a root of $p(m) = 0$;

$f^*(x) = (A_0 + A_1x + \dots + A_sx^s)x^r \sin qx + (B_0 + B_1x + \dots + B_sx^s)x^r \cos qx$, if $a + bi$ [with $\tan q = \pm(b/a)$] is a root of $p(m) = 0$ of multiplicity r .

15.11 REMARK ON THE EXTENSION OF TABLE 15.10

Suppose that $g(x) = g_1(x) + g_2(x) + \dots + g_n(x)$, where particular solutions for $[p(E)]f(x) = g_i(x)$ can be found in Table 15.10 (or else-

where). Then a particular solution for $[\mathbf{p}(\mathbf{E})]\mathbf{f}(x) = \mathbf{g}(x)$ can be obtained by adding the particular solutions of each of the equations $[\mathbf{p}(\mathbf{E})]\mathbf{f}(x) = \mathbf{g}_i(x)$.

15.12 EXAMPLES

(1) $[\mathbf{E}^2 + 2\mathbf{E} + 5]\mathbf{f} = 5x^2 + 8x + 6$. Since $\mathbf{g}(x)$ is a polynomial of degree 2, and since 1 is not a root of $m^2 + 2m + 5 = 0$, formula (2) of Table 15.10 tells us that a particular solution of the equation has the form $A_0 + A_1x + A_2x^2$. Substituting this in the given equation, we obtain

$$(8A_0 + 4A_1 + 6A_2) + (8A_1 + 8A_2)x + 8A_2x^2 = 5x^2 + 8x + 6.$$

Equating like powers of x ,

$$8A_2 = 5,$$

$$8A_2 + 8A_1 = 8,$$

$$6A_2 + 4A_1 + 8A_0 = 6.$$

Solving, we get

$$A_0 = \frac{3}{32}, \quad A_1 = \frac{3}{8}, \quad A_2 = \frac{5}{8}.$$

Adding our particular solution to the general solution form found in Example 1 of Sec. 15.5, we find that the general solution form for the given equation is

$$\begin{aligned} \mathbf{f}(x) = (\sqrt{5})^x [C_1 \sin x(\arctan - 2) + C_2 \cos x(\arctan - 2)] \\ + \frac{5}{8} + \frac{3x}{8} + \frac{3x^2}{32}. \end{aligned}$$

(2) $[\mathbf{E}^2 + 6\mathbf{E} + 9]\mathbf{f} = (-3)^x$. In Example 3 of Sec. 15.5, we found that -3 is a double root of $m^2 + 6m + 9 = 0$. Thus, the particular solution according to (5) of Table 15.10 has the form $Ax^2(-3)^x$. Substituting this for \mathbf{f} in the given equation, we obtain

$$\begin{aligned} A(-3)^2(-3)^x(x^2 + 4x + 4) + A(-3)(-3)^x(x^2 + 2x + 1) \\ + A(-3)^x x^2 = (-3)^x. \end{aligned}$$

The coefficients of $x^2(-3)^x$ and $x(-3)^x$ are zero. Equating coefficients of $(-3)^x$, we have $33A = 1$, or $A = \frac{1}{33}$. Using this particular solution and the general solution form found in Example 3 of Sec.

15.5, the general solution form of the given equation is

$$f(x) = (C_1 + C_2x)(-3)^x + \frac{1}{3}x^2(-3)^x.$$

(3) $[E - 3]f = 2 \sin 2x$. Using (3) of Table 15.10, a particular solution of the equation has the form $A \sin 2x + B \cos 2x$. Substituting this into the equation, we obtain

$$A \sin (2x + 2) + B \cos (2x + 2) - 3A \sin 2x - 3B \cos 2x = 2 \sin 2x.$$

Using the argument sum properties of Sec. 6.4, we obtain

$$(A \cos 2 - B \sin 2 - 3A) \sin 2x + (A \sin 2 + B \cos 2 - 3B) \cos 2x = 2 \sin 2x.$$

Equating coefficients of $\sin 2x$ and $\cos 2x$ and solving for A and B , we get

$$A = \frac{\cos 2 - 3}{5 - \cos 2} = -0.63, \quad B = \frac{-\sin 2}{5 - \cos 2} = -0.08$$

to two decimal places. Since the general solution form of the reduced equation is $C(3)^x$, the general solution form of the given equation is

$$f(x) = C(3)^x - 0.63 \sin 2x - 0.08 \cos 2x.$$

16

OTHER DIFFERENCE EQUATIONS

(1) $\mathbf{E}f(x) - \mathbf{h}(x)f(x) = \mathbf{g}(x)$ (linear first-order equation).

Let $\mathbf{H}(x) = \mathbf{h}(x) \cdot \mathbf{h}(x-1) \dots \mathbf{h}(0)$. If $\mathbf{H}(x)$ is never 0,

$$\text{GSF: } f(x) = \mathbf{H}(x-1) \left(\Delta^{-1} \frac{\mathbf{g}(x)}{\mathbf{H}(x)} + C \right).$$

If for some nonnegative integers $\mathbf{H}(x) = 0$, one can solve for $f(x)$ in a domain of nonnegative integers $a \leq x \leq b$ for which $\mathbf{H}(x) \neq 0$. In this case, let $\mathbf{H}_a(x) = \mathbf{h}(x) \cdot \mathbf{h}(x-1) \dots \mathbf{h}(a)$.

$$\text{GSF: } f(x) = \mathbf{H}_a(x-1) \left[\Delta^{-1} \frac{\mathbf{g}(x)}{\mathbf{H}_a(x)} + C \right].$$

(2) $f(x) \cdot \mathbf{E}f(x) + af(x) + b = 0$, a and b real numbers.

Let m be a root of the equation $z^2 + za + b = 0$.

Let $\mathbf{h}(x)$ be the general solution form of

$$\left[\mathbf{E} + \frac{m}{m+a} \right] \mathbf{h}(x) = -\frac{1}{m+a}.$$

$$\text{GSF: } f(x) = \frac{1}{\mathbf{h}(x)} + m.$$

(3) $f(x) \cdot \mathbf{E}f(x) + a\mathbf{E}f(x) + bf(x) + c = 0$, a , b , and c real numbers.

Case 1: $(a + b)^2 \neq 4c$. Let m and n be roots of the equation $z^2 - (a - b)z + c - ab = 0$

$$\text{GSF: } f(x) = \frac{C\mathbf{m}^{x+1} + \mathbf{n}^{x+1}}{C\mathbf{m}^x + \mathbf{n}^x} - a.$$

Case 2: $(a + b)^2 = 4c$.

$$\text{GSF: } f(x) = \frac{a - b}{2(C + x)} - \frac{a + b}{2}.$$

$$(4) \quad \mathbf{p}(x)\mathbf{f}(x) \cdot \mathbf{E}\mathbf{f}(x) + \mathbf{q}(x)\mathbf{E}\mathbf{f}(x) + \mathbf{r}(x)\mathbf{f}(x) = 0.$$

Let $\mathbf{h}(x)$ be the general solution form of the equation.

$$\mathbf{E}\mathbf{h}(x) + \frac{\mathbf{q}(x)}{\mathbf{r}(x)} \mathbf{h}(x) = -\frac{\mathbf{p}(x)}{\mathbf{r}(x)}.$$

$$\text{GSF: } f(x) = \frac{1}{\mathbf{h}(x)}.$$

(5) $\mathbf{E}^2\mathbf{f}(x) + \mathbf{a}(x)\mathbf{E}\mathbf{f}(x) + \mathbf{b}(x)\mathbf{f}(x) = \mathbf{c}(x)$. Let $\mathbf{v}(x)$ be a particular solution of the reduced equation

$$\mathbf{E}^2\mathbf{f}(x) + \mathbf{a}(x)\mathbf{E}\mathbf{f}(x) + \mathbf{b}(x)\mathbf{f}(x) = 0.$$

This can frequently be found by trial and error. Let $\mathbf{p}(x) = 1 + \mathbf{a}\mathbf{E}\mathbf{v}(x)/\mathbf{E}^2\mathbf{v}(x)$ and $\mathbf{q}(x) = \mathbf{c}/\mathbf{E}^2\mathbf{v}(x)$.

Let $\mathbf{h}(x)$ be the general solution form of the equation $\mathbf{E}\mathbf{h}(x) + \mathbf{p}(x)\mathbf{h}(x) = \mathbf{q}(x)$, and call the arbitrary constant C_1 .

$$\text{GSF: } f(x) = \Delta^{-1}\mathbf{h}(x) + C_2.$$

$$(6) \quad a_0(\mathbf{E}\mathbf{f}(x))^n + a_1(\mathbf{E}\mathbf{f}(x))^{n-1}\mathbf{f}(x) + a_2(\mathbf{E}\mathbf{f}(x))^{n-2}\mathbf{f}^2(x) + \dots + a_n\mathbf{f}^n(x) = 0.$$

Solve the equation

$$a_0m^n + a_1m^{n-1} + \dots + a_{n-1}m + a_n = 0.$$

Let the distinct (not necessarily real) roots be denoted m_1, m_2, \dots, m_k and their multiplicities r_1, r_2, \dots, r_k . Let

$$\mathbf{f}_i(x) = (C_1 + C_2x + \dots + C_{r_i}x^{r_i-1})m_i^x.$$

GSF: $\mathbf{f}(x)$ is given implicitly by

$$[y - \mathbf{f}_1(x)][y - \mathbf{f}_2(x)] \dots [y - \mathbf{f}_k(x)] = 0,$$

where $y = \mathbf{f}(x)$.

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